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REVIEW PAPER

Small failure probability: principles, progress and perspectives

Ikjin Lee[1] · Ungki Lee[1] · Palaniappan Ramu[2] · Deepanshu Yadav[2] · Gamze Bayrak[3] · Erdem Acar[3]

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Abstract
Design of structural and multidisciplinary systems under uncertainties requires estimation of their reliability or equivalently the probability of failure under the given operating conditions. Various high technology systems including aircraft and nuclear power plants are designed for very small probabilities of failure, and estimation of these small probabilities is computationally challenging. Even though substantial number of approaches have been proposed to reduce the computational burden, there is no established guideline to decide which approach is the best choice for a given problem. This paper provides a review of the approaches developed for small probability estimation of structural or multidisciplinary systems and enlists the criterion/metrics to choose the preferred approach amongst the existing ones, for a given problem. First, the existing approaches are categorized into the sampling-based, the surrogate-based, and statistics of extremes based approaches. Next, the small probability estimation methods developed for time-independent systems and the ones tailored for time-dependent systems are discussed, respectively. Then, some real-life engineering applications in structural and multidisciplinary design studies are summarized. Finally, concluding remarks are provided, and areas for future research are suggested.

Keywords Extreme value statistics · High reliability · Machine learning · Rare event · Sampling · Small failure probability · Surrogate model

1 Introduction

1.1 Motivation of small probability or rare event

Rare event can be defined as an event that occurs at low frequency, thereby associated with a small probability. Wikipedia states that rare events encompass natural phenomena (major earthquakes, tsunamis, asteroid impacts, etc.), anthropogenic hazards (industrial accidents, financial and commodity market crashes, etc.), as well as phenomena for which natural and anthropogenic factors interact in complex ways (epidemic disease spread, global warming-related changes in climate and weather, etc.). In engineering, the term "rare event" is often applied to catastrophic failures, including aircraft accidents, collapse of structures (e.g., bridges, dams, etc.) and failure of nuclear power plants.

In the case of aircraft engineering, errors in the navigation system, natural disasters, and failure of components rarely occur but can lead to fatal accidents (Löbl and Holzapfel 2015). To ensure structural safety, structural failures that rarely occur due to various uncertainties in external excitations and structural factors should be prevented (Zhou and Li 2022). Due to the small failure probability related to the structure, the lifespan of the structure may be shortened and a collapse accident may occur. For nuclear power plants, there are uncertainties such as natural circulation, pressure, and convection due to incomplete knowledge as well as geometrical dimensions, material properties, and natural disasters (Zio and Pedroni 2010a, b). When the load on a nuclear power plant exceeds its capacity due to various uncertainties, serious accidents such as the Chernobyl disaster and the Fukushima nuclear disaster can occur even with a small probability. To prevent the accidents aforementioned, estimation of small failure probability that can be obtained

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Erdem Acar
acar@etu.edu.tr

[1] Mechanical Engineering Department, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon 34141, Republic of Korea

[2] Advanced Design, Optimization and Probabilistic Techniques Laboratory, Department of Engineering Design, Indian Institute of Technology Madras, Chennai 600036, India

[3] Department of Mechanical Engineering, TOBB University of Economics and Technology, 06560 Ankara, Turkey

through a very large number of simulations or experiments is required to detect failure caused by uncertainties with small probability. The estimation of small failure probability estimation is computationally challenging and the computational cost required for estimation of small failure probability is often not feasible when high-fidelity simulation is involved to secure the confidence of the estimation.

1.2 Definition of small probability or rare event

There is no consensus on the value of the probability that defines a rare event. According to Lagnoux (2006), a rare event is defined by a very low probability of occurrence, between 10^{-9} and 10^{-12}. In the Foreword section of Morio and Balesdent (2016), Haftka associates the rare event with a low probability, typically less than 10^{-6}. Balesdent et al. (2016) along with Wang and Wang (2016) attribute the probability of failure being less than 10^{-5} to a rare event. In this review, an event with a probability of occurrence being less than 10^{-5} is considered to be a rare event for a time-independent event, whereas an event with a probability of occurrence being less than 10^{-3} is considered to be a rare event for a time-dependent event.

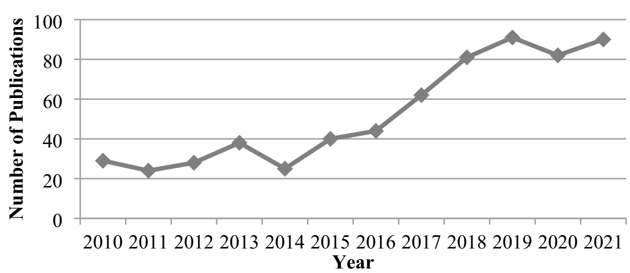

Fig. 1 Publish or Perish study of year-wise publications using Google Scholar database

In order to track the literature relevant to small probability estimation, a 'Publish or Perish' study was performed using Google Scholar database with the keywords 'low', 'small', 'failure', 'probability', 'structural', 'design'. The year-wise publication during the last decade for the mentioned keyword search is presented in Fig. 1. The increasing trend in the year-wise publications indicates that there is significant growth and interest in the estimation methods for small failure probability. Current work is intended to review the paper/articles on various methods associated with the low failure probability estimation. Since Google Scholar database keeps on updating from time to time, the actual numbers of publication may change but the trend remains the same.

1.3 Brief categorization of the approaches and the organization of the paper

For accurate estimation of rare event probabilities, various approaches have been developed. In this paper, these approaches are categorized into three: (1) sampling-based approaches, (2) surrogate-based approaches, and (3) statistics of extremes based approaches (see Fig. 2).

Advanced sampling approaches such as importance sampling, subset simulation, asymptotic simulation are few methods among advanced sampling-based approaches that have been proven to provide more accurate and more efficient estimations of rare event probabilities than the crude Monte Carlo method. The sampling-based approaches are reviewed in Sect. 2. Surrogate-based approaches include the use of traditional metamodels and machine learning models. Widely used surrogate models include Kriging, support vector regression, and neural networks. Rare event

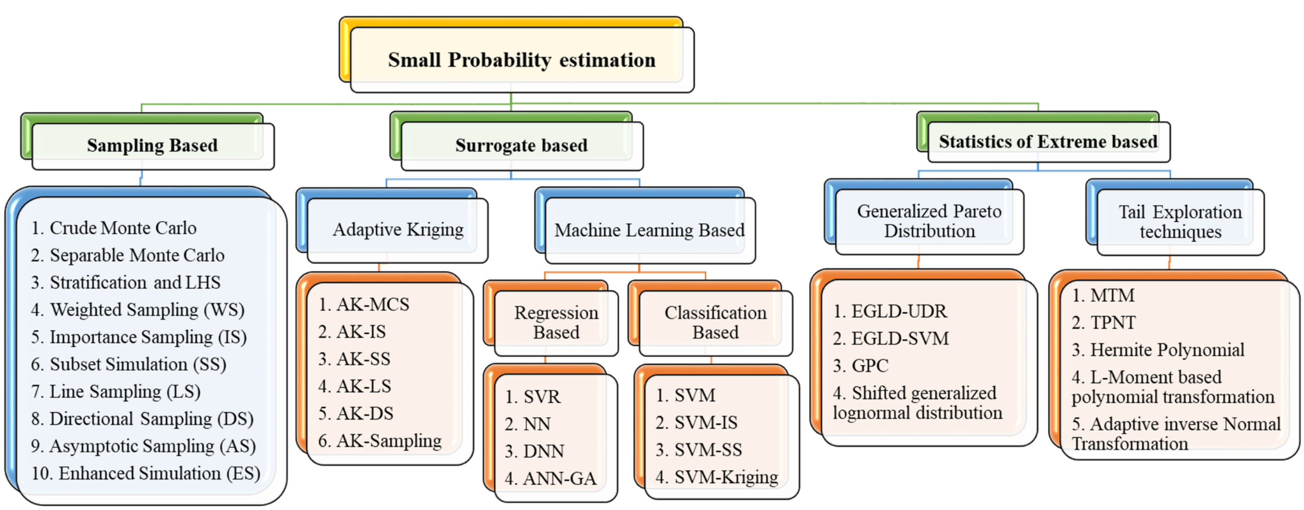

Fig. 2 Brief categorization of the approaches for estimation of small failure probability

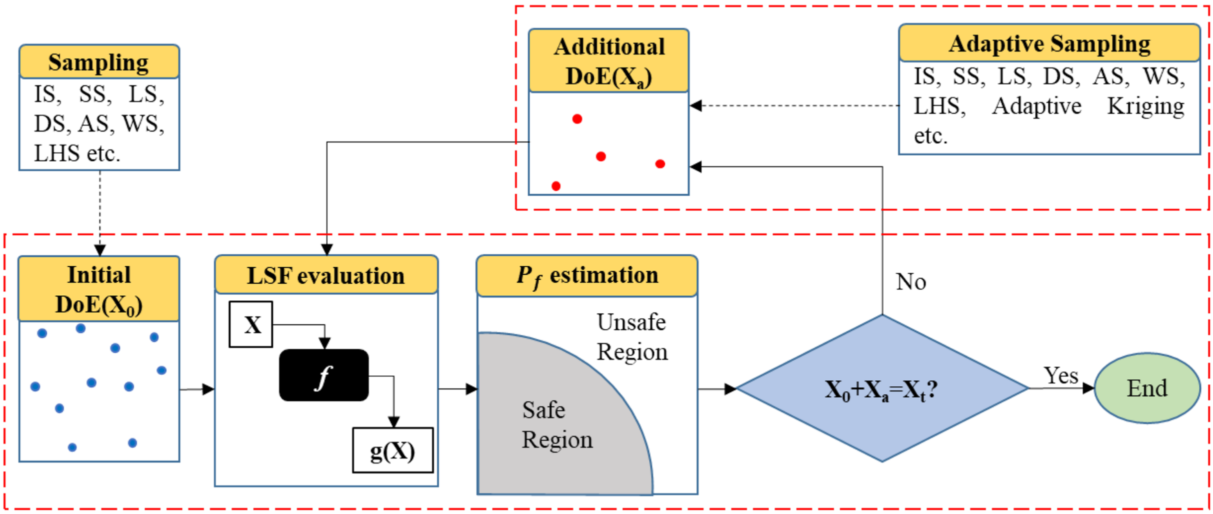

Fig. 3 General framework for sampling-based methods for estimation of small failure probability. X_0 is the initial sample, X_a is the adaptive or additional sample and X_t is the total samples available for LSF evaluation or budget in terms of LSF

probability estimation using surrogate-based approaches is detailed in Sect. 3. The use of statistics of extremes for rare event probability estimation is often conducted using generalized Pareto distribution, extreme value distribution or other asymptotic distributions. The approaches based on the statistics of extremes are reviewed in Sect. 4. The rare event probability prediction methods tailored for time-dependent systems are discussed in Sect. 5. Use of rare event probability estimation methods applied to industrial application type engineering problems is reviewed in Sect. 6. Finally, the paper culminates with some concluding remarks and suggestions for future research provided in Sect. 7.

2 Sampling-based approaches

Probability of failure of a structural or multidisciplinary system is defined as the probability for exceeding a limit. Mathematically, it is defined as

$$P_f = \Pr\left[g(\mathbf{x}) \leq 0\right] = \int I\left[g(\mathbf{x}) \leq 0\right] f(\mathbf{x})d\mathbf{x} = E\left[I\left[g(\mathbf{x}) \leq 0\right]\right] \tag{1}$$

where I is an indicator function that takes a value of 1 if [.] is true and 0 if [.] is false, g is the limit-state function (or performance function), f is the joint probability density function (PDF) of the input random variables, and E is the expectation operator. Since the analytical calculation of this multi-dimensional integral is burdensome, sampling-based techniques are often used.

Sampling-based techniques are based on sampling the input variables according to their probabilistic properties and propagating the uncertainty to the corresponding output(s). Crude Monte Carlo (CMC) is the straightforward application of this approach. In CMC, independent and identically distributed (iid) samples $X_1, \ldots, X_N$ are generated with the joint PDF of $\mathbf{X}$, and the corresponding outputs (or responses) $g_1, \ldots, g_N$ are computed by using a response function $g(\mathbf{X})$. Then, the probability of failure P_f can be estimated from

$$\hat{P}_f^{CMC} = \frac{1}{N}\sum_{i=1}^{N} I\left[g(X_i)\right] \tag{2}$$

The variance of the estimation $\hat{P}_f^{CMC}$ can be computed from (Rubinstein and Kroese 2016):

$$Var\left(\hat{P}_f^{CMC}\right) = \frac{\hat{P}_f^{CMC}\left(1 - \hat{P}_f^{CMC}\right)}{N} \tag{3}$$

It is clear from Eq. (3) that for a given number of sampling points N, the accuracy decreases rapidly with decreasing P_f. For example, for a P_f of 10^{-6}, 10^8 (100 million) simulations are required for 10% accuracy and 4×10^8 simulations are required for 5% accuracy. Since CMC is not convenient for estimating small probabilities of rare events in terms of the excessive number of simulations required, variance reduction methods and other efficient simulation techniques have been developed to reduce the computational cost by using a priori information about the problem of interest. A general framework for sampling-based methods for estimation of small failure probability is shown in Fig. 3. Note that these methods can also be used efficiently to estimate larger probabilities without any limitation or a special treatment.

2.1 Separable Monte Carlo (SMC)

The SMC method is a conditional MC method since it may use the cumulative distribution function (CDF) of a control variable in the limit state to estimate the probability of failure. In the conditional MC method (Ayyub and Chao-Yi 1992), the conditional expectation.

$E[H(x)|y]$ is computed for the probability of failure estimation, noting that $E[H(x)] = E\left[E\left[H(x)|y\right]\right]$. It is proven that the conditional MC always leads to variance reduction compared to CMC (Rubinstein and Kroese 2016).

