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REVIEW PAPER

Modeling, analysis, and optimization under uncertainties: a review

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Abstract
Design optimization of structural and multidisciplinary systems under uncertainty has been an active area of research due to its evident advantages over deterministic design optimization. In deterministic design optimization, the uncertainties of a structural or multidisciplinary system are taken into account by using safety factors specified in the regulations or design codes. This uncertainty treatment is a subjective and indirect way of dealing with uncertainty. On the other hand, design under uncertainty approaches provide an objective and direct way of dealing with uncertainty. This paper provides a review of the uncertainty treatment practices in design optimization of structural and multidisciplinary systems under uncertainties. To this end, the activities in uncertainty modeling are first reviewed, where theories and methods on uncertainty categorization (or classification), uncertainty handling (or management), and uncertainty characterization are discussed. Second, the tools and techniques developed and used for uncertainty modeling and propagation are discussed under the broad two classes of probabilistic and non-probabilistic approaches. Third, various design optimization methods under uncertainty which incorporate all the techniques covered in uncertainty modeling and analysis are reviewed. In addition to these in-depth reviews on uncertainty modeling, uncertainty analysis, and design optimization under uncertainty, some real-life engineering applications and benchmark test examples are provided in this paper so that readers can develop an appreciation on where and how the discussed techniques can be applied and how to compare them. Finally, concluding remarks are provided, and areas for future research are suggested.

Keywords Uncertainties · Optimization · Modeling · Characterization · Propagation · Analysis · Quantification

1 Introduction

1.1 Motivation of the review

Though uncertainty has been incorporated into many studies on design and optimization of structural and multidisciplinary systems, the current literature lacks a combined review of uncertainty modeling, uncertainty analysis, and design optimization under uncertainty in a single review paper. Hence, this paper aims to (1) provide an in-depth review of uncertainty treatment practices such as uncertainty modeling, uncertainty analysis, and design under uncertainty; (2) suggest areas for future research; and (3) complement existing reviews on similar topics such as reliability analysis by Rackwitz (2001), sampling-based methods for uncertainty and sensitivity analysis by Helton et al. (2006), reliability-based optimization by Valdebenito and Schuëller (2010), uncertainty handling theories by Li et al. (2012), uncertainty representation by Zio and Pedroni (2013), and others. In addition to the in-depth review on uncertainty treatment in engineering fields, practical engineering applications and benchmark test examples are listed in this paper so that readers can easily understand where to apply the topics explained in the paper or how to compare them.

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1.2 Definition of uncertainty

The term uncertainty has different meanings depending on the domain of application. In economics, Knight (1921) used the term uncertainty to describe the unquantifiable knowledge about some possible occurrence, as opposed to the presence of quantifiable risk. In terms of computational modeling and simulation perspective, Oberkampf et al. (2002) used the term total uncertainty to describe potential deficiency in any phase or activity of the modeling process. The US National Research Council's committee on improving risk analysis approaches defined uncertainty as a general concept that reflects our lack of sureness about something or someone, ranging from just short of complete sureness to an almost complete lack of conviction about an outcome (Council et al. 2009). It is seen that the content and intent affect the definition of uncertainty in a particular field.

In this paper, we use the definition provided by Nikolaidis et al. (2004b), where a useful functional definition of uncertainty is given as "information/knowledge gap between what is known and what needs to be known for optimal decisions, with minimal risk." Based on this definition, the uncertainty is classified into two categories: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty refers to the inherent uncertainty due to probabilistic variability, and is also known as statistical uncertainty. This aleatory uncertainty is irreducible and usually characterized by a probability distribution. On the other hand, epistemic uncertainty stems from the lack of knowledge such as inadequate understanding of the underlying processes, incomplete knowledge of the phenomena, or imprecise evaluation of the related characteristics (Alleman 2014). Epistemic uncertainty is also known as systematic uncertainty, and is reducible if we have more information on the system. Epistemic uncertainty can be characterized by methods such as probability bounds analysis, fuzzy logic, or Dempster–Shafer theory. This paper will review uncertainty modeling, uncertainty analysis, and design under uncertainty according to the definition of uncertainty and its categorization explained above.

1.3 Organization of the paper

The remainder of the paper is organized as follows. Section 2 provides a fundamental background of uncertainty treatment—uncertainty modeling, uncertainty analysis, and design under uncertainty—in engineering applications. Section 3 presents a review of uncertainty modeling practices. Uncertainty categorization (or classification), uncertainty handling (or management), and uncertainty characterization activities are reviewed in this section. Characterized uncertain inputs are to be propagated through the analysis models to obtain uncertain outputs. Section 4 provides a review of analysis models, methods, and tools. Probabilistic techniques (e.g., sampling-based, stochastic, analytical, and dimension reduction) to deal with aleatory uncertainty and non-probabilistic techniques (e.g., interval analysis, convexity approaches) to deal with epistemic uncertainty are covered in this section. Output uncertainty information can be used in reliability-based design optimization (RBDO) of engineering systems. The current manuscript predominantly reviews time- and space-independent approaches, though we discuss a few time-dependent literature in Sect. 4.1.5. Section 5 provides a review of RBDO according to the purpose of research, reliability estimation, and type of uncertainty. Analytical approaches using most probable point (MPP) and sampling approaches for RBDO under aleatory uncertainty as well as various approaches to deal with RBDO under both aleatory and epistemic uncertainty induced by insufficient data are covered in this section. Section 6 reviews benchmark examples and application problems in the literature. Finally, Sect. 7 provides some concluding remarks on the current status of uncertainty treatment practices and possible future research directions.

2 Fundamentals

2.1 Uncertainty modeling

In this section, we aim to provide some basic formulations concerning uncertainty modeling, analysis, and optimization. An experienced researcher in uncertainty treatment might choose to skip this section. Commonly used uncertainty handling theories are probability theory (the most used), evidence theory, fuzzy set theory, possibility theory, interval analysis, info-gap decision theory, and hybrid approaches.