The SMC method is applicable when the limit state function can be separated into response and capacity terms, and they are stochastically independent random variables. Then, the probability of failure can be formulated as (Smarslok et al. 2010):

$$P_f = \int F_C(r) f_R(r) dr, \tag{4}$$

where F_C is the CDF of the capacity and f_R is the PDF of the response. Then, P_f estimation can be obtained by using iid response samples r_i

$$\hat{P}_{smc} = \frac{1}{N} \sum_{i=1}^{N} F_C(r_i) \tag{5}$$

Alternate formulations are also available and discussed in Smarslok et al. (2010). Chaudhuri and Haftka (2013) combined SMC method with importance sampling method to further improve its accuracy. They considered a composite plate example and a tuned mass damper example, and find that SMC and importance sampling reduced the error individually by factors of two to five, and the combination reduced it further by about a factor of two. The main drawback of the SMC method is that it requires the limit-state function to be expressed in a separable form, otherwise this method is not applicable.

2.2 Stratified sampling and Latin hypercube sampling

Stratified sampling and Latin hypercube sampling (LHS) use the common idea that the sampling space can be divided into strata (or hypercubes) and that only a few of the many possible samples be selected in each strata (or hypercube) for probability of failure estimation (Melchers and Beck 2018). For a particular choice of strata, the sample size can be obtained in an optimal manner (Rubinstein and Kroese 2016).

Even though the stratified sampling and LHS methods substantially reduce the number of limit-state function evaluations compared to the CMC method, their computational cost is still high for rare event probability estimation. Stratified sampling and LHS can be used together with other techniques to improve their efficiency. Olsson et al. (2003) combined LHS with importance sampling and showed that more than 50% of the computing effort can be saved by using LHS instead of CMC in importance sampling. Similarly, Vaisman (2021) combined stratified sampling with subset simulation to improve the efficiency of the subset simulation. The details of importance sampling and subset simulation are discussed in the later subsections.

2.3 Weighted sampling (WS)

The WS method is based on determining the probability of failure by using the concept of the weight index for the generated samples (by using any of the simulation methods). The weight indices for a given sample can be computed as the product of the probability density function (pdf) of the random variables present in that sample. Next, the samples are computed for limit state function evaluation. The tail region is determined by the sign of the limit state function. Finally, the failure probability is computed as the fraction of the weight indices located in the failed region. Associated equations and references are provided in Appendix A. The major advantage of the WS method is that it can estimate the most probable point (MPP) with high accuracy and without any excessive computation.

The WS method has been used in various engineering reliability prediction studies, including reliability analysis of a bridge crane (Li et al. 2021) as well as mathematical test problems (Rashki et al. 2012; Efraimidis and Spirakis 2006; Okasha 2016; Rashki 2021).

2.4 Importance sampling (IS)

The IS method is based on the idea that an auxiliary PDF distribution $h_V(x)$ can be used to generate more samples in the failure region (Rubinstein and Kroese 2016). A weight term is then incorporated in P_f estimation to account for the use of $h_V(x)$ instead of the original PDF $f_X(x)$ to generate the samples. Associated equations and discussions are presented in Appendix A.

To improve the efficiency of the IS method (i.e., to reduce the number of limit state function calculations), adaptive sampling strategies and surrogate models have been utilized. Appendix B lists the studies that used the original IS method, its adaptive sampling variants, its surrogate-based variants, and also its variants enhanced with both adaptive sampling and surrogate models.

2.5 Subset simulations (SS)

The SS method is based on expressing a small failure probability as a product of larger conditional probabilities by introducing intermediate failure events, thereby converting a small failure probability problem into a series of larger probability problems that are easier to handle (Au and Beck 2001). In the implementation of this method, samples conditional on intermediate failure events are adaptively generated to gradually populate from the large probability regions to small probability regions. The SS method was also named as the adaptive multilevel splitting technique by various authors (Balesdent et al. 2015; Bréhier et al. 2015, 2016; Cérou and Guyader 2007; Lagnoux and Lezaud 2017, Vaisman et al. 2017; Wadman et al. 2014). In addition, a simulation method based on thermodynamic integration and parallel tempering (TIPT), which was proposed to estimate small failure probabilities (Xiao et al. 2019b) is also similar to subset simulation. Both of these methods convert a complex problem into a series of simple problems and use Markov chain Monte Carlo methods to solve these simple problems. In the SS method, all the Markov chains are constructed sequentially and the latter chains depend on the former chains. In the TIPT method, all the Markov chains are constructed simultaneously and can assist each other in all directions. Associated equations and discussions are presented in Appendix A.

The SS method utilizes the relationship between the input random variables and the output variable(s) as a black box. That is, the limit-state function is not needed to be expressed explicitly. This privilege is attractive for complex systems where it is difficult to use other methods (e.g., the IS method) that require prior information (e.g., the auxiliary PDF distribution h_V(x) in the IS method). The main drawback of the SS method is that the geometric structure of the limit state surface is not modeled, which can slow down the procedure or even lead to incorrect estimates (Breitung 2019).

To improve the efficiency of the SS method, various adaptations have been made. Surrogate models such as Kriging (Huang et al. 2016; Ling et al. 2019a, b; Tong et al. 2015; Wang and Shafieezadeh 2021; Xiao et al. 2019a; Xu et al. 2020), neural networks (Papadopoulos et al. 2012; Xia et al. 2017), support vector machines (Bourinet et al. 2011), hybrid polynomial correlated function expansion (Chakraborty and Chowdhury 2017), multiple response Gaussian process (Qian et al. 2021), high dimensional model representation (Wei et al. 2019b) and surrogate models combined with dimension reduction methods (Jiang et al. 2021) have been used to reduce the number of limit-state function calculations. The SS method has also been combined with other simulation methods such as the IS method and hybrid SS methods have been developed (Chen and Li 2017; Rashki 2021; Song et al. 2021a; Tong et al. 2015; Wagner et al. 2020; Wang et al. 2015). Furthermore, the failure threshold was modified and the variabilities of input variables were amplified (Cheng et al. 2022). Appendix B provides the literature relevant to the regular and improved subset simulation method.

2.6 Line sampling

Line sampling (LS) employs a line to obtain information on the failure region and efficiently calculates the failure probability by adding samples on a hyperplane, which is perpendicular to the important direction heading towards the failure region (Pradlwarter et al. 2007). In the LS method, the failure probability of the original high-dimensional problem in the standard normal space is estimated through several conditional one-dimensional failure probabilities. Associated equations and discussions are presented in Appendix A.

There have been several attempts to estimate small failure probabilities based on LS. De Angelis et al. (2015) proposed an advanced line sampling (ALS) approach that adaptively adjusts the important direction during simulation. Peng et al. (2015a) proposed an artificial bee colony-based line sampling (ABCLS) method that combines the artificial bee colony (ABC) algorithm and line sampling. The ABCLS method quickly finds the important direction of line sampling by solving a multi-constrained optimization model. Shi et al. (2016) introduced LS to a problem including multiple failure modes, and estimated a gravity dam's failure probability and the sensitivity of reliability. Shayanfar et al. (2017) proposed an adaptive line sampling method that adaptively updates the important direction as the MPP of the limit state surface changes. Under the interpretation that the ALS estimator is a special case of a combination of line sampling estimators, Papaioannou and Straub (2021) proposed a combination line sampling (CLS) that optimizes the ALS estimator through an alternative combination of estimators.

2.7 Directional simulation (DS)

The DS method is one of the variance reduction techniques used when the failure probability is small (Ditlevsen et al. 1990; Nie and Ellingwood 2004; Zhang et al. 2020a). The DS method is implemented in the standard normal space **U**, and the accuracy of the DS method depends on the determination of the direction vectors on the unit hypersphere (Nie and Ellingwood 2000). The failure probability is estimated as a sum of chi square distribution estimates at points along the unit direction. Associated equations and discussions are presented in Appendix A.

Various methods based on DS have been studied to estimate small failure probabilities. Nie and Ellingwood (2004) introduced a deterministic point set as sample points instead of randomly generated sample points for directional sampling, and identified that Fekete point set is useful in terms of accuracy and efficiency. Grooteman (2011) proposed an adaptive directional importance sampling (ADIS) method that combines an adaptive response surface approach with a directional sampling scheme in which the most important directions are sampled exactly. Zuniga et al. (2012) proposed an adaptive directional stratification (ADS) method that combines stratified sampling and directional sampling. Shayanfar et al. (2018) combined importance sampling and directional sampling and proposed a closed form update rule to obtain a sampling function that performs sampling in random directions.

The DS method has been successfully applied to system reliability problems (Nie and Ellingwood 2004; Guo et al. 2020). The DS method has been combined with other techniques to improve their efficiency. Zuniga et al. (2011, 2012) combined the DS method with stratified sampling and proposed the so-called adaptive directional stratification method. They applied their proposed method to a flood model and a nuclear reactor pressurized vessel model, to practically demonstrate their interest in real industrial examples. Zhu et al. (2017) used the DS method to calculate the small probability of slope failure. They compared the results of DS, IS and SS methods, and found that the DS method leads to the same accuracy level by using a smaller number of sampling, indicating that it is more efficient than the other two methods. Guo et al. (2020) combined the DS method with the IS method and active learning Kriging model. The main advantage of their proposed method was its ability to have great computational efficiency and deal with small failure probability problems. Also, the efficacy of the DS method in dealing with multi-failure model reliability problems was used to apply the proposed method to system reliability analysis in a successful manner. Zhang et al. (2021) combined DS with adaptive Kriging to reduce the size of the sample pool by generating samples of the direction vector, uniformly distributed in the unit hypersphere. It was shown that the DS greatly accelerates the learning process by significantly reducing the sample pool compared to adaptive Kriging combined with MCS (AK-MCS), especially for small failure probability.

2.8 Extrapolation methods

2.8.1 Asymptotic sampling (AS)

The AS method is an efficient simulation-based technique used for estimating the small failure probabilities of structures. The concept of asymptotic sampling utilizes the asymptotic behavior of the reliability index with respect to the standard deviations of the random variables. In this method, the standard deviations of the random variables are artificially inflated using a scale parameter to obtain smaller reliability indices, known as "scaled" reliability indices. Subsequently, a functional relationship is established between the scale parameters and scaled reliability indices. Finally, the actual reliability index is predicted using the established functional relationship.

Sichani et al. (2011a, 2011b) applied the asymptotic sampling method on high dimensional structural dynamic problems and first passage probability of high-dimensional nonlinear systems. Zhangchun et al. (2013) discovered that the use of a single extrapolation model was not robust. Inspired by the multiple tail median formulation (Ramu et al. 2010), where the median of multiple tail model predictions was used, they proposed to generate multiple extrapolation models and use the mean value of the reliability predictions of these models. In a follow-up study, Zhangchun et al. (2014) proposed a new mean extrapolation technique that involves six extrapolation models to estimate the actual reliability index. Acar (2016) increased the effectiveness of the asymptotic sampling by re-formulating the extrapolation formulation for highly safe structures with separable limit state functions. The accuracy and performance of the asymptotic sampling method are affected by various factors including the sampling method used, the values of the scale parameters, the number of support points, and the formulation of extrapolation models. Bayrak and Acar (2021) made a critical evaluation of the performance of the asymptotic sampling method for highly safe structures, and established some guidelines to improve the performance of the asymptotic sampling method. They found that generating the random variables by Sobol sequences and using the 6-model mean extrapolation formulation gave slightly more accurate results. Besides, the optimum initial scale parameter was approximately around 0.3 and 0.4, and the optimum number of support points is typically four for all problems. They also found that as the reliability level increases, the optimum initial scale parameter value decreases, and the optimum number of support points increase.

The "scaled-sigma sampling" method, proposed by Sun et al. (2015), is close to asymptotic sampling. This method generates random samples from a distorted distribution for which the standard deviation (i.e., sigma) is scaled-up. Sun et al. (2015) used this method to conduct reliability analysis of rare circuit failure events.

2.8.2 Enhanced simulation (ES)

Asymptotic sampling is based on the asymptotic behavior of failure probabilities as the standard deviation of the random variables tends to zero. Enhanced simulation (ES), on the other hand, uses the asymptotic behavior of failure probabilities as the limit state function is shifted away from the mean values of the random variables.

In the ES method, instead of artificially inflating the standard deviations of the random variables, an artificial limit-state function is formulated by introducing a scaling parameter that also shifts the limit-state function (Naess et al. 2009, 2012). This method was applied to reliability analysis of corroding pipelines by Leira et al. (2016).

2.9 Other sampling-based methods

Bao and Cassandras (1995) combined sample-path-based derivative estimation techniques with rational approximation techniques to solve rare event probability problems. They applied their proposed rational approximation approach to solve buffer overflow probability estimation.

Cadini and Gioletta (2016) developed a Bayesian Monte Carlo approach to estimate small failure probabilities. The Bayesian framework allowed an effective use of all the information available, i.e., the computer code evaluations and the input uncertainty distributions, and the analytical formulation of the Bayesian estimator guaranteed the construction of a computationally lean algorithm.

2.10 Discussion on sampling-based methods

The choice of the most suitable method for a given problem is dependent on the dimension of the problem, multi-modality of the limit-state function, the ease of implementation and the computational budget. Table 1 compares the sampling-based methods based on these different features, and provides a guideline to the user to choose the most appropriate technique for a given problem. In Table 1, each shaded circle is a score. As the score increases, the method performs better in that feature.