In probability theory, first a sample space Ω is defined that relates to the set of all possible outcomes. For each element $x \in \Omega$, the probability function $f(x)$ is associated, which satisfies

$$f(x) \in [0, 1] \text{ for all } x \in \Omega; \text{ and } \sum_{x \in \Omega} f(x) = 1 \tag{1}$$

Each subset of Ω is called an event E, and the probability of an event is defined as

$$\Pr(E) = \sum_{x \in E} f(x) \tag{2}$$

In evidence theory, first a space of mass is defined as $m : 2^X \rightarrow [0, 1]$, where X is the universal set including all possible states, and 2^X is the set of all the subsets of X. The mass function is also called *basic belief assignment*. For a subset $S \in 2^X$, $m(S)$ is derived from the evidence that supports S:

$$\sum_{S \in 2^X} m(S) = 1 \tag{3}$$

Evidence from different sources are combined to arrive at a degree of belief. Belief is the summation of all the evidence that fully supports S, and plausibility is the summation of all the evidence that partly or fully supports S. That is,

$$\text{belief}(S) = \sum_{T \subseteq S} m(T), \quad \text{plausibility}(S) = \sum_{T \cap S \neq \phi} m(T) \tag{4}$$

The probability of a set $S \in 2^X$ falls into the range of $[\text{belief}(S), \text{plausibility}(S)]$. In fuzzy set theory, the notion of a regular crisp set is extended by introducing a membership function. In this theory, there is a gradual rather than sharp transition between non-membership and full membership. For each element $x \in \Omega$, the membership function $\mu_A : 2^X \to [0, 1]$ depicts the degree of membership. The membership function μ_A and the $set(A)$ constitutes a fuzzy set.

In possibility theory, two measures are attached to one event, namely a 'necessity measure' and a 'possibility measure.' Both are membership functions that can take values between 0 and 1. If an event A is completely necessary, its necessity measure is one ($N(A) = 1$), and the possibility measure of its complement event is zero ($\Pi(\overline{A}) = 0$). Similarly, if an event is completely possible, its possibility measure is one ($\Pi(A) = 1$), and necessity measure of its complement event is zero ($N(\overline{A}) = 0$). To characterize the uncertainty of event A, both of these measures are needed. The necessity degree describes the indications supporting the event, and 1 minus the possibility degree describes the indications weighing against it.

The interval analysis aims at placing upper and lower bounds for the range of a function defined in terms of uncertain variables. Real intervals are typically used as given below, where $a = -\infty$, and $b = +\infty$ are allowed.

$$[a, b] = \{x \in \mathbb{R} | a \leq x \leq b\} \tag{5}$$

In info-gap decision theory, uncertainty level α of a parameter x is modeled by using the envelope model as

$$U(\alpha, \tilde{x}) = \left| \frac{x - \tilde{x}}{\tilde{x}} \right| \leq \alpha \tag{6}$$

where $\tilde{x}$ is the point estimate of x, and $U(\alpha, \tilde{x})$ is the set of all values of x whose deviation from $\tilde{x}$ will never be more than $\alpha\tilde{x}$. The decision maker does not know the values of x and α. Two decision concepts are used, namely robustness and opportuneness. The robustness strategy satisfices the outcome and maximizes the immunity to error, and this strategy is different from outcome optimization. The opportuneness strategy, on the other hand, seeks windfalls at minimal uncertainty.

2.2 Uncertainty analysis

In the context of structural design, one usually solves different variants of the following equation:

$$y = f(\mathbf{x}) \tag{7}$$

where $\mathbf{x}$ is the vector of input variables which are mostly independent variables. However, if input variables are dependent, they can be converted to independent variables through Rosenblatt transformation. y is the output or response and f is the function that relates $\mathbf{x}$ and y. In real life, most of these inputs are uncertain and it is imperative to design against these uncertainties. As a first step, the uncertainties in $\mathbf{x}$ are handled based on the class of uncertainty as discussed earlier. The characterized uncertainties are then propagated through f. f oftentimes is not available explicitly and researchers typically use approximations such as surrogates or metamodels to obtain an emulator for f. Establishing f requires an $\mathbf{x}$ and the corresponding y and typically follows the form:

$$y = \mathbf{w}^T \mathbf{x} + v \tag{8}$$

The weights $\mathbf{w}$ are essentially the coefficients and one finds them using an optimization formulation on least square or maximum likelihood approaches, while v is a constant. Though the regression itself is linear, complex functions can be approximated by using different basis functions such as radial basis and Gaussian process. Obtaining y is usually an expensive process and requires a physical experimental set up or a computational analysis model. Once the surrogate is built and uncertainty is propagated, the resultant y is also random which calls for uncertainty handling approaches such as reliability or robustness-based design to account for the uncertainty in y while making decisions based on the response.

Depending on the amount and type of data available, the approaches used to address the uncertainties are either probabilistic or non-probabilistic in nature. The amount of data that one can seek and the type of data influence the choice of design of experiment which is the preceding step to model building and analysis. Majority of the approaches that possess an underlying probabilistic essence fall under one or a combination of the following: sampling, stochastic, analytical, and dimension reduction techniques. Sampling-based approaches are usually variants of the Monte Carlo simulation (MCS) where each realization is propagated through the model. The model itself is usually a surrogate that can be built on one shot design of experiment, adaptively refined, ensembled, or be multi-fidelity. Once the model is available, crude MCS can be used to estimate probability of an event (E) as follows:

$$\Pr(E) \simeq \frac{1}{N}\sum_{j=1}^{N} I[G(\hat{x}_j \leq 0)] \tag{9}$$

where N is the number of samples, I is an indicator function which takes a value of 1 if [.] is true and 0 if [.] is false. G is the limit-state function and x_j is the jth random realization from PDF $f_x()$. Since evaluating I is usually expensive, one would like to keep the evaluation count as low as possible. Several surrogate models such as noise-free Gaussian process (GP), which is also called Kriging, radial basis function (RBF), etc. are widely used. Despite the use of the surrogate model, more efficient samplings than the crude MCS are necessary such as importance sampling (IS), line sampling, and subset simulation. In addition, various sequential sampling methods such as efficient global reliability analysis and local adaptive sampling have been proposed to further improve the efficiency of the surrogate modeling.

Since estimating small failure probabilities requires large samples, techniques such as IS, subset simulation, and adaptive sampling techniques that progressively place samples in the regions of interest are developed and used. Polynomial chaos expansion presented in (10) is one of the widely used stochastic approaches to propagate the uncertainties.

$$y \approx \sum_{i=1}^{p} C_i\varphi_i(x) = \sum_{i=1}^{p} C_i\Pi_{j=1}^{d} P_j(x_j) \tag{10}$$

where p is the number of coefficients, C_i is the expansion coefficients, φ is the multivariate polynomial that is obtained as a product of d-univariate orthogonal polynomials P_j which allows use of powerful statistical properties. There are also developments in stochastic collocation type of approaches. Popular analytical approaches include first-order reliability method (FORM), second-order reliability method (SORM), and dimension reduction method (DRM) according to how a limit-state function is approximated. Major inroads were made in contributing to the reliability estimates such as probabilistic performance measure, probabilistic sufficiency factor, and percentile measure. These inverse measures operate in performance space and aid in stable convergence. When the dimensions are large, even surrogates suffer from the curse of dimensionality. Hence, dimension reduction approaches are preferred. The underlying idea is to combine the dimensions in a linear sense along a dimension of larger variation. While approaches such as principal component analysis were used in the past, recent techniques such as active subspace are being widely adopted. The primary idea in active subspace is to rotate the coordinates such that the directions of the strongest variation are aligned with the rotated coordinates and the model built and analysis performed along the most important rotated coordinates. P-box based approaches are used to obtain conservative estimates when scarce samples are available and probabilistic approaches are required.