3 Surrogate-based approaches

Assessing small failure probabilities is time-consuming and challenging as very large numbers of simulated samples are required to identify failure regions. As noted earlier, if the order of magnitudes of the failure probability is 10^{-p}, MCS generally requires 10^{p+2} to 10^{p+4} samples (Cadini et al. 2017; Yun et al. 2021). To reduce the expensive calculation of MCS, variance-reduced simulation methods have been extensively developed, such as IS, SS, LS, and DS among other approaches. Although the number of function evaluations is reduced through these methods, thousands of function evaluations are still required when calculating small failure probability, hence the problem of calculation burden still remains. To solve this problem, surrogate model approaches that predict performance function values by replacing computationally expensive original functions with approximated models have been employed in reliability analysis or estimation of small failure probability.

A Kriging model is widely used as a surrogate model because it provides exact prediction for the simulation points and estimates prediction variance for other sample points. In order to build an accurate Kriging model with a small number of function evaluations, adaptive Kriging, which progressively updates the Kriging model with new samples obtained through the learning function, has been developed and many studies have proposed adaptive Kriging using various adaptive learning schemes to reduce the computational burden required for small failure probability estimation. In addition, other machine learning-based methods such as neural network (NN), support vector regression (SVR), support vector machine (SVM), and so on (Sun et al. 2017) are suitable for matching highly nonlinear performance functions, and they are commonly used to deal with small failure probabilities. The general framework and summary for surrogate-based techniques for small failure probability estimation are shown Figs. 4 and 5, respectively.

Surrogate-based approaches other than Kriging or machine learning are also often used to assess small failure probabilities. Wagner et al. (2022) applied stochastic spectral embedding method (Marelli et al. 2021) to sequentially expand the residual in the subdomains of the input parameter space. The resulting partition of the input space decomposes the failure probability into a set of easy-to-compute conditional failure probabilities. While applying stochastic spectral embedding method for rare event probability estimation, they proposed a set of modifications that include specialized refinement domain selection, partitioning and enrichment strategies. Dhulipala et al. (2022) proposed a framework for active learning with multi-fidelity modeling for probability estimation of rare events. Their framework operates fusing the low-fidelity (LF) prediction with a Gaussian process correction term, filtering the corrected LF prediction to decide whether to call the high-fidelity (HF) model and, for enhanced accuracy of subsequent corrections, adapting the Gaussian process correction term after an HF call. They also proposed dynamic active learning functions to improve the proposed algorithm's robustness for estimation of smaller failure probabilities.

In this section, studies that estimate small failure probabilities using surrogate-based approaches are reviewed: Sects. 3.1 and 3.2 discuss studies based on adaptive Kriging methods and machine learning-based methods, respectively.

3.1 Adaptive Kriging methods for rare probability of failure

The basic idea of the Kriging framework assumes that the response of the performance function is the realization of a stochastic process (Matheron 1973). Kriging is an exact interpolation method in which predictions at DoE points are equal to observed values, and DoE can be updated to

Table 1 Comparison of sampling-based methods in terms of different features

Method	Dimensionality	Multimodality	Implementation	Computational efficiency	Remarks
CMC	●●●	●●●	●●●	●	- Not suitable for small probabilities
SMC	●●●	●●●	●●◐	●●	- Limit state function should be in separable form
WS	●●●	●●●	●●◐	●●	- Can estimate the MPP with high accuracy and without any excessive computation
IS	●●	●●	●◐	●●●	- Requires knowledge of variance of the distribution sample - Adaptive sampling can further improve its computational efficiency
SS	●●◐	●●	●◐	●●●	- Geometric structure of the limit state surface is not modeled, which can slow down the procedure or even lead to incorrect estimates. - Tuning of its parameters requires experience
LS	●●	●	●●	●●	- Implementation is simple as long as the important direction can be determined easily
DS	●	●●●	●●◐	●●	- The efficiency and accuracy is dependent on the number of directions generated
AS/ES	●●	●	●●	●●●	- Tuning of its parameters requires experience

improve the accuracy of the Kriging model by using Kriging variance to quantify the uncertainty of the unexplored DoE region.

Although the Kriging model reduces the computational cost of reliability analysis, it is still difficult to quantitatively measure the accuracy of the Kriging model. Also, the fidelity of the Kriging model is highly affected by the selection

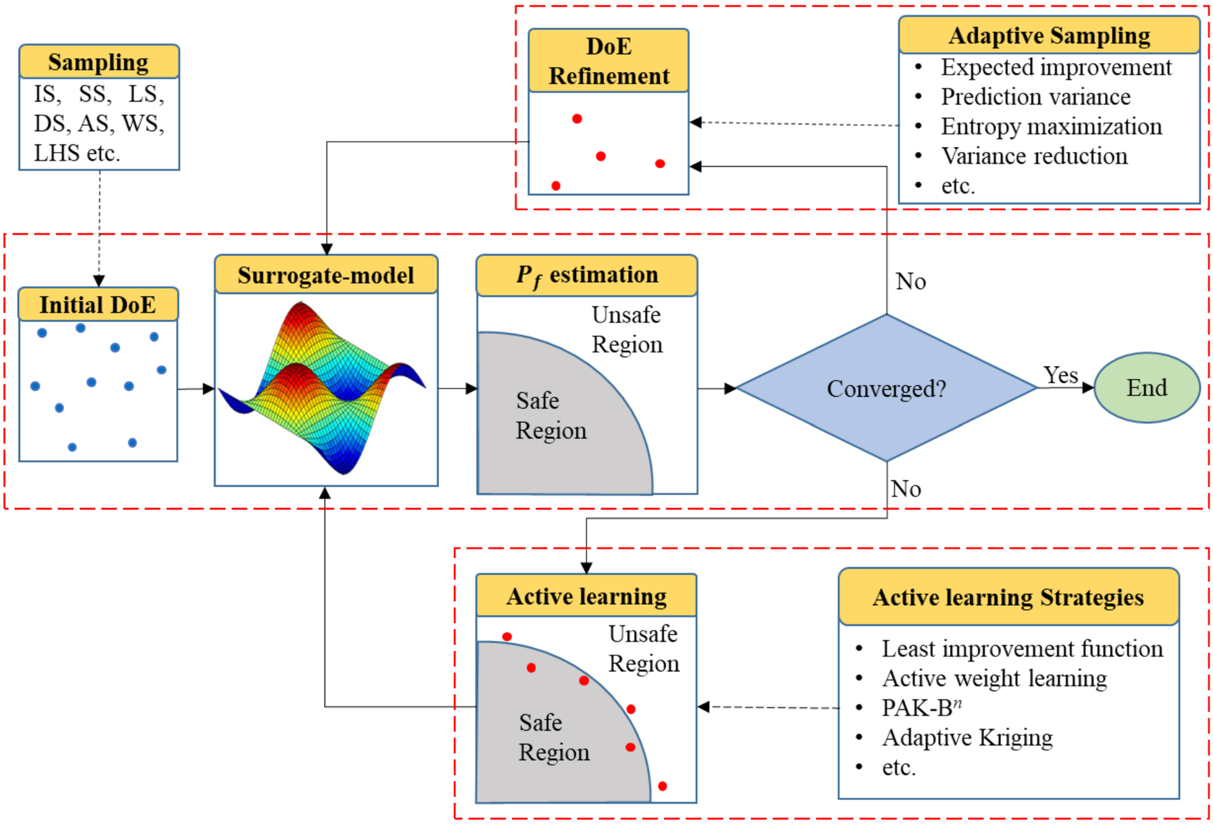

Fig. 4 General framework for surrogate-based techniques for small failure probability estimation

of DoE and its sample size (Wang and Wang 2016; Guo et al. 2020; Song et al. 2021c). In order to build an accurate Kriging model with a small number of DoE points, various adaptive Kriging methods that sequentially update DoE based on learning functions have been developed. Bichon et al. (2008) proposed an efficient global reliability analysis (EGRA) that adds samples near the limit state with large variance to DoE based on the expected feasibility function (EFF). Echard et al. (2011) proposed an active learning reliability method combining Kriging and MCS (AK-MCS) based on the learning function U that determines the next training sample considering the trade-off between Kriging mean and variance. Lv et al. (2015) proposed a learning function H that finds samples with large information entropy near the limit state. Dang et al. (2021) used upper bound posterior variance contribution, Dang et al. (2022) combined expected misclassification probability contribution and k-means clustering as learning functions. El Haj and Soubra (2020) utilized the weighted k-mean clustering for multi-point enrichment by adding the new training samples to update the Kriging metamodel. In this algorithm, larger weights are assigned to the samples with high information values according to the learning function.

However, for small failure probability problems, the size of the sample population required to obtain the converging probability of failure is extremely large. Since the candidate training samples need to be evaluated by the current Kriging model in order to select the next training sample required for updating the Kriging model. Hence, the computational efficiency of adaptive Kriging methods can be limited to some extent (Yun et al. 2020).

3.1.1 New learning functions and update strategies

To enable efficient estimation of small failure probability, various adaptive Kriging methods introduce new learning functions and update strategies. Studies that developed adaptive Kriging methods with new learning functions and update strategies are summarized in Table 2. These studies focus on selecting the next training sample using the information of each sample point.

3.1.2 Variance reduction techniques

Active learning reliability methods that combine adaptive Kriging and variance reduction techniques have been developed to reduce the candidate sample pool size. To

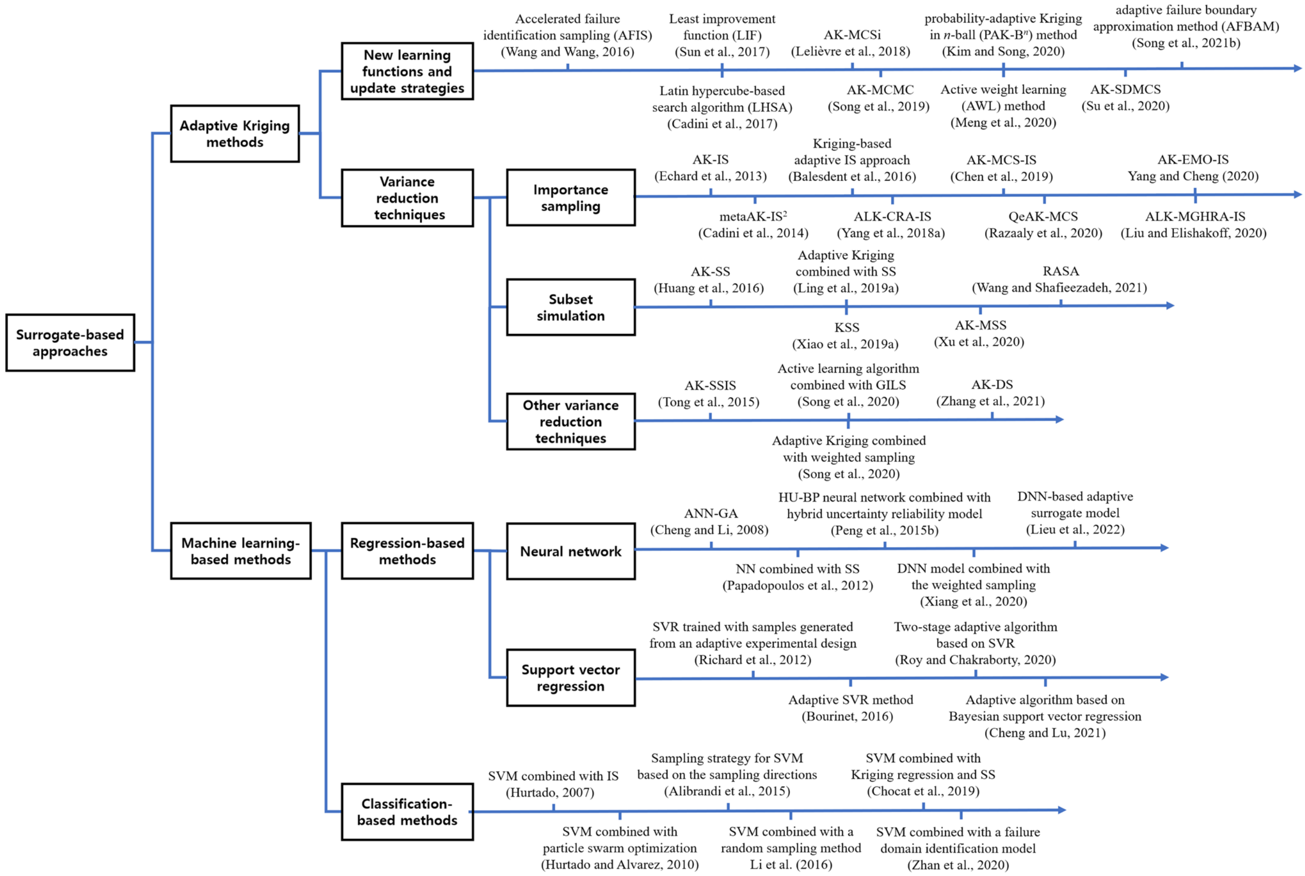

Fig. 5 Summary of surrogate-based approaches

improve the computational efficiency of small failure probability problems, IS is widely used because a large number of simulated samples are located in the failure region (Liu et al. 2020). Although SS significantly reduces the number of required function evaluations as compared to MCS, it still suffers from a number of time-consuming function evaluations required for the estimation of small failure probabilities, and there have been many attempts to couple adaptive Kriging with SS. In addition, various other variance reduction techniques are combined with adaptive Kriging to improve the efficiency and accuracy of small failure probability estimation. Studies that combine adaptive Kriging methods and variance reduction techniques are summarized in Table 3.