When the data available are scarce and usually in terms of bounds or intervals, non-probabilistic approaches are widely used. When the available information is in the form of interval variables, interval approaches are used to model the uncertainties and perform optimization. Interval approaches are usually conservative but better than deterministic designs. Convex models combine concepts of interval and convexity concepts to develop uncertainty representation. The idea of convexity approaches is to bound the uncertain domain using different geometric shapes and use their properties for quantifying uncertainties.

2.3 Design optimization under uncertainties

RBDO is to find a reliable optimum while satisfying probabilistic constraints. It has gained wide popularity in engineering applications through accurate reliability analysis under various uncertainties. Depending on the uncertainty it deals with, design optimization under uncertainty is classified into two categories: (1) design optimization under aleatory uncertainty also called RBDO and (2) design optimization under both aleatory and epistemic uncertainties. RBDO under aleatory uncertainty basically assumes that input statistical models and limit-state functions are fully known and perfectly accurate. On the other hand, when epistemic uncertainty caused by lack of knowledge and insufficient data is involved in the design optimization, it is extremely difficult to know exact input statistical models and output simulation models in the real world.

In general, RBDO is formulated as

$$\begin{aligned} \min_{\mathbf{d},\mu_{\mathbf{X}}} \quad & \mathrm{cost}(\mathbf{d}, \mu_{\mathbf{X}}, \mu_{\mathbf{P}}) \\ \text{subject to} \quad & \Pr(G(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leq 0) \geq \mathrm{Re}_{\mathrm{target}} \\ \text{where} \quad & \Pr(G(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leq 0) \equiv \int_{G(\mathbf{d},\mathbf{X},\mathbf{P})\leq 0} f_{\mathbf{x},\mathbf{p}}(\mathbf{X}, \mathbf{P}) d\mathbf{x} d\mathbf{p} \end{aligned} \tag{11}$$

where G is the limit-state function, $\mathbf{d}$ is deterministic design variable, $\mathbf{X}$ is the random variable, $\mathbf{P}$ is the random parameter, and $\mathrm{Re}_{\mathrm{target}}$ is the target reliability. The probability measure Pr in (11) is equivalent to a multidimensional integration where $f_{\mathbf{x},\mathbf{p}}(\mathbf{X}, \mathbf{P})$ is a joint probability density function (PDF) of random variables and parameters

2.3.1 Design optimization under aleatory uncertainty

The multidimensional integration in (11) is difficult or even impossible to accurately compute. Hence, RBDO under aleatory uncertainty approximates either the limit-state function (analytical approach) or the numerical integration (sampling approach). To alleviate the computational cost for the double-loop RBDO in (11), decoupled loop such as sequential optimization and reliability assessment (SORA) and single loop such as single-loop approach and single-loop single vector methods have been proposed where the probabilistic constraints can be transformed to deterministic constraints. To alleviate the computational cost for the sampling approach, efficient and accurate surrogate modeling methods and various sequential sampling methods have been proposed.

2.3.2 Design optimization under both aleatory and epistemic uncertainties

In this study, epistemic uncertainty in design optimization is categorized into input and output model uncertainties. The purpose of design optimization under epistemic uncertainty is not to reduce the epistemic uncertainty but quantify it for reliability estimation. There are two ways of dealing with the epistemic uncertainty in the input statistical model: (1) non-probabilistic approach and (2) probabilistic approach. In the non-probabilistic approach, the typical RBDO formulation in (11) may not be applicable since the input statistical model is not described as PDF. Instead, interval analysis, evidence theory, possibility theory, and fuzzy set theory described using membership functions can be used to describe the epistemic uncertainty. Using these, mixed-variable design optimization where interval and random variables coexist, evidence-based design optimization, and non-probabilistic RBDO have been proposed. On the contrary, the probabilistic approach attempts to describe the incomplete input statistical model using non-parametric kernel density estimation (KDE), the Bayesian approach combined with the bootstrapping method, and parametric distributions with conservative parameter estimation. In this case, confidence-based design optimization utilizes the confidence of reliability to replace the probabilistic constraints in (11).

Output models, such as surrogate models or simulation models, may not perfectly emulate simulation models or experiments, respectively, and it can lead to inaccurate reliability analysis. Thus, design optimization under surrogate model uncertainty has been proposed, which utilizes the prediction variance calculated from a Kriging model to obtain the distribution of reliability. In other words, the uncertainty induced by an inaccurate surrogate model is taken into account to prevent the overestimation of reliability. On the other hand, the simulation model always has discrepancy with experimental results. It is necessary to calibrate the simulation model to estimate unknown model parameters and discrepancy and validate it. Thus, model calibration and validation methods such as Bayesian model calibration and optimization-based model calibration have been studied. The seminal work, called KOH framework (Kennedy and O'Hagan 2001; Roy and Oberkampf 2011), described various uncertainties in simulation model and gives a Bayesian approach to deal with the uncertainties, especially for model calibration, and thus it may be helpful to understand the model uncertainties. Even though the simulation model is calibrated and validated, simulation model uncertainty cannot be perfectly eliminated since experimental data may be limited in the real world. Thus, the simulation model uncertainty has to be quantified and taken into account in design optimization.

3 Uncertainty modeling

Uncertainty modeling constitutes uncertainty categorization (or classification), uncertainty handling (or management), and uncertainty characterization (Fig. 1). Uncertainty categorization refers to the classification of uncertainty into different categories. This classification is an important step before uncertainty handling and quantification because specific methods on uncertainty handling and quantification are suitable for specific class of uncertainty. Uncertainty handling refers to the management of uncertainties by various theories such as probability theory, evidence theory, fuzzy set theory, possibility theory, interval analysis, and info-gap decision theory. Uncertainty characterization refers to statistical description of input uncertainties. It includes distribution fitting (e.g., PDF, membership function), parameter estimation (e.g., mean value, standard deviation, quantile), and correlation modeling of the input uncertainties. Note that uncertainty characterization is the first stage of uncertainty quantification (UQ) that also involves uncertainty analysis and propagation, which will be covered in Sect. 4.

3.1 Uncertainty categorization (or classification)

Uncertainty is usually classified into aleatory uncertainty and epistemic uncertainty (Hoffman and Hammonds 1994; Rowe 1994; Hora 1996; Ferson and Ginzburg 1996; Paté-Cornell 1996; Ferson et al. 2004; Acar et al. 2006; Sankararaman and Mahadevan 2011; Li et al. 2012). In this distinction, epistemic uncertainty includes both the non-deterministic behavior due to the lack of knowledge (e.g., mathematical modeling approximations), and also the recognizable deficiency that is not due to lack of knowledge (e.g., computer programming errors).