3.2 Machine learning-based methods

Machine learning, a form of applied statistics that statistically estimates complicated functions based on computational power, is prevalent in academic and industrial areas (Goodfellow et al. 2016; Xu and Saleh 2021). In line with this trend, there have been efforts to perform reliability analysis by employing various machine learning techniques. Among the categories of machine learning, supervised learning, which maps input vectors and outputs based on pairs of given datasets, is commonly used for reliability analysis. The prediction purpose of supervised learning is to play the role of a surrogate model in predicting the response of an input not included in the dataset. Generally, two major subcategories—regression and classification—constitute supervised learning (Miorelli et al. 2021). Since machine learning-based methods do not involve sample-wise prediction error as in Kriging, it is difficult to employ the adaptive schemes actively used in Kriging; however, various methods specialized for machine learning-based methods have been developed to deal with small failure probabilities.

3.2.1 Regression-based methods

Regression problems aim to estimate quantitative responses, and machine learning models such as NN and SVR are used in reliability analysis. In NN based on the neural structure of the brain, neurons in each layer are connected to neurons in subsequent layers, and the weight of each connection is determined through learning (Elhewy et al. 2006). A deep neural network (DNN) with multiple hidden layers is

Table 2 Summary of adaptive Kriging methods with new learning functions and update strategies

Reference	Method	Summary	Number of updated samples
Wang and Wang (2016)	Accelerated failure identification sampling (AFIS)	Sequentially identifies rare failure sample points while omitting the evaluation of the majority of sample points in the safe region	Single sample
Cadini et al. (2017)	Latin hypercube-based search algorithm (LHSA)	Identifies at least one input sample point in a failure region required to help the efficient start of an adaptive procedure for refinement of Kriging model	Single sample
Sun et al. (2017)	Least improvement function (LIF)	Measures how much the accuracy of estimated failure probability will be improved by considering both statistical information of the Kriging model and the joint probability density of input variables	Single sample
Schöbi et al. (2017)	Active learning algorithm coupled with PC-Kriging	Polynomial-Chaos Kriging (PC-Kriging) and AK-MCS are combined	Multiple samples
Lelièvre et al. (2018)	AK-MCSi	Sequentially performs MC simulation by dividing the large MC population into several smaller populations	Multiple samples
Song et al. (2019)	AK-MCMC	AK and Markov chain Monte Carlo simulation (MCMC) are combined Kriging model is updated through a learning function considering the distance from the existing DoE samples and the Kriging variance	Single sample
Meng et al. (2020)	Active weight learning (AWL)	Assigns different weight indices to samples on the limit state function considering the important degree of each sample point	Single sample
Kim and Song (2020)	Probability-adaptive Kriging in n-ball (PAK-Bn)	Uses a new learning function to identify important points that are located near the limit-state surface and have a significant impact on the failure probability	Single sample
Su et al. (2020)	AK-SDMCS	AK and spherical decomposition-MCS (SDMCS) are combined	Single sample
Yu et al. (2020)	RCA-PCK	PC-Kriging model and radial centralized adaptive sampling strategy are combined	Multiple samples
Song et al. (2021b)	Adaptive failure boundary approximation method (AFBAM)	Kriging and a new adaptive learning strategy based on uniform sampling are combined	Single sample
Zhou and Li (2022)	Hierarchical partitioning (HP) strategy	Constructs the adaptive Kriging model via two steps The Kriging model is used to approximate the relationship existing between extreme values and random variables in the system	Single sample

gaining popularity for regression of high dimensional and highly nonlinear problems (Zhang et al. 2018). The SVR is based on a principle that minimizes the error of the fit while maintaining the flatness of the regression surface to prevent overfitting, and has been revealed to show improved response prediction accuracy with a small number of DoE points (Vapnik 1995; Roy et al. 2019). In addition, a notable property of SVR is that it can reduce the burden arising from the increase in the dimension of the problems (Vapnik 1995).

For problems of small failure probabilities, there have been studies employing machine learning-based regression models to act as surrogate models. After an adequate surrogate model is defined with the regression model, MPP or variance reduction techniques are employed to estimate the failure probability. Adaptive strategies are also used to iteratively improve the accuracy of the regression model. Regression-based methods are summarized in Table 4.

Table 3 Summary of adaptive Kriging methods with variance reduction techniques

Reference	Method	Summary	Type of variance reduction technique
Echard et al. (2013)	AK-IS	Extends AK-MCS by replacing MCS with IS	IS
Cadini et al. (2014)	metaAK-IS2	Replaces the FORM stage of AK-IS with the metamodel refinement step of the meta-IS algorithm proposed by Dubourg et al. (2013)	IS
Tong et al. (2015)	AK-SSIS	AK and subset simulation importance sampling (SSIS) are combined	IS, SS
Balesdent et al. (2016)	Kriging-based adaptive IS approach	Finds the optimal auxiliary distribution for IS using the preceding IS estimations	IS
Huang et al. (2016)	AK-SS	Kriging model and SS are combined	SS
Yang et al. (2018a, b)	ALK-CRA-IS	Updates the Kriging model by treating IS samples populating most probable failure regions (MPFRs) as candidate points A concentric ring approaching (CRA) method is proposed to identify the MPFRs	IS
Yun et al. (2018)	AK-MIS	AK and modified importance sampling (AK-MIS) are combined Employs the importance weight function to designate samples in the important area as candidate samples used to select the next training sample	IS
Chen et al. (2019)	AK-MCS-IS	Replaces the initial large population with two or more populations and determines the next training sample by an iterative approach based on points not in the original populations	IS
Barkhori et al. (2019)	Kriging-aided cross-entropy-based adaptive importance sampling (KCE-AIS)	Kriging model and cross-entropy-based adaptive importance sampling are combined	IS
Ling et al. (2019a)	Adaptive Kriging coupled with SS	Employs SS to transform small failure probabilities into a series of larger conditional failure probabilities of the intermediate failure events	SS
Xiao et al. (2019a)	Kriging-based subset simulation (KSS)	Updates Kriging model based on the samples in the first and last levels of SS by employing a strategy that finds samples located around the projection outlines on the limit-state surface	SS
Razaaly et al. (2020)	QeAK-MCS	Estimates extreme quantiles based on importance sampling method using an isotropic-centered Gaussian distribution to generate candidate training samples for the DoE	IS
Yang and Cheng (2020)	AK-EMO-IS	Introduces evolutionary multimodal optimization (EMO) to explore multiple MPPs Updates the adaptive Kriging by performing importance sampling on multiple MPPs	IS
Liu et al. (2020)	AK-ALIS	AK and adaptive linked importance sampling (ALIS) are combined	IS
Guo et al. (2020)	ALK-DIS	AK and directional importance sampling (DIS) are combined	IS
Yun et al. (2020)	AK-ARBIS	AK and adaptive radial-based importance sampling (ARBIS) are combined	IS
Liu and Elishakoff (2020)	ALK-MGHRA-IS	Kriging-based importance sampling and more general hybrid reliability analysis (MGHRA) method are combined	IS
Xu et al. (2020)	AK-MSS	Replaces the original sample population of AK-MCS with the sample population generated by modified subset simulation (MSS)	SS
Song et al. (2020)	GILS	Active learning algorithm and global imprecise line sampling (GILS) are combined	LS

Table 3 (continued)

Reference	Method	Summary	Type of variance reduction technique
Yang et al. (2021)	ALK-MAIS-TCR	Active learning Kriging and multimodal adaptive important sampling (MAIS) are combined	IS
Guo et al. (2021)	ALK-SIS	Active learning Kriging and system importance sampling (SIS) are combined	IS
Yun et al. (2021)	Error-based stopping criterion (ESC)	Uses the maximum relative error of failure probability estimation in the AK-IS method to improve the efficiency of updating	IS
Li et al. (2021)	AK coupled with WS	AK and WS are combined	WS
Zhang et al. (2021)	AK-DS	Adaptive Kriging and DS are combined	DS
Wang and Shafieezadeh (2021)	Reliability analysis using subset simulation and adaptive Kriging (RASA)	Decomposes reliability problems into a number of sub-reliability problems and adaptively adjusts each subset's intermediate failure probabilities and the number of candidate samples	SS

3.2.2 Classification-based methods

Among the various subcategories of supervised learning, classification involves qualitative responses that can be obtained from categorical data (Xu and Saleh 2021). Therefore, classification-based methods are suitable for reliability analysis since it requires the distinction between safe region and unsafe region. As one of the classification-based methods, the SVM is motivated by statistical learning theory and has been successfully applied to two-category classification problems including reliability analysis (Rocco and Moreno 2002). SVM is generally formulated to find a hyperplane that maximizes the width of the gap in two categories, and is effective for high dimensional problems. The advantage of SVM is that it can directly estimate the sign of the performance function, thereby reducing the computational effort required to estimate the performance function value (Xiong and Tan 2017).

To address small failure probabilities with SVM, researchers have developed methods that introduce variance reduction techniques or focus training samples on important regions where failure is prone to occur. Classification-based methods are summarized in Table 5.

3.3 Discussion on surrogate-based approaches

This section presents a guideline for estimating small failure probabilities using surrogate-based approaches. A flowchart for the guideline is shown in Fig. 6. As aforementioned, surrogate-based approaches can be divided into two types: adaptive Kriging methods and machine learning-based methods. For highly nonlinear problems, machine learning-based methods can be appropriate, and classification-based methods are recommended when categorical data are available. In the case of adaptive kriging methods, small failure probabilities can be estimated based on new learning functions and update strategies, and in another way, adaptive kriging combined with various variance reduction techniques can be used for estimation. Among the variance reduction techniques, importance sampling can be adopted if MPP is available, and subset simulation can be adopted if conditional failure probabilities are available.

4 Statistics of extremes based approaches

4.1 Generalized extreme value theory and tail equivalence

Rare event prediction translates to low failure probability estimation, also referred to as extreme value estimation requires sufficient data in the tails of the distribution to model the extremes. This is often not possible and tail modeling techniques based on extreme value theory are an attractive alternative to predict the probability of extreme events. Maes and Breitung (1993) qualify two CDFs to be tail equivalent when the approximated error of small probabilities goes to zero as the abscissa tends to infinity. The theory comprises a principle for model extrapolation based on the implementation of mathematical limits as finite-level approximations. For an iid random variable and its CDF, F, it can be shown that there exist sequences (a_n), (b_n) and a random variable z with CDF H such that-

$$\lim_{n\to\infty} F^n(a_n z + b_n) \approx H(z) \tag{6}$$

for a non-degenerate distribution function, H belongs to one of the following extreme value distribution families: (1) Gumbel distribution, (2) Frechet distribution, (3) Weibull distribution.

Table 4 Summary of regression-based methods

Reference	Method	Summary	Type of machine learning model
Cheng and Li (2008)	Artificial neural network based genetic algorithm (ANN-GA)	Estimates failure probability by finding the minimum reliability index based on the genetic algorithm	NN
Papadopoulos et al. (2012)	NN coupled with SS	Estimates failure probability based on NNs trained sequentially at each SS level	NN
Richard et al. (2012)	SVR-based regression model	Employs SVR trained with samples generated from an adaptive experimental design that can be rotated based on the gradient of the SVR model	SVR
Dai et al. (2012)	SVR coupled with adaptive Markov chain simulation and IS	Generates samples located in the most likely failure region, and estimated the failure probability by constructing a local surrogate model based on SVR	SVR
Peng et al. (2015b)	Hybrid uncertainty reliability analysis method based on ANN	Hybrid uncertainty reliability model and back propagation (HU-BP) neural network are combined Estimates failure probability after finding the MPPs corresponding to the upper and lower bounds of the reliability index	NN
Bourinet (2016)	Adaptive SVR	Explores the safe region using surrogate models with moderate accuracy and improves the accuracy of surrogate models after they get close enough to the limit state	SVR
Xia et al. (2017)	NN coupled with SS and explicit time-domain method (ETDM)	Reduces the required number of samples by combining SS and modified Metropolis–Hastings (MMH) algorithm	NN
Xiang et al. (2020)	NN coupled with WS	Updates NN model by selecting the next training sample located near the limit state surface	NN
Roy and Chakraborty (2020)	Two-stage adaptive algorithm based on SVR	Builds SVR model through an initial DoE generated based on the space-filling design Adds the next training sample so that it is separated from the existing DoE points	SVR
Cheng and Lu (2021)	Adaptive algorithm based on Bayesian SVR	Bayesian SVR model and MCMC are combined Updates each intermediate failure surface based on the expected risk function	SVR
Lieu et al. (2022)	DNN-based adaptive surrogate model	Adds more important samples near the limit state	NN

It is to be noted that regardless of the underlying distribution, the above three types are the only possible limits for the distribution of the normalized maxima as stated by the 'three-type theorem': Frechet–Fisher–Tippett theorem (Gnedenko 1948). In this respect, this theorem is to extreme value theory what the central limit theorem is to central statistics. All 3 distributions are expressed in terms of a, b, α which are the scale, location and shape parameters, respectively. Frechet–Fisher–Tippett theorem implies that the normalized maxima have a limiting distribution that must be one of the three types of extreme value distribution. These three types of extreme value distribution have been combined into a single three-parameter family (Jenkinson 1955; Hosking et al. 1985) known as Generalized Extreme Value (GEV) distribution which is a function of location, scale and shape parameter.

4.2 Generalized Pareto distribution (GPD)

In engineering applications, rather than maxima, the interest is to address the excesses over the threshold. In order to address this, researchers introduced the notion of 'threshold exceedances' where all maximums should help for the evaluation of the tail by extracting more information in the tail of the distribution than just that given by the largest order statistics (Pickands Theorem 1975).