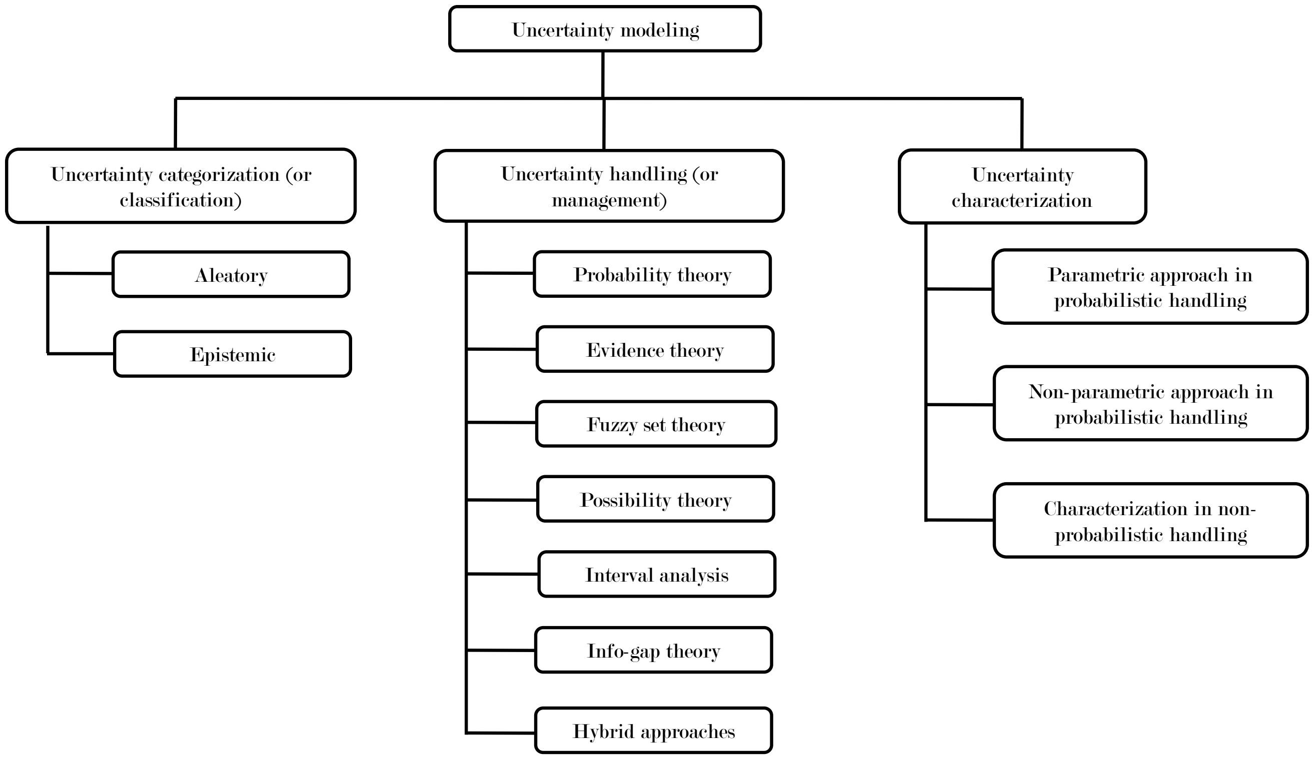

Fig. 1 Uncertainty modeling tree structure

Aleatory uncertainty also referred to in the literature as variability, stochastic uncertainty, inherent uncertainty, and irreducible uncertainty is recognized as the inherent randomness originating from the natural variability of the physical system. Aleatory uncertainty cannot be eliminated or reduced by collecting more information or gathering more knowledge. Epistemic uncertainty also referred to in the literature as subjective uncertainty, informative uncertainty, and reducible uncertainty is recognized as non-deterministic behavior due to lack of knowledge of the physical system along with the ability of modeling and measuring the physical system. Unlike aleatoric uncertainty, epistemic uncertainty can be reduced through quality control (Acar et al. 2007), structural testing (Acar et al. 2010), non-destructive inspection (Kale and Haftka 2008), and sometimes can even be eliminated.

There also exist studies that use more than two classes for uncertainty categorization. For instance, Oberkampf et al. (2002) classified uncertainty as variability, uncertainty, and error. In that classification, variability describes the inherent variation associated with the physical system under consideration. Uncertainty is defined as a potential deficiency in any phase or activity of the modeling process that is due to lack of knowledge. Error is defined as a recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge.

3.2 Uncertainty handling (or management)

The main theories used for uncertainty handling (or management) can be considered as the following: (1) probability theory, (2) evidence theory, (3) fuzzy set theory, (4) possibility theory, (5) interval analysis, (6) info-gap decision theory, and (7) hybrid approaches. The main difference among these approaches relate to the techniques used for describing the uncertainty in input parameters. The origins of different approaches are presented in this section, and advances on these approaches are discussed in the next section.

3.2.1 Probability theory

Probability theory is the oldest and the most widely used uncertainty handling theory. Parsons and Hunter (1998) note that this theory dates back to several hundred years and it is difficult to state where the definitive account may be found. The earliest known forms of probability and statistics were developed by Arab mathematicians studying cryptography in the eight century according to Broemeling (2011). The classical interpretation is known to be completed by Laplace (1812) according to Hájek (2019), in probability theory, it is assumed that the input parameters are random variables with a known PDF or cumulative distribution function.

In structural mechanics, geometric parameters, loading, and material properties are random and are usually represented by PDF or cumulative distribution function. Most manufacturing operations produce normally distributed dimensions if they are performed in a controlled manner (Rao 1992). The dead loads (e.g., gravity load) are often represented with normal distribution (with 10 % coefficient of variation), and the live loads are usually modeled with extreme type I distribution (Ellingwood 1980). It is advised that the type-II extreme value (or Frechet) distribution could be used to describe the yearly maximum wind velocity at any location (Thom 1960).

Hess et al. (2002) analyzed the data bank generated by Kaufman and Prager (1990) by using the computer program BestFit® (1995) to explore which PDF were most representative of the sample data for the material properties of steel used in marine applications. They found that lognormal, Weibull, and extreme value distributions were good choices for describing the yield strength of steel, whereas Weibull and normal distributions well represented the ultimate strength as well as the elastic modulus of steels. There also exists studies on probabilistic modeling of fatigue endurance of steel. Mischke (1987) found that the fatigue life data of steel alloys can be well represented with Weibull distribution and lognormal distribution.