Table 5 Summary of classification-based methods

References	Method	Summary
Hurtado (2007)	SVM coupled with IS	Estimates the failure probability combining SVM with IS
Bourinet et al. (2011)	SS-SVM	SVM and SS are combined in an active learning scheme
Alibrandi et al. (2015)	SVM-based response surface	Evaluates the failure probability using SVM and a novel second-order response surface
Li et al. (2016)	Multi-input multi-output SVM	Combines LHS and uniform sampling to perform reliability analysis on multiple limit state functions
Xiong and Tan (2017)	Adaptive SVM	Selects the next training sample that improves the current SVM the most among the candidate samples in the important region
Chocat et al. (2019)	Adaptive regression and classification based on subset simulation (ARC-Subset)	Kriging regression, SVM, and SS are combined
Zhan et al. (2020)	One-class SVM-based scheme	Constructs a failure domain identification model based on one-class SVM to estimate the probability of failure under imbalanced samples

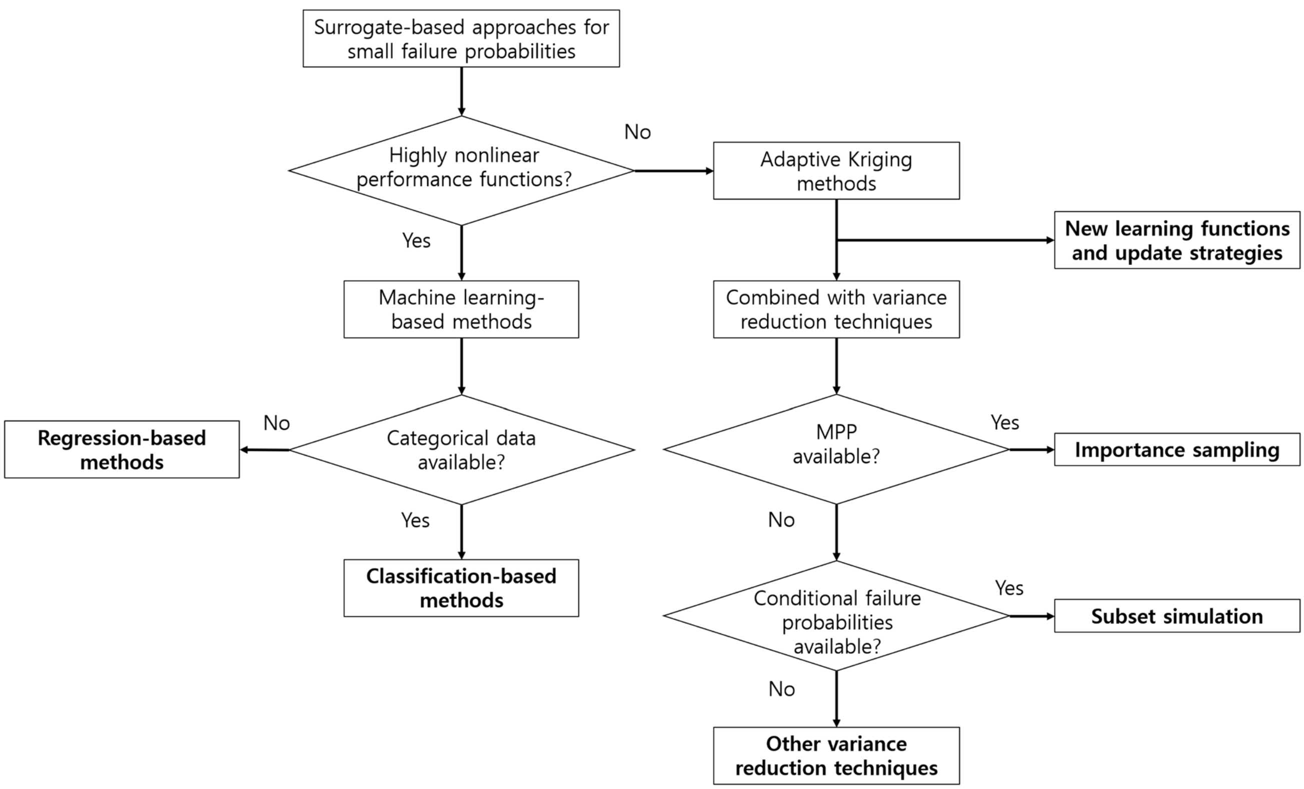

Fig. 6 Guideline for surrogate-based approaches

The concept of GPD is presented in Fig. 7. Let y be a model output which is random and u be a large threshold of y. The observations of the model output y that exceed the threshold u are called 'exceedance'. The conditional distribution function $F_u(z|y>u)$ of the exceedance given that the data y is greater than the threshold u, can be modeled by the GPD. Here, $z=y-u$.

Let approximation of conditional distribution $F_u(z|y>u)$ using GPD be $\hat{F}_{\xi,\psi}(z)$, then for a large enough u, the distribution function of $F_u(z|y>u)$, is approximately written as (Coles 2001):

$$\hat{F}_{\xi,\psi}(z)=\begin{cases}1-\left\langle 1+\frac{\xi}{\psi}z\right\rangle_+^{-\frac{1}{\xi}} & \text{if } \xi\neq 0\\ 1-\exp\left(-\frac{z}{\psi}\right) & \text{if } \xi=0\end{cases}, \tag{7}$$

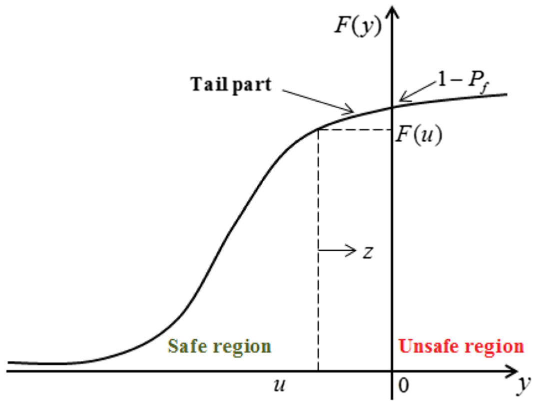

Fig. 7 Concept of GPD for tail modeling

where ξ and ψ are the shape and scale parameters, respectively. $\langle A \rangle_+ = \max(A, 0)$ and $z > 0$. Shape parameter ξ plays a key role in assessing the weight of the tail. The above equation can be seen as a limiting distribution as u increases. Tails can be classified based on ξ as: (1)$\xi > 0$, heavy tail (Pareto-type tail), (2) $\xi = 0$, medium tail (exponential type tail), and (3) $\xi < 0$, light tail (Beta-type tails).

Small failure probability corresponds to the tail of a distribution which can be approximated as GPD is discussed in Ramu et al. (2010). Parameter estimation methods such as maximum likelihood estimation and least square regression are used to obtain the estimates of the shape and scale parameters. The general framework and summary for statistics of extreme-based methods for estimation of small failure probability are shown in Figs. 8 and 9, respectively.

4.2.1 Threshold selection

The final estimates such as the quantile, extreme values etc. depend on the shape and scale parameter estimation that again depends on the threshold selection (Caers and Maes 1998; McNeil and Saladin 1997). Threshold selection is a trade-off between bias and variance of the estimates. Boos (1984) suggests that the ratio of the number of tail data over the total number data should be 0.02 for ($50 < N < 500$) and 0.1 for ($500 < N < 1000$). Hasofer (1996) suggests using $1.5\sqrt{N}$ samples in the tail region. Caers and Maes (1998) propose to use a finite sample mean square error (MSE) as a criterion for estimating the threshold. They use the threshold value that minimizes the MSE. In a similar fashion Beirlant et al. (1996) find an optimal threshold by minimizing an approximate expression for asymptotic mean square error. The other methods include plotting the quantile, shape or scale factor or any quantity of interest with respect to different thresholds and looking for stability in the curve (Bassi et al. 1998; Coles 2001, pp: 84–86). In order to avoid the issue of threshold selection in EVT, Albrecher et al. (2020) define the class of matrix Mittag–Leffler distributions and use it to model the extremes. De Carvalho et al. (2021) develops a Bayesian regression model for the conditional left and right tails of a heavy-tailed response. This model permits the covariates to be significant for the lower values but not for the tail and vice versa while bypassing the selection of threshold values. Pipiras (2020) argue for physics enabled extremes prediction rather than a data driven peak over thresholds method or mixed models approach for extremes prediction. Xu et al. (2022a, b) proposed an adaptive mixture of normal-inverse Gaussian distribution (A-MNIGD) for structural reliability to represent the unknown distribution

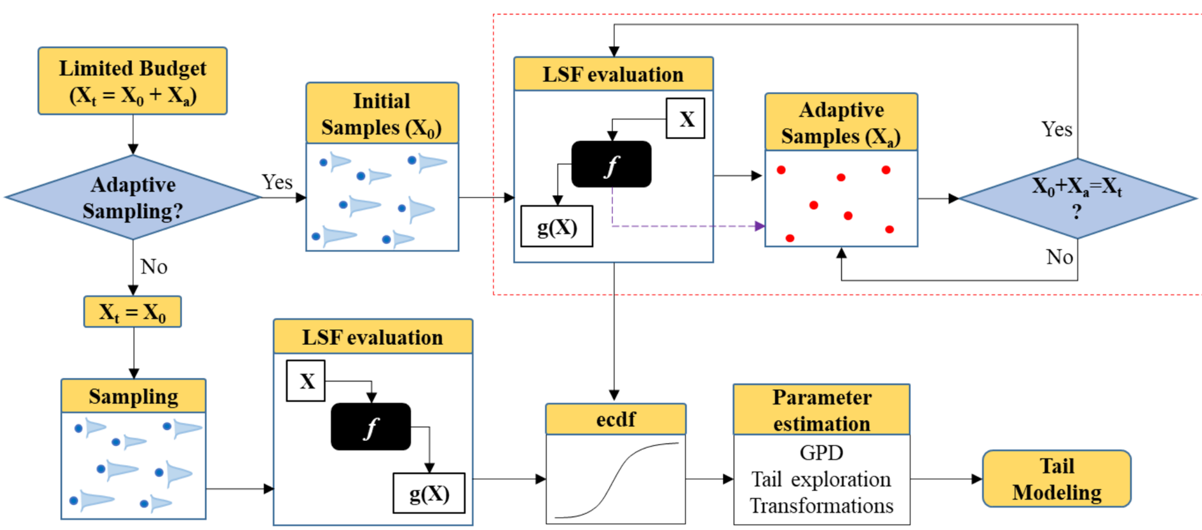

Fig. 8 General framework for statistics of extreme-based methods for estimation of small failure probability. X_0 is initial sample, X_a is the adaptive or additional sample and X_t is total samples available for LSF evaluation or budget in terms of LSF

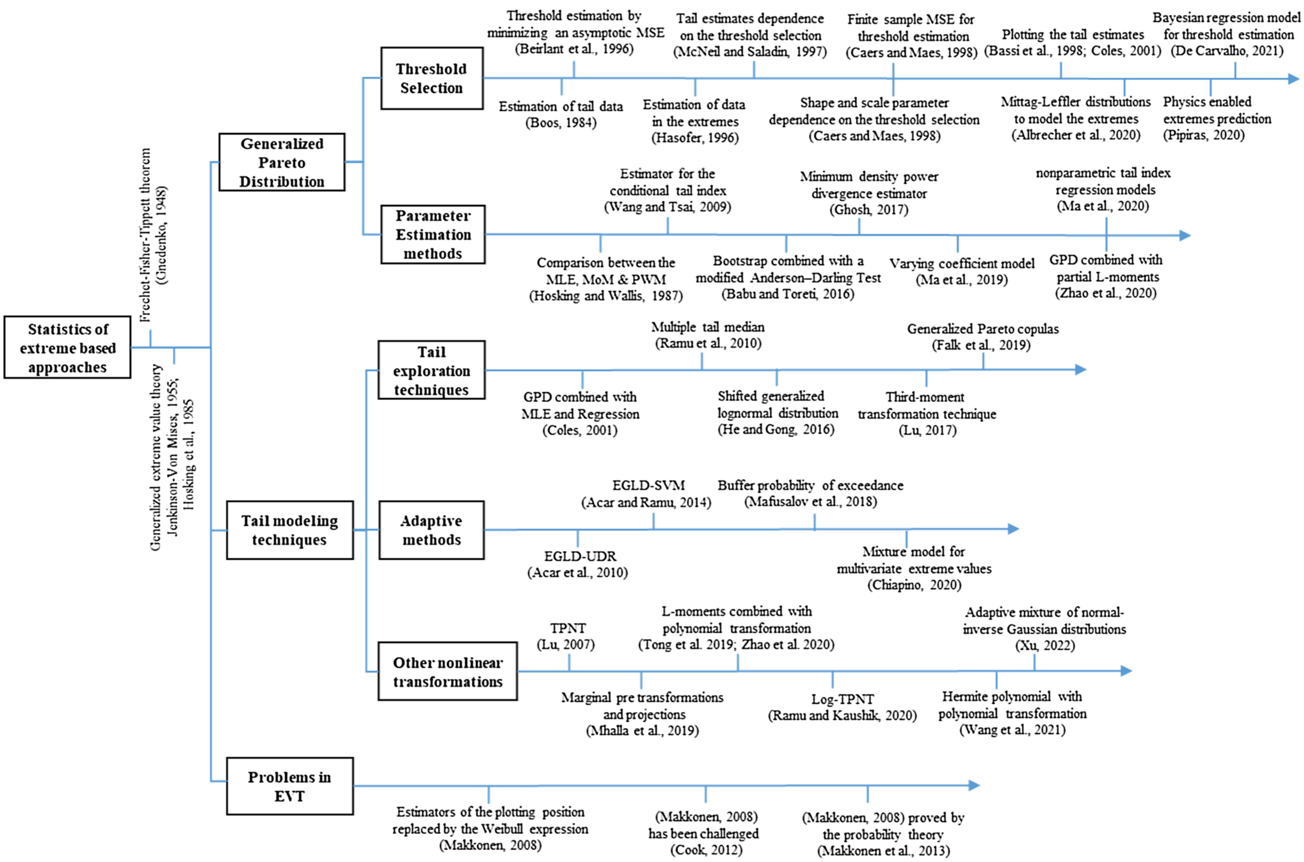

Fig. 9 Summary of statistics of extreme-based approaches

of the limit state function (LSF), according to the limit condition. Cai et al. (2022) proposed a novel method for evaluating the distribution of performance functions (DPF) in terms of expectation that is computed by combining the Sobol sequence with the point estimate method. Li et al. (2022) introduced the concept of conditional reliability index (equivalently conditional probability of failure) by considering the uncertainties of distribution parameters in the evaluation of structural reliability and proposed a novel approach by integrating the Smolyak-type quadrature formula with the cubic normal distribution to determine the percentile value of the conditional probability of failure.

4.2.2 Parameter estimation

Among the several existing techniques for parameter estimation, maximum likelihood estimation (MLE) and least square regression are the well-known and widely used techniques. Other parameter estimation techniques include method of moments (MoM), probability weighted moments (PWM), elemental percentile method, etc. Generally, MLE method is widely used and accepted by researchers but it suffers from some limitations. The asymptotic properties of the maximum likelihood estimators are preserved as long as the scale factor $\xi > -0.5$. Although MLE are obtainable but do not have the standard asymptotic properties for $-1 < \xi < -0.5$. MLE is not obtainable when $\xi < -1$ (Coles 2001, pp. 54–55). Beirlant and Goegebeur (2004), Castillo et al. (2005) discuss these methods in detail. Hosking and Wallis (1987) report a comparison between the MLE, MoM & PWM and conclude that MoM and the PWM are more reliable compared to the MLE method unless the sample size is greater than 500. Babu and Toreti (2016) propose a general bootstrap procedure combined with a modified Anderson–Darling Test as goodness-of-fit for heavy-tailed distribution such as GPD. Ghosh (2017) proposed a new general estimator of the tail index called the minimum density power divergence estimator (MDPDE) that provides the robust estimate of the Tail Index through a suitable exponential regression model (ERM) by minimizing the density power divergence (DPD). Ma et al. (2019) propose the varying coefficient model where the coefficients of covariates are allowed to change with other variables via smooth functions. This work is a generalization of parametric tail index regression model, proposed by Wang and Tsai (2009) as estimator for the conditional tail index in random covariate context, to the varying coefficient setting. Ma et al. (2020) propose a method for the estimation of the tail index in the presence of a random covariate, where the conditional distribution of the variable of interest is of Pareto-type. They used a logarithmic function to link the tail index to the nonlinear predictor induced by covariates, which forms the nonparametric tail index regression models. Zhao et al. (2020) combine GPD with partial L-moments to arrive at computationally efficient parameters for the GPD. Cabral et al. (2022) asymptotically compares several estimators of the extreme value index including the Hill estimator, asymptotically unbiased Hill estimator, and recent generalized means estimator, based on the moments of the upper order statistics, through a Monte Carlo simulation study. Zhu et al. (2022) proposed a new and efficient estimation method for extreme conditional quantiles of functional quantile regression with heavy-tailed distributions by first estimating the intermediate conditional quantiles using the regression in orthonormal plane obtained using eigenvalue decomposition of the sample covariance, and then extrapolating the intermediate conditional quantile estimate to extreme tails.

4.3 Tail modeling techniques

Traditionally, tails of statistical distributions have been modeled using the GPD principle (Castillo 2012) and MLE or least square regression are used as parameter estimation techniques. However, based on the limitations discussed above, alternate approaches to model the tails have been proposed. Ramu et al. (2010) subject the data to simple transformations such as inverse standard normal CDF and logarithm to model different parts of the tails as an ensemble and use its median as the compromise estimate. Acar et al. (2010) approximates CDF using the extended generalized lambda distribution (EGLD) whose statistical moments are obtained from the univariate dimension reduction (UDR) method. They note that this approach is very sensitive to tail probabilities. Acar (2011) combined EGLD with dimension reduction method, Acar (2013) along with Acar and Ramu (2014) combined EGLD and SVM to identify regions corresponding to the tails of the distribution and sample there additionally. Zhao and Lu (2007) propose explicit fourth-moment standardization function utilizing the idea of the third order polynomial normal transformation (TPNT) using the first four central moments that is found to be accurate enough to include independent random variables with unknown CDFs/PDFs in reliability analysis. The proposed method provides more appropriate normal transformation and inverse transformation results compared to the third-moment function, Fisher-Cornish expansion, or Winter-stein formula. Lu et al. (2017) proposed a third-moment transformation technique for transforming the correlated variables into independent standard normal variables while Wang et al. (2021b) uses Hermite polynomial with polynomial normal transformation. Tong et al. (2019) and Zhao et al. (2020) combine L-moments approach and polynomial transformation approach for better prediction of small failures. Ramu and Kaushik (2020), inspired by Fleishman (1978), Hong and Lind (1996) and Hong (2011), modelled the tails as a cubic function of a normal random variable but in the logarithmic probit space to obtain better estimates of the low failure probability compared to the competing approaches. They call their approach the log-TPNT. Several approaches exist for modelling the multivariate case as well. Winterstein and MacKenzie (2013) compares the models based on moments versus L-moments. This paper also compares the moment-based models based on Hermite transformations versus maximum entropy and argues that the L-moments and four-moment maximum entropy models may be inappropriate to model broader-than-Gaussian cases. He and Gong (2016) employ a shifted generalized lognormal distribution to approximate the tails of the univariate extreme distributions, in which the model parameters are estimated by an extrapolation method. Then, the tails of the multivariate extreme distributions of the nonlinear response are determined by using the Nataf model. Mhalla et al. (2019) developed a framework based on marginal pre transformations and projections of d-dimensional random vector X along the directions of the unit simplex, that lead to convenient univariate representations of multivariate exceedances based on the exponential distribution. Mafusalov et al. (2018) introduced the buffer probability of exceedance as an upper bound to the probability of exceedance. Falk et al. (2019) introduced generalized Pareto copulas (GPC) using the concept of D-norms. The family of GPC together with the set of univariate generalized Pareto distributions (GPD) enables multivariate GPD computations. Chiapino et al. (2020) introduces the mixture model for multivariate extreme values. This approach describes the distribution of extremal which permits assigning to any extreme point a posterior probability for each anomaly type. The extremal index (EI) is the parameter that describes and quantifies the clustering characteristics of the extreme values in many stationary sequences. Gomes and Neves (2020) introduced a new blocks estimator to estimate the extremal index (EI) using disjoint blocks and sliding blocks. Xu et al. (2022a) proposed an adaptive mixture of normal-inverse Gaussian distributions to represent the unknown distribution of the limit state function to obtain failure probability. Xu et al. (2022b) proposed an adaptive polynomial skewed-normal transformation (A-PSNT) model that uses a tail error criterion to select the most appropriate order for the PSNT model, which makes the proposed method an adaptive one. Also, the A-PSNT method can effectively reconstruct the probability distribution to predict the failure probability with rare events accurately. Zhao et al. (2022) develop an updating strategy to construct Kriging model based on extreme value function response. The first four moments of the extreme value function are estimated using a weighted approach based on sparse grid numerical integration (WA-SGNI). The bounds of time-variant failure probability are evaluated by combining the adaptive Kriging model and WA-SGNI.

4.4 Problems in the extreme value analysis

Makkonen (2008) outlined the means to avoid the problems in the extreme value analysis arising due to the widely used wrong probability plotting positions and asymptotic behavior of the theoretical extreme value distributions. Two distinct problems in the extreme value analysis are addressed in Makkonen (2008): (i) assessing the probability positions for the order ranked extremes is commonly done incorrectly, (ii) the belief in the applicability of the extreme value theory is so strong that the analysis is commonly done even when a good fit should not be expected due to the asymptotic nature of the theory.

Makkonen (2008) argued that the estimators of the plotting position in EVA should be abandoned and replaced by the Weibull expression $m/(N+1)$. Cook (2012) challenged this development. Later, Makkonen et al. (2013) proved by the probability theory that the Weibull expression provides the rank probability exactly showing that no estimators of the plotting positions are necessary.

4.5 Discussion on statistics of extremes-based methods

The choice of the most suitable method for a given problem is dependent on the dimension of the problem, multimodality, non-linearity of the limit-state function and the computational budget. Table 6 compares the statistics of extremes-based methods based on these different features, and provides a guideline to the user to choose the most appropriate technique for a given problem. In Table 6, each shaded circle is a score. As the score increases, the method performs better in that feature. Figure 10 provides the guideline for selecting appropriate tail modeling technique for a given problem.

5 Methods tailored for time-dependent systems

The failure probabilities observed in time-dependent systems are much larger than those of the time-independent systems. In this section, the probability of failure is treated as a small probability when it is smaller than 10^{-3} (rather than 10^{-5} for the case of a time-independent system). This section discusses the rare event probability estimation of time-dependent or stochastic systems. The sampling-based methods tailored for time-dependent systems are first discussed, followed by the discussion of surrogate-based methods.

5.1 Sampling-based methods

The time-dependent reliability is often regarded as the probability that the structural random response process does not exceed the specified failure threshold within a specified time period. In this regard, Poisson outcrossing rate methods (Der Kiureghian 2000; Andrieu-Renaud et al. 2004; Sudret 2008) using the first-order reliability method (FORM) are extensively used. For instance, being one of the most popular methods, PHI2 calculates the outcrossing rate using a parallel system composed of a pair of limit state functions at successive time instants (Andrieu-Renaud et al. 2004). The outcrossing rate is estimated as the bivariate Gaussian integral using FORM. Since the Gaussian CDF is usually denoted by the Greek letter Phi and the index 2 is for the bivariate case, this method was named as PHI2. Sudret (2008) found that the performance of PHI2 is very sensitive to time increments, and proposed a modified version, PHI2+. Even though PHI2+ uses the analytical gradient solution of the bivariate normal integral relative to time and provides robustness, it can still lead to large errors for small failure probabilities, and in the presence of dependency of outcrossing events. Hu and Du (2013a) proposed the JUR/FORM method to take into account the joint outcrossing events. They showed that the accuracy of JUR/FORM is superior to PHI2+. Gong and Frangopol (2019) presented a new time-dependent reliability method, NEWREL, where the time-dependent reliability is formulated as a large-scale series system consisting of time-independent response functions obtained by discretizing time-dependent continuous response functions within the forecast time period, instead of analyzing outcrossing rates. They found that their proposed method is as efficient as PHI2+, but outperforms PHI2+ in terms of accuracy.

There also exist other approaches than outcrossing rate-based methods for small probability estimation of time-dependent systems. Chen and Li (2007) proposed a new approach for dynamic reliability assessment based on probability density evolution method. A virtual stochastic process is firstly constructed such that the extreme value equals the value of the virtual stochastic process at a certain instant of time. Further, the first-passage reliability problem is investigated from the view of the extreme value distribution instead of the level-crossing process. Therefore, the reliability could be evaluated requiring neither the joint PDF of the response and its velocity, nor the assumptions on properties of the level-crossing events. Hu and Du (2013b) proposed a sampling approach to

Table 6 Comparison of statistics of extreme-based methods based on-dimensionality, multi-modality, non-linearity and computational efficiency

Method	Dimensionality	Multi-modality	Non-linearity	Computational efficiency	Remarks
GPD	●●●●●	●	●	●●●●●	- threshold estimation is required - Not suitable for multi-modal and non-linear limit state function - MLE based parameter estimation doesn't preserve asymptotic properties for all the ranges of shape parameter
MTM	●●●●●	●◐	●●◐	●●●●◐	- suitable for non-linear limit state function - provides the better tail estimates
EGLD-UDR	●●◐	●◐	●●	●●●◐	- very sensitive to the tail probability - not suitable for the non-linear limit state functions
EGLD-SVM	●●◐	●◐	●●◐	●●●◐	- applicable if capacity and response pdf are independent to each other - suitable for non-linear limit state functions
Shifted Generalized Log-Normal Distribution	●●●◐	●●◐	●●●◐	●●◐	- suitable for non-linear random responses - applicable for multi-variate response case as well - extrapolation method is used for parameter estimation
TPNT	●●●◐	●◐	●●●	●●●◐	- suitable for non-linear limit state functions as well - includes independent random variables with unknown CDFs/PDFs - TPNT based on L-moments performs better as compare to C-moments
Log-TPNT	●●◐	●●●◐	●●●◐	●●●◐	- suitable for both non-linear and multi-modal limit state functions - condition for monotonically increasing polynomial increases the complexity - includes independent random variables with unknown CDFs/PDFs
Hermite - based	●●●	●●●◐	●●●●◐	●◐	- computationally costly - many parameter estimation requires - accurate for unknown CDF/PDF with independent and uncorrelated random variables
L-moment based	●●●◐	●◐	●●◐	●●●	- provides stopping rule to estimate threshold - the estimators are computationally efficient - doesn't perform well if shape parameter is larger than 0.5 - only applicable for the shape parameters between 0 and 1
Adaptive inverse NT	●●●◐	●●	●●●●◐	●◐	- only applicable for unimodal and bimodal LSF - computational effort is moderate to high - not suitable for estimating failure probability greater than 10^{-4}
A-PSNT	●●●●●	●●●	●●●●	●◐	- suitable for P_f<$5x10^{-4}$ - sot suitable in recovering the probability density with multiple peaks - suitable for modeling unknown probability distributions