Bayes theorem is based on probability theory (Bayes 1991). It relates two or more events through conditional probabilities to make inferences. It is often used when there is a lack of direct information about an event. It describes the probability of an event, based on prior knowledge and provides a way to update the probability of that event (posterior probability) given new or additional evidence. Jiao and Moan (1990) used Bayesian theory to investigate the effect of proof tests on structural safety. Beck and Katafygiotis (1998) addressed the problem of updating a probabilistic structural model using dynamic test data from structure by using Bayes theorem. Acar et al. (2010) used Bayes theorem to update the failure stress distribution of an aircraft structural element based on results of the element tests. It is also worthy to note that a probabilistic graphical model entitled Bayesian network (Mahadevan et al. 2001; Pearl 2014) is established to symbolize the random variables and their conditional dependencies by using a probabilistic directed acyclic graphical model.

In a probabilistic approach, probability distribution is used to characterize uncertainty. In some cases (e.g., scarce data), it may not be possible to specify the precise values of the input distribution parameters, precise probability distributions, and dependencies between input parameters. In these cases, probabilistic approach is not effective, and other uncertainty handling theories are developed to address some of these limitations.

3.2.2 Evidence theory

The evidence theory was first developed by Dempster (1967) and extended by Shafer (1976). Therefore, this theory is also called the Dempster–Shafer theory. This theory combines evidence from different sources and arrives at a degree of belief by taking into account all the available evidence. In this theory, belief and plausibility are defined as the lower and upper boundary, and a set of belief and plausibility distribution functions is used to describe the input uncertainty.

While handling subjective uncertainty arising from experts, Bayes theorem requires prior and error assumptions, and the obtained results are sensitive to these assumptions. Evidence theory, on the other hand, does not require these assumptions. Soundappan et al. (2004) provides a comparison of Bayesian approach and evidence theory in handling epistemic uncertainty.

3.2.3 Fuzzy set theory

Fuzzy set theory is developed by Zadeh (1965) and it models epistemic uncertainty through fuzzy sets with membership functions. Fuzzy set theory extends the notion of a regular crisp set and expresses classes with vague boundaries such as tall, good, and important. It provides a natural way of dealing with problems in which the source of uncertainty is the absence of sharply defined criteria of class membership rather than the presence of random variables (Zadeh 1973). In this theory, the transition between non-membership and full membership is gradual rather than sharp, and a fuzzy set is represented by stating its membership function, where a degree of membership in the interval [0,1] is given to every element in the set (Zimmermann 2001).

3.2.4 Possibility theory

Possibility theory is based on the notion of fuzzy sets, and it was first presented by Zadeh (1978). It utilizes two measures attached to one event, namely a “necessity measure” and a “possibility measure.” Both are membership functions that can take values between 0 and 1 (Dubois and Prade 1988).

Possibility theory is analogous to probability theory as they both use the [0,1] interval for their measures as the range of their respective functions (Zimmermann 2001). The fundamental difference between probability and possibility is that probability is a measure of the frequency of occurrence of an event, while possibility is used to quantify the meaning of an event (Agarwal and Nayal 2015). Probability distribution functions are required to add to 1, while for possibility distributions the largest values are required to be 1. Therefore, possibility can be seen as an upper probability.

3.2.5 Interval analysis

Roots of interval analysis can be traced back to Moore (1966). It is a branch of the numerical analysis that allows us to compute closed intervals for the exact values of integrals. In interval analysis, the uncertain parameter is denoted by a simple range, and if a preference function is to describe the desirability of using different values in this range, then fuzzy theory can be used (Rao and Berke 1997).

When the uncertainties in structural system are modeled with random variables, the failure probability and reliability index are deterministic values. However, the failure probability (or reliability index) becomes an interval number with the lower and upper bounds if interval variables are included in the structural system (Gao et al. 2011).

Convex method (Elishakoff et al. 1994a), anti-optimization method (Elishakoff et al. 1994b), perturbation method (Chen and Yang 2000), interval finite element method (Muhanna et al. 2005), MCS method (Sim et al. 2007), and affine arithmetic (Degrauwe et al. 2010) have been combined with interval operations to analyze structures with interval parameters.

3.2.6 Info-gap decision theory

Info-gap decision theory is developed by Ben-Haim (2001). It is a non-probabilistic decision theory for prioritizing alternatives and making choices and decisions under deep uncertainty (Ben-Haim 2006). An "info-gap" is the disparity between what is known and what needs to be known for a responsible decision. Info-gap models of uncertainty represent uncertainty in parameters and in the shapes of functional relationships.

Info-gap decision theory offers two decision concepts: robustness and opportuneness. The robustness strategy satisfices the outcome and maximizes the immunity to error, and this strategy is different from outcome optimization. The opportuneness strategy, on the other hand, seeks windfalls at minimal uncertainty. Info-gap decision theory has been used in truss optimization, (Kanno and Takewaki 2006), pipeline reliability improvement (Cicala and Irias 2014), energy hub management of electric vehicles (Soroudi and Keane 2015), wind power uncertainty analysis (Soroudi et al. 2017), airplane landing gear design (Platz and Götz 2017), freshwater management in coastal aquifers (Ranjbar and Mahjouri 2019), etc.

3.2.7 Hybrid approaches

Different theories used for uncertainty handling can be combined to take advantage of the ability of each theory. Ferson and Ginzburg (1996) combined probability theory and interval analysis to produce probability boxes (p-boxes); Tonon et al. (2001) combined probabilistic, fuzzy, and anti-optimization approaches; Guyonnet et al. (2003) combined probabilistic and possibilistic approaches; Jiang et al. (2012a) combined probabilistic and interval analysis approaches; Chutia (2017) combined probabilistic and fuzzy approaches. Jiang et al. (2017) present a literature review on probability-interval hybrid uncertainty analysis for structures with both aleatory and epistemic uncertainties.

3.3 Uncertainty characterization

Uncertainty is characterized by using a probability distribution function (e.g., PDF) when it is handled by probability theory. While using probability theory, a parametric or a non-parametric approach can be used (McFarland and Mahadevan 2008; Kim et al. 2019). Brief details of these approaches are given below, followed by the studies on correlation modeling. Finally, uncertainty characterization in non-probabilistic uncertainty handling approaches is presented in this sub-section.

3.3.1 Parametric approach in probabilistic handling

Parametric approach consists of two steps. In the first step, a proper distribution function is determined (Kang et al. 2019b), which could be performed based on expert knowledge (Soundappan et al. 2004) or by using a model selection method such as Akaike information criterion (Akaike 1974), Bayesian information criterion (Burnham and Anderson 2004), and Bayesian method (Schwarz 1978; Noh et al. 2010). In the second step, the parameters of the distribution function are estimated by using a goodness of fit test such as Anderson–Darling test (Anderson and Darling 1952), Kolmogorov–Smirnov test (Kolmogorov 1933; Smirnoff 1939; Kolmogoroff 1941), or chi-squared test (Ayyub and McCuen 2016). The main downside of the parametric approach is that the use of incorrect distribution function may lead to erroneous results.

In many structural problems, the input random variables are correlated (Annis 2004; Nikolaidis et al. 2004a). However, these variables have often been assumed to be independent because of the difficulty in constructing the joint distribution of correlated input variables (Noh et al. 2010). Even when the correlation is taken into account, usually the joint Gaussian distribution has been used while the correct joint distribution could be non-Gaussian (Nataf 1962; Noh et al. 2009). In that case, copulas (functions that couple multivariate distribution functions to their one-dimensional marginal distribution functions) can be used (Noh et al. 2010). Similarly, intrusive (Paulson et al. 2017) or non-intrusive (Lin et al. 2020) polynomial chaos method can be used to deal with correlated random variables, where the multivariate orthogonal polynomial basis corresponding to the correlated input random variables is constructed by solving the moment-matching equations based on the correlation statistical moments. In some problems, point and interval samples might be available for the estimation of distribution parameters. For those problems, the correlations among interval distribution parameters can be modeled using ellipse models (Xiao et al. 2020).

3.3.2 Non-parametric approach in probabilistic handling

In non-parametric approach, on the other hand, probability distribution is determined directly from the data. Histograms or KDE are widely used in non-parametric approach (McFarland and Mahadevan 2008; Cho et al. (2016c)). The non-parametric approach is recommended over parametric approach if the random variables follow non-parametric distributions or the number of given data is insufficient, even though the true distribution of the data is a parametric distribution (Kang et al. 2017b). However, if the number of data is very small (e.g., less than 10 samples), the non-parametric approach is very sensitive to the quality of the given data and it may lead to erroneous results (Kang et al. 2019a). Non-parametric approach can be combined with an interval analysis to overcome the limitations of the non-parametric approach (Kang et al. 2018).

For the correlation modeling for non-parametric approaches, multivariate non-parametric KDE is often used (Wang and Wang 2015a). The KDE is similar to the use of empirical probability mass function, but each point of the mass function is replaced with a continuous, symmetric distribution centered at that point. The scale parameter of the symmetric distribution (i.e., the bandwidth) has a substantial effect on the performance of the KDE. Ahmad (1982), Wand and Jones (1994), and Duong and Hazelton (2003) used fixed bandwidth, whereas Zhang (2011) and Zougab et al. (2014) used adaptive (or variable) bandwidth in KDE. In univariate case, the bandwidth is a scalar, where it turns into a matrix (e.g., the covariance matrix). The selection of the bandwidth or the covariance matrix can be done through cross-validation (Bowman 1984; Duong and Hazelton (2005)), maximum likelihood (Wang 2007; Konečná and Horová 2019), Bayesian approach (Zhang et al. 2006; Zougab et al. 2014), or method of penalizing functions (Bashtannyk and Hyndman 2001).

3.3.3 Uncertainty characterization in non-probabilistic handling

Uncertainty is characterized by using a membership function when it is handled by fuzzy set theory. Cheng and Chen (1997) determined the membership function such that the corresponding fuzzy event has maximum entropy, based on the fact that a larger entropy of an information system indicates more information contained in the system (Martin and England 1981). Civanlar and Trussell (1986) presented a guideline to construct the membership functions for fuzzy sets whose elements have a defining feature with a known PDF, and showed that their method is capable of generating membership functions in accordance with the possibility–probability consistency principle. Jang (1993) presented an adaptive-network-based fuzzy inference system, a fuzzy inference system implemented in the framework of adaptive networks. By using a hybrid learning procedure, adaptive-network-based fuzzy inference system can construct an input–output mapping based on both human knowledge (in the form of fuzzy if-then rules) and stipulated input–output data pairs. Hong and Lee (1996) proposed a general learning method as a framework for automatically deriving membership functions and fuzzy if-then rules from a set of given training examples to rapidly build a prototype fuzzy expert system. Simon (2002) presented a modified form of gradient descent and Kalman filter methods for optimization of asymmetric triangular membership functions. Determination of fuzzy membership functions through genetic algorithm (Arslan and Kaya 2001), ant colony optimization (Jiang et al. 2008), and particle swarm optimization (Omizegba and Adebayo 2009) was also presented. Hasuike and Katagiri (2016) constructed the appropriate membership function based on size of fuzzy set and mathematical programming. Jalota et al. (2017) constructed membership function for uncertain portfolio parameters by using a credibilistic framework.

4 Uncertainty analysis

In this section, we discuss the uncertainty analysis models, methods, and tools. The quantified uncertainties using uncertainty characterization or modeling techniques that were discussed in the previous section need to be propagated through the analysis models to obtain random responses or outputs. The resulting uncertain output needs to be quantified and is commonly referred to as model uncertainty analysis (Ghanem et al. 2017). Based on system complexities, the analysis models could be simple closed-form analytical functions or computer models such as finite element models, computational fluid dynamic models, or physical experiments. Uncertainty analysis involves accessing these analysis models repeatedly to obtain a random characterization of the output. That is, in a sampling perspective, for each realization of the random input, the analysis model needs to be accessed to obtain the corresponding response. The interest in the output includes finding the moments, estimating tail probabilities, and computing the PDFs towards design under uncertainty such as robust or reliability-based design.