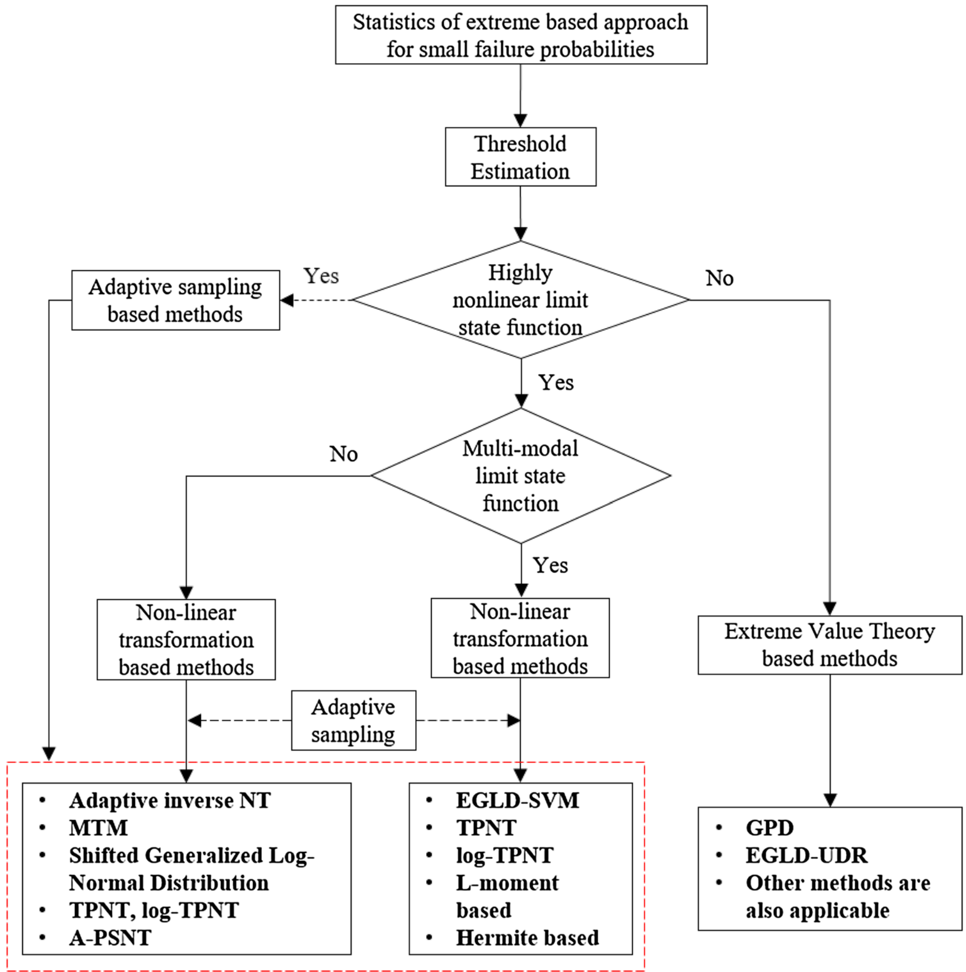

Fig. 10 Guideline for statistics of extreme-based approaches

estimate the distributions of the extreme value of the stochastic process. The extreme value is then used to replace the corresponding stochastic process. Consequently, the time-dependent reliability analysis is converted into its time-invariant counterpart. Finally, FORM is applied to calculate the probability of failure over a given period of time. Yang et al. (2017) proposed a cross-entropy-based adaptive importance sampling method for the efficient computation of time-dependent reliability of deteriorating structures using the stochastic-process-based method. LHS with proper correlation control is used to extend cross-entropy-based importance sampling, a method previously discussed in the earlier section for time-independent reliability problems, for time-dependent reliability analysis as well. They showed that their proposed method led to more efficient solutions for the time-dependent reliability problems related to structural systems with multiple important regions. Lin and Su (2021) proposed an efficient MCS-based approach for dynamic reliability analysis of jacket platforms subjected to random wave loads. To overcome the difficulty involved in the MCS, the explicit time-domain method is used for the required time–history analyses of jacket platforms, in which truncated explicit expressions of critical responses with regards to the contributing loading terms are first established and then used for numerous repeated sample analyses. They showed that the use of the explicit time-domain method greatly enhances the computational efficiency of MCS.

5.2 Surrogate-based methods

Since the time-dependent reliability analysis is more time-consuming than the time-independent reliability analysis, many studies have employed surrogate models to approximate the time-dependent response with respect to random variables and time. To deal with small failure probabilities in time-dependent reliability problems, various adaptive schemes and variance reduction techniques have been combined with surrogate models as in time-independent reliability analysis. The summary of surrogate-based approaches for time-dependent systems is shown in Fig. 11 and Table 7.

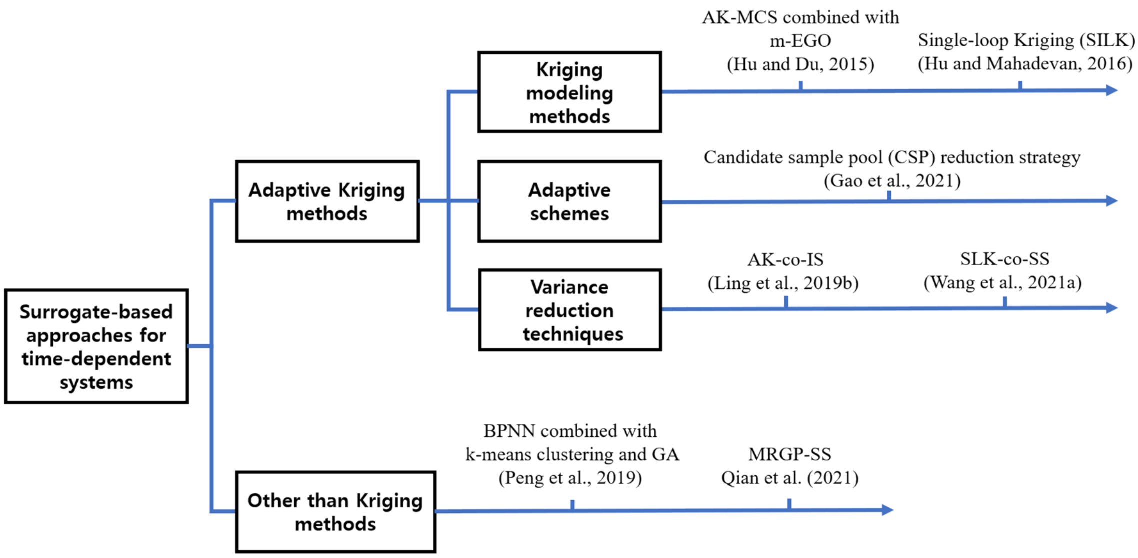

Fig. 11 Summary of surrogate-based approaches for time-dependent systems

Table 7 Summary of surrogate-based approaches for time-dependent systems

References	Method	Summary
Hu and Du (2015)	Mixed efficient global optimization (m-EGO) method	Builds a surrogate model by considering the interaction between random variables and time through simultaneous extraction of samples of random variables and time Combines m-EGO method and AK-MCS
Hu and Mahadevan (2016)	Single-loop Kriging (SILK)	Updates single-loop surrogate model using a learning function based on the properties of the time-dependent reliability problem
Ling et al. (2019b)	AK-co-IS, AK-co-SS	Combines active learning Kriging coupled with IS Combines active learning Kriging coupled with SS
Gao et al. (2021)	Candidate sample pool (CSP) reduction strategy	Removes the samples whose states are correctly recognized by the current Kriging model from the CSP
Wang et al. (2021a)	SLK-co-SS	Single-loop Kriging model is coupled with SS Converts the small failure probability into a product of larger conditional probabilities using time-dependent intermediate failure events
Peng et al. (2019)	BPNN coupled with k-means clustering and GA	Updates the BPNN by identifying DoE points that improve expected improvement
Qian et al. (2021)	Multiple response Gaussian process (MRGP)-SS	Combines MRGP and SS Approximates the extreme value using the best value in current initial samples

6 Application problems

This section provides examples of engineering reliability prediction studies for the rare event probability estimation of stochastic systems. The SMC method has been used in various engineering reliability prediction studies, including probability of failure estimation of aircraft structural components (Acar 2011), probability of failure prediction of an aircraft wingbox (Kaddour and Lord 2012), estimation of probability of first-passage of linear dynamic systems (Norouzi and Nikolaidis 2017), probability of failure evaluation of corroded pipelines (Lee et al. 2013; Seghier et al. 2018), model misspecification in financial engineering (Agarwal et al. 2018). The IS method along with its adaptive sampling and surrogate-based hybrid variants have been utilized in various industrial application type engineering reliability prediction studies, including launch vehicle fallout zone probability estimation (Balesdent et al. 2013, 2016; Chabridon et al. 2018; Derennes et al. 2019; Morio and Balesdent 2016), reliability assessment of radioactive waste repositories (Cadini et al.

2015), fatigue crack initiation of a blade support (Echard et al. 2013), probability estimation of queueing networks (Garvels 2011; Kroese and Rubinstein 2004; Kuruganti and Strickland 1997; Mahdipour and Rahmani 2009a, b; Mahdipour et al. 2014; Sandmann 2004, 2007; Shultes 2002), probability estimation of circuit yield (Hagiwara et al. 2014), failure probability estimation of static random-access memory cells (Kanj et al. 2006; Shahid 2012), failure probability estimation of memory circuits (Shi et al. 2018, 2019, 2020), design of aircraft landing gear (Liu et al. 2020), probability of failure estimation of thermal–hydraulic passive system (Pedroni and Zio 2017), estimation of portfolio credit risk (Qiu and Wang 2015), probability estimation of biochemical systems (Roh 2019), reliability estimation for a passive residual heat removal system (Wang et al. 2015), reliability analysis of power plants (Wang et al. 2011; Wang 2018), reliability estimation of a missile wing (Yang et al. 2021; Yun et al. 2021), reliability estimation of an aero-engine turbine disk (Yun et al. 2020). The SS method and its improved variants has been utilized in various industrial application type engineering reliability prediction studies, including launch vehicle fallout zone probability estimation (Balesdent et al. 2015), strip footing design (Ahmed and Soubra 2014), reliability estimation of the critical temperature of spacecraft components (Au and Thunnissen 2007), estimation of seismic risk (Au and Beck 2003; Xia et al. 2017), reliability estimation of a radioactive waste repository (Cadini et al. 2012), probabilistic fatigue assessment (Du et al. 2021), buckling probability estimation of thin-walled cylindrical launchers (Elegbede and Normand 2012), reliability-based hydraulic transmission mechanism design (Meng et al. 2015), failure probability of a thermal–hydraulic passive system (Pedroni and Zio 2017), reliability estimation of underground pipelines (Tee et al. 2013), reliability estimation of concrete filled steel tubular columns (Thai et al. 2021), structural reliability prediction of a piezoelectric energy harvester (Xiao et al. 2019a), reliability estimation of vehicle–track coupled systems (Zhang et al. 2019), and failure probability estimation of static random-access memory cells (Peng et al. 2020). The LS method was applied to reliability estimation of buried steel pipes subjected to seismic effect (Ebenuwa and Tee 2019).

7 Concluding remarks

Design optimization of structural and multidisciplinary systems under uncertainties involves reliability analysis through the calculation of the probability of failure. Advanced complex technology systems with precise design warrant the estimation of small failure probabilities. Although many studies have proposed approaches that reduce the computational burden in estimating the small probability of failure, guidelines for deciding the appropriate approach to a given problem have not been established. This paper provides a comprehensive review of approaches developed for the estimation of small failure probabilities of structural or multidisciplinary systems. The existing approaches can be classified into three categories: (1) the sampling-based approaches, (2) the surrogate-based approaches, and (3) the statistics of extremes-based approaches. For each approach, the developed methodologies and the published literature are summarized. Then, small failure probability estimation methods tailored for time-dependent systems are reviewed and examples of real-life engineering applications in structural and multidisciplinary design studies for small failure probabilities are explained. The following conclusions and future opportunities can be derived through this review.

1. For some problems, the limit state function may not be available, rather a fixed set of output values may be available. In such a case, neither the sampling-based approaches nor the surrogate-based approaches work. Statistics of extremes-based approaches can be used to estimate small probabilities. If this fixed set of output values are also accompanied with a fixed set of input variables, then the surrogate-based approaches can also be used. If the problem is high-dimensional, then statistics of extremes based approaches are preferable over surrogate-based approaches.
2. For high-dimensional problems ($d \geq 20$) with available limit-state function, the SS and LS methods are well-suited. For medium-dimensional ($10 \leq d < 20$) problems, along with the SS and LS methods, DS method is also suitable. For low-dimensional problems, the IS method is also suitable along with the other methods.
3. For multimodal problems, the LS method is not preferable since it is used together with the FORM/SORM methods, which suffer from multi-modality. Other methods are suitable.
4. For highly nonlinear problems, the surrogate-based approaches as well as the LS and DS methods are not preferable. Other methods are suitable.
5. Sampling-based approaches are often used to estimate the probability of failure since no analytical calculation of multi-dimensional integration is required. In estimating small failure probabilities, variance-reduced simulation methods and other efficient simulation techniques have been developed to reduce the com-

putational burden instead of CMC requiring excessive number of simulations.