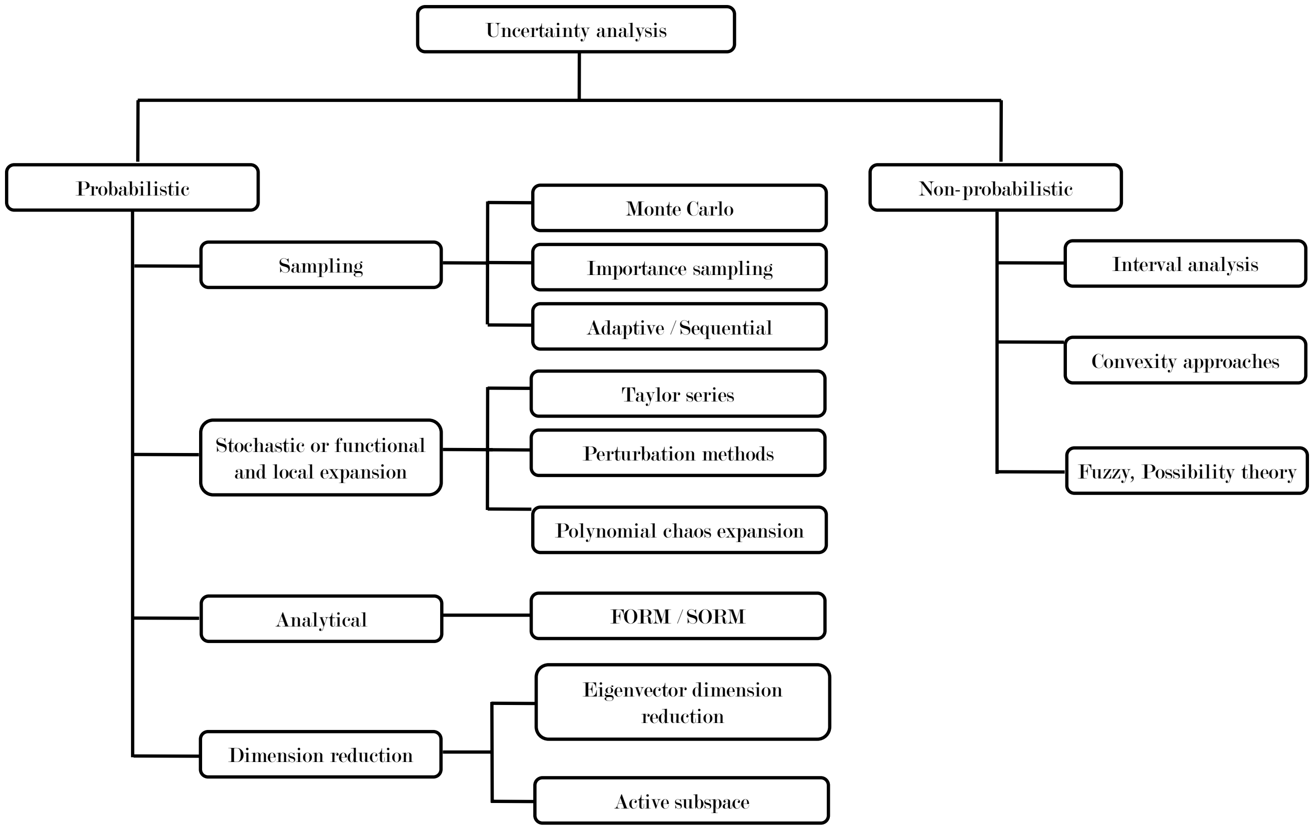

Fig. 2 Uncertainty analysis tree structure

The uncertainty analysis tree structure is presented in Fig. 2. A wide variety of uncertainty analysis approaches can be broadly classified into (1) probabilistic and (2) non-probabilistic techniques. Depending on the availability of probabilistic information, output requirement, and complexity of the analysis model, either of the above techniques can be used. Any of the techniques discussed in the subsections below can be used on the models directly or on metamodels constructed based on design of experiment points.

4.1 Probabilistic techniques

There are different approaches that fall under the probabilistic techniques umbrella (Lee and Chen 2009). Required probabilistic information can be modeled using different approaches as discussed in the previous section. The approaches in probabilistic techniques for uncertainty analysis can be broadly classified into 4 categories as shown in Fig. 2. All categories are discussed in the following subsections.

4.1.1 Sampling-based approaches

Sampling-based approaches predominantly include MCS techniques (Madsen et al. 2006) or its variants such as separable Monte Carlo (Smarslok et al. 2010), IS (Cadini et al. 2015), multilevel Monte Carlo approaches (Giles 2008), and adaptive sampling techniques (Liu et al. 2018b). In the event of knowing the probability information of the inputs, MCS is directly used in the crude form if the analysis model is computationally cheap to evaluate. Oftentimes, the analysis model is replaced by a metamodel (Li et al. 2012). Popular metamodels include polynomials (Hosder et al. 2001), Kriging (Booker et al. 1999), support vector regression (SVR) (Basudhar and Missoum 2008), or their ensembles (Goel et al. 2007; Viana et al. 2009). Recent developments in metamodels for uncertainty treatment include approaches such as deep GP (Radaideh and Kozlowski 2020) and game theory-driven approaches (Chen et al. 2020). Basudhar and Missoum (2008) and Basudhar et al. (2008) applied support vector machine (SVM) for solving limit-states with discontinuous limit-states as well. In large variable problems, beyond the usual screening process to identify influential inputs, Iooss and Le Gratiet (2019) have suggested to model the non-influential inputs as another GP. Using the joint metamodel, the uncertainties are then propagated. Long run times in end-to-end complex designs with uncertainties are avoided by using a combination of grouped sensitivity analysis, expert opinions to certify surrogate models, and verification and validation techniques (Allaire et al. 2014). In the context of multidisciplinary problems, Baudoui et al. (2012) suggest processing the uncertainty locally in each discipline with minor changes to the initial multidisciplinary system. Hoseyni et al. (2014) use a hybrid qualitative and quantitative approach for uncertainty assessment. It is to be noted that there are other reduced order modeling techniques such as projection-based approaches (Benner et al. 2015), Krylov spaces (Freund 2003), and Centroidal voronoi tessellations which are typically used in applications such as computational fluid dynamics, control theory, and dynamics. While literature discussed above employs models of particular fidelity throughout the analysis, there are also techniques that use multi-fidelity approaches. Ng and Willcox (2014) discuss strategies for uncertainty propagation with variable fidelity models. Peherstorfer et al. (2018) discuss several variable fidelity approaches for uncertainty analysis and optimization.

Since failure probabilities are typically small, MCS requires many model evaluations until samples in the tails of the distributions are sampled for an accurate estimate. IS is a variance reduction technique that permits reduction of number of model evaluations by utilizing a targeted sampling strategy. Dubourg et al. (2013) use IS along with metamodels to estimate the augmented probability. They then add a correction term to remove bias in the event of the metamodel not being accurate and demonstrate the approach's efficiency up to 100 variables. Chaudhuri and Haftka (2013) use separable MCS and IS for reliability estimation. Zhang and Taflanidis (2019) use Kriging to approximate the function in both the design variable and random variable space so that UQ and Pareto front can be achieved simultaneously. They combine the metamodel with IS to sample only in the regions of interest and selectively propagate the uncertainties. Peherstorfer et al. (2016) propose a multi-fidelity IS method where they build a surrogate model for the high-fidelity simulations during the construction of the biasing distribution. Then, a high-fidelity model is evaluated at the samples drawn from the biasing IS distribution that allows to estimate small failure probabilities. Papaioannou et al. (2015) propose a sequential importance sampling approach for estimating reliability. Here, the samples from the random variable distributions are translated to samples from an approximately optimal IS density. Sample transition happens through a sequence of intermediate distributions based on a resample-move scheme. This is further used (Papaioannou et al. 2018) for computing the sensitivity of reliability estimates. Cadini et al. (2015) combine metamodels and IS approach with an adaptive sampling scheme to estimate failures as less as 10^{-6} while achieving an order of magnitude reduction in the number of required runtime. A similar approach is proposed by Echard et al. (2013) while using an active Kriging metamodel and by Cadini et al. (2014) for estimating multiple low probability failure regions. Yang et al. (2018) combined active Kriging with IS for low failure probability estimates. Zhang et al. (2020) proposed adaptive Kriging and IS for system reliability analysis.