6. Since variance-reduced simulation methods still require a large number of simulations to estimate small failure probabilities, surrogate-based approaches have been employed to replace computationally expensive simulations with approximated models. Adaptive Kriging methods that update the Kriging model with various learning functions and strategies accurately estimate small failure probabilities with a small number of simulations.
7. Adaptive Kriging methods combined with various variance reduction techniques reduce the large candidate sample pool size required for estimation of small failure probabilities. Methods combined with IS update both importance sampling density and Kriging model, and methods combined with SS update the Kriging model iteratively based on samples in intermediate levels of SS.
8. Future studies need to focus on cases where samples with various fidelities are used to build or update Kriging model. In actual applications, multi-fidelity data can often be provided, and methods that integrate multiple Kriging models with different fidelities need to be developed in order to increase the accuracy of the estimation of small failure probabilities. In addition, it is expected that the accuracy and efficiency of estimation can be improved by considering the fidelity of the data in determining the location of the next sample point required to update the Kriging model.
9. As the computational power increases, surrogate-based approaches assisted with machine learning techniques have been developed to deal with small failure probabilities. Among the categories of machine learning, supervised learning is commonly used to estimate the probability of failure, and two major subcategories—regression and classification—constitute supervised learning. Regression-based methods employ NN and SVR, and classification-based methods use SVM.
10. In machine learning-based methods, there is a need for strategies that sequentially update machine learning models to estimate small failure probabilities efficiently and accurately. Various update strategies and learning functions in adaptive Kriging are expected to help in developing the methods which provide new sample points that can improve the accuracy of machine learning models.
11. Small failure probability estimation can be referred to as extreme value estimation, and various tail modeling techniques for statistical distributions are used to estimate small failure probability.
12. For time-dependent or stochastic systems, sampling-based and surrogate-based methods are used to estimate small failure probabilities. Sampling-based methods employ outcrossing rate methods and efficient sampling approaches, whereas surrogate-based methods combine surrogate models with various adaptive schemes and variance reduction techniques.

Appendix A: Associated equations and references for sampling-based approaches

Weighted sampling method

The weight indices can be computed from:

$$W(i) = \prod_{j=1}^{s} f_j(i), \tag{8}$$

where $f_j(i)$ is the value of the PDF of the jth random variable for the ith sample. Also, an index function is used to distinguish the samples located in the failed region from those in the safe region:

$$I(i) = \begin{cases} 1\, if\ g_i < 0 \\ 0\, if\ g_i \geq 0 \end{cases}, \tag{9}$$

where g_i is the value of the limit-state function for the i^{th} sample. Then, probability of failure is predicted as the sum of the weight indices for the samples located in the failed region divided by the sum of the weight indices for all samples:

$$P_f = \frac{\sum_{i=1}^{N} I(i).W(i)}{\sum_{i=1}^{N} W(i)} \tag{10}$$

Importance sampling method

The probability of failure, P_f, can be formulated as

$$P_f = \int \ldots \int I[G(x) \leq 0] \frac{f_x(x)}{h_v(x)} h_v(x) dx \tag{11}$$

Then, P_f can be estimated by using the samples drawn from the importance sampling function h_V

$$\hat{P}_f^{IS} = \frac{1}{N} \sum_{i=1}^{N} \left\{ I\left[G\left(v_i\right) \leq 0\right] \frac{f_x\left(v_i\right)}{h_v\left(v_i\right)} \right\} \tag{12}$$

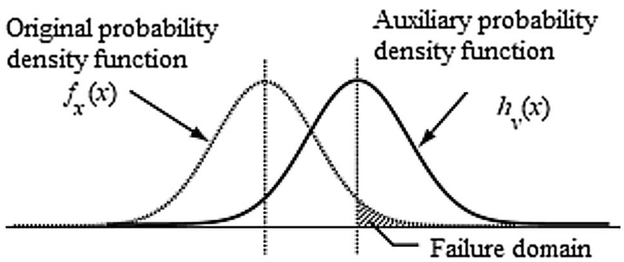

Fig. 12 Concept of importance sampling

Figure 12 shows that the auxiliary PDF $h_V(x)$ allows the sampling of DoE in the tail region of the original limit state function that reduces the sample pool size required for estimation of low failure probability. However, the exact information of auxiliary PDF $h_V(x)$ is needed for the accurate estimation of small failure probability.

Subset simulation method

In the SS method, the probability of failure event (F) is expressed as a product of conditional probabilities of some chosen intermediate failure events ($F_1, F_2, \ldots, F_m$). Letting $F_1 \supset F_2 \supset \ldots \supset F_m = F$ such that $F_k = \bigcap_{i=1}^{k} F_i, k = 1, ..., m$, the probability of a failure event can be estimated as

$$\begin{aligned} P_F = P(F_m) &= P\left(\bigcap_{i=1}^{m} F_i\right) = P\left(F_m | \bigcap_{i=1}^{m-1} F_i\right) P\left(\bigcap_{i=1}^{m-1} F_i\right) \\ &= P(F_m|F_{m-1}) P\left(\bigcap_{i=1}^{m-1} F_i\right) \ldots \\ &= P(F_i) \prod_{i=1}^{m-1} P(F_{i+1}|F_1) \end{aligned} \tag{13}$$

In computing P_F using Eq. (13), the probabilities $P(F_1), \{P(F_{i+1}|F_1) : i = 1, \ldots, m-1\}$ are required. The probability $P(F_1)$ can be estimated from Eq. (14) through MCS as

$$P(F_1) \approx \tilde{P}_1 = \frac{1}{N} \sum_{k=1}^{N} I_{F_1}(\theta_k), \tag{14}$$

where $\{\theta_k : k = 1, \ldots, N\}$ are iid samples simulated according to their probability distribution. The conditional probabilities can be computed using MCS, where the conditional distribution of θ that it lies in F_i is required. This can be achieved by a Markov chain Monte Carlo method based on the Metropolis algorithm (Au and Beck 2001).

Line sampling method

In the standard normal space $\mathbf{U}$ consisting of n independent standard normal random variables, the important unit vector indicating the most probable point (MPP) is denoted as α. By employing the rotation matrix $\mathbf{R}$ in which the first row is α, and $\mathbf{R}^T\mathbf{R} = \mathrm{I}$, the transformed space $\mathbf{V}$ can be defined through the linear mapping $\mathbf{V} = \mathbf{RU}$. The first standard normal random variable V_1 is parallel to α, and the remaining variables $\mathbf{V}_{2:n}$ constitute an $(n-1)$ dimensional vector perpendicular to α. From $\mathbf{V}_{2:n}$, a set of samples $\boldsymbol{\nu}^i_{2:n}$, $i = 1,\ldots,N$ are generated and the failure probability can be estimated as

$$\hat{P}_f^{LS} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{\Phi}(-d^i), \tag{15}$$

where $\mathbf{\Phi}$ is the standard normal cumulative distribution function and d^i is the distance between the rotated limit state function $G_{\mathbf{R}}$ and $\boldsymbol{\nu}^i_{2:n}$.

Directional simulation method

A common way to generate the direction vectors is to generate N sample points according to the joint PDF of standard normal vector $\mathbf{U}$ and normalize all sample points to unit length to obtain, $i = 1,\ldots,N$. The failure probability in polar coordinates is given as (Zhang et al. 2021)

$$P_f = \int_{\mathbf{B}} \left\{ \int_{R > r(\mathbf{b})} f_{R|\mathbf{B}}(r) dr \right\} f_{\mathbf{B}}(\mathbf{b}) d\mathbf{b}, \tag{16}$$

where $\mathbf{B}$ is the unit direction vector uniformly distributed on the unit hypersphere; $f_{R|\mathbf{B}}(r)$ denotes the conditional PDF of polar radius R with the direction $\mathbf{B}$; $r(\mathbf{b})$ is the distance between the origin and the point on the limit state surface at the direction $\mathbf{B}$; and $f_{\mathbf{B}}(\mathbf{b})$ is the uniform distribution on the n-dimensional unit hypersphere. Since $R^2 = \sum_{i=1}^{n} X_i^2$ follows the chi-squared distribution in standard normal space, the failure probability can be estimated as

$$\hat{P}_f^{DS} = \frac{1}{N} \sum_{i=1}^{N} \left(1 - \chi^2\left(r^2\left(\mathbf{b}_i^{DS}\right)\right)\right), \tag{17}$$

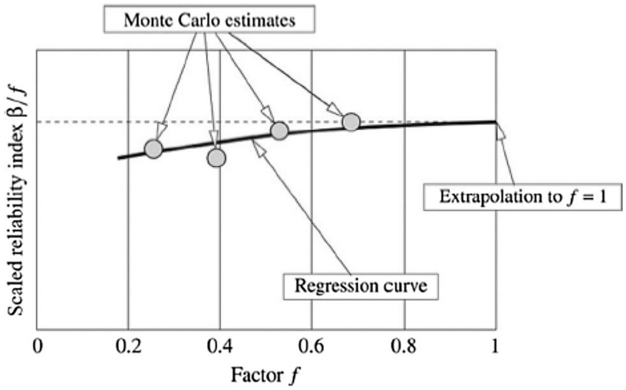

Fig. 13 The concept of asymptotic sampling (Bucher 2009)

where χ^2 is the chi-squared cumulative distribution function.

Asymptotic sampling method

Based on the asymptotic behavior of the reliability index with respect to the standard deviation scale parameter, Bucher (2009) assumed the following functional relationship between the reliability index and the standard deviation scale parameter f

$$\beta = Af + \frac{B}{f} \tag{18}$$

Notice that as $f \to \infty$ (that is, as $\sigma_f \to 0$) the reliability index $\beta \to \infty$ so that the asymptotic behavior is ensured. Coefficients A and B are determined from least squares regression analysis based on the estimates for different values of f smaller than 1. That is, a set of "support points" $[f_i, \beta(f_i)]$ shown in Fig. 13 is used in the regression. To assign equal weights to all support points for the regression analysis, Eq. (18) can be rewritten in terms of a scaled reliability index as follows:

$$\frac{\beta}{f} = A + \frac{B}{f^2} \tag{19}$$

Enhanced simulation method

In the ES method, instead of artificially inflating the standard deviations of the random variables, an artificial limit-state function is formulated by introducing a scaling parameter λ (Naess et al. 2009, 2012)

$$g(\lambda) = g - \mu_g(1 - \lambda), \tag{20}$$

where μ_g is the mean value of the limit-state function g. The behavior of the failure probability with respect to the scaling parameter is assumed to be in the following form:

$$p_f(\lambda) \approx q(\lambda) exp\{-a(\lambda - b)^c\}, \tag{21}$$

where the function $q(\lambda)$ is slowly varying compared with the function $exp\{-a(\lambda - b)^c\}$. This method was applied to reliability analysis of corroding pipelines by Leira et al. (2016).

Appendix B: Reference papers for the original and improved versions of the IS and SS studies

See Tables 8 and 9.

Table 8 The original and improved IS studies

The IS version	Reference studies
Regular IS or IS with optimal sampling distribution	Bhamidi et al. (2015), Blanchet et al. (2011), Cao et al. (2011), Hagiwara et al. (2014), Hassanaly and Raman (2019), Ho et al. (2016), Homem-de-Mello and Rubinstein (2002), Homem-de-Mello (2007), Juneja and Shahabuddin (2001), Kanj et al. (2006), Kroese and Rubinstein (2004), Kuhn et al. (2018), Kuruganti and Strickland (1997), Liu et al. (2015), Mahdipour et al. (2009a, 2009b, 2014), Morio (2010), Qiu and Wang (2015), Qiu et al. (2007, 2008), Radev and Lokshina (2007), Roh (2019), Rubinstein (2006), Sandmann (2004, 2007), Shahid (2012), Shultes (2002), Wei et al. (2012) and Xu et al. (2020)
Adaptive sampling IS	Botev and Kroese et al. (2008); Botev et al. (2013, 2016), Chabridon et al. (2018), Chan et al. (2012), Derennes et al. (2019), El Masri et al. (2021), Garvels (2011), Jacquemart and Morio (2016), Morio and Balesdent (2016), Shi et al. (2018, 2020), Wang (2018) and Zhao et al. (2010, 2011)
Adaptive and surrogate-based IS	Barkhori et al. (2019), Balesdent et al. (2013, 2016), Cadini et al. (2014), Cadini et al. (2015), Chen et al. (2019), Echard et al. (2013), Guo et al. (2020), Li et al. (2011), Ling and Lu (2021), Liu and Elishakoff (2020), Liu et al. (2019), Pedroni and Zio (2017), Razaaly and Condego (2018, 2020), Razaaly et al. (2020), Shi et al. (2019), Xiao et al. (2020), Yang and Cheng (2020), Yang et al. (2018a, 2018b), Yun et al. (2018, 2020), Zhang et al. (2019) and Zhang et al. (2020b)
Hybrid IS	Chen and Li (2017), Pedroni and Zio (2017), Song et al. (2021a), Tong et al. (2015), Wagner et al. (2020), Wang et al. (2011) and Wang et al. (2015)

Table 9 Studies based on the regular and improved version of the SS method

The SS version	Reference studies
Regular SS (including its parallel or system reliability version)	Au and Beck (2001), Au and Beck (2003), Au and Thunnissen (2007), Bréhier et al. (2015, 2016), Cadini et al. (2012), Cérou and Guyader (2007), Ching et al. (2005), Du et al. (2021), Elegbede and Normand (2012), Green (2017), Hsu and Ching (2010), Hua et al. (2015), Huang et al. (2017), Lagnoux and Lezaud (2017), Li et al. (2015), Meng et al. (2015), Santoso et al. (2011), Tee et al. (2013), Thai et al. (2021), Vaisman et al. (2017), van den Eijnden et al. (2017), Wadman et al. (2014), Zaharija et al. (2020), Zhang et al. (2019) and Zio and Pedroni (2008, 2009a, 2009b, 2010a, b)
Surrogate-based SS	Bourinet et al. (2011), Chakraborty and Chowdhury (2017), Huang et al. (2016), Jiang et al. (2021), Ling et al. (2019a), Papadopoulos et al. (2012), Qian et al. (2021), Wang and Shafieezadeh (2021), Wei et al. (2019a, b), Xia et al. (2017), Xiao et al. (2019a) and Xu et al. (2020)
Hybrid SS	Chen and Li (2017), Pedroni and Zio (2017), Rashki (2021), Song et al. (2021a), Tong et al. (2015), Wagner et al. (2020) and Wang et al. (2015)
Adaptive SS	Cheng et al. (2022)

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Replication of results In this review paper, we do not provide any results to replicate.

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