In addition to IS, other adaptive sampling techniques which usually focus on building accurate metamodels locally are also used for uncertainty analysis. Volpi et al. (2015) develop a dynamic RBF approach where the stochastic kernel is defined by an uncertain tuning parameter. The effect of tuning parameters on prediction are determined by UQ methods. Prediction uncertainty and parallel infill are used for adaptive sampling and effectively reduce the number of high-fidelity simulations. Weinmeister et al. (2018) combine polynomial chaos expansion (PCE), Kriging and adaptive sampling for UQ. While most of the adaptive sampling schemes operate on the idea of prediction variance from the constructed metamodel, Romero et al. (2004) proposed to use progressive lattice sampling to incrementally add samples for accurate metamodels. A similar approach was discussed by Helton and Davis (2003). Metamodel-based adaptive sampling was proposed by Bichon et al. (2008) using efficient global optimization procedure which ensures accuracy of the limit-state. Another suite of algorithms focus on estimating the sensitivities of the probability estimates (Jensen et al. 2009; Valdebenito and Schuëller 2010). Taflanidis and Beck (2008a, 2008b) propose the stochastic subset optimization approach where the subset simulation approach (Au and Beck 2001) is employed to explore both uncertain parameter space and design variable space simultaneously. The fundamental idea behind subset simulation is to express the failure probability as a product of conditional probabilities which are usually larger and can be estimated with lesser samples. Subset simulation and its extensions were extensively used to estimate small failure probabilities (Zuev et al. 2012; Meng et al. 2015a; Papaioannou et al. 2015; Šehić and Karamehmedović 2020). Li and Cao (2016) present Matlab codes for the algorithm. However, there are limitations as well to this approach as pointed by Breitung (2019).

4.1.2 Stochastic or functional expansion and local expansion approaches

Spectral finite element and approaches such as PCE can be classified under this category. PCE is based on representing a random variable by a series of polynomial chaos basis. Chaos here refers to the Gaussian random process. The generalized PCE generates an orthogonal basis based on distribution characteristics of input random variables. Carefully chosen orthogonal basis yields approximate structural response after multiple iterations. PCE can also be viewed as a metamodel because the coefficients need to be computed. Widely used approaches for the coefficient estimation are least squares and projection approaches. However, neither of the approaches escape the curse of dimensionality. In order to reduce the computational effort when the dimensions increase, a series of sparce PCE approaches are proposed (Blatman 2009; Blatman and Sudret 2010a, b; Chen et al. 2018; Xu and Wang 2019). The sparse PCE techniques help in reducing the number of expensive computations. Researchers have combined dimension reduction technique and sparse PCE to emulate the reduced output (Nagel et al. 2020) while sensitivity information is obtained by postprocessing the coefficients. Zhou et al. (2019b) use sparse PCE for sensitivity analysis. Rajabi (2019) compares PCE and GP emulation in the context of ground water applications. They observe that PCE provides better accuracy in moment and tail estimates with less variance across different runs. Researchers develop a metamodel by combining PCE, dimension reduction approach, and information-theoretic entropy (He et al. 2020). A multi-fidelity PC approach is developed by extending the multilevel co-Kriging multi-fidelity modeling framework in the deterministic domain to the stochastic one (Wang et al. 2019a). Zhang et al. (2019) obtain the multimodal input distributions through a GP and propagate them through the response using an integration of the sparse grid numerical method and maximum entropy method. Wu et al. (2017) use a sparse grid collocation technique to build metamodel and a combination of Bayesian analysis and Markov chain Monte Carlo is used to solve the inverse UQ problem. Teckentrup et al. (2015) suggest a multilevel stochastic collocation approach to deal with random inputs. Proper orthogonal decomposition is combined with PCE for efficient UQ of complex acoustic wave problems with large number of output physical variables (El Moçayd et al. 2020). Kumar et al. (2020) use PCE on complex geometrically irregular spatial domains and the surrogate models are constructed using stochastic collocation. The local expansion-based methods include approaches such as Taylor series or perturbation method (Der Kiureghian 1996; Ghanem and Spanos 1991). Such methods suffer under large variability of inputs and non-linearity of performance functions (Lee and Chen 2009). While using the advanced first-order second moment approach to estimate reliability, the suboptimization process to estimate MPP is accelerated by using the Neumann expansion technique (Lee and Kwak 1995) to deal with random state equations.

4.1.3 Analytical approaches

Analytical approaches such as FORM and SORM approximate the reliability estimation and allow for analytical uncertainty analysis. They typically require the limit-state functions and the distribution of input parameters. These approaches are broadly classified as single-loop, double-loop, and decoupled approaches (Valdebenito and Schuëller 2010; Aoues and Chateauneuf 2010; Bichon et al. 2008). The double-loop approach has the optimization outer loop and reliability estimation inner loop. In the inner loop, reliability estimates such as Reliability Index (Tu et al. 2001), probabilistic sufficiency factor (Ramu et al. 2006), probabilistic performance measure (Youn et al. 2003), or percentile performance measure (Du et al. 2004) are estimated. All these approaches focus on estimating the MPP in a forward reliability estimation or inverse reliability approach. Detailed discussions are presented in Valdebenito and Schuëller (2010).

There have been numerous studies on efficient strategies to deal with a non-linear high-dimensional limit-state function for MPP search in RBDO. In the beginning, there were two different approaches for RBDO: reliability index approach (Hasofer and Lind 1974) and performance measure approach (PMA) (Tu et al. 1999). Several developments have been proposed for reliability index approach: Santosh et al. (2006) applied the Armijo rule to optimal step length selection for the Hasofer–Lind and Rackwitz–Fiessler method, and the HLRF–BFGS optimization algorithm was proposed exploiting the BFGS updates to approximate the Hessian matrix since the conventional Hasofer–Lind and Rackwitz–Fiessler algorithm can be treated as the sequential quadratic programming method where the Hessian matrix of Lagrangian is approximated by an identity matrix (Periçaro et al. 2015). In addition, stability transformation method for chaos control (CC) of MPP search using Hasofer–Lind and Rackwitz–Fiessler is proposed by Yang (2010), In PMA, Youn et al. (2003) proposed the hybrid mean value method integrating conventional advanced mean value method with conjugated mean value method to exploit both methods selectively according to whether a limit-state function is convex or concave. In addition, several strategies for efficient RBDO are developed in enriched performance measure approach (PMA+) (Youn et al. 2005a). A hybrid chaos control for PMA is proposed by integrating advanced mean value and modified chaos control procedures to find MPP more efficiently and robustly (Meng et al. 2015b). Modified chaos control further improves the convergence by extending the iterative point of CC method to the constraint boundary (Yang et al. 2020). Keshtegar and Hao (2017) proposed hybrid self-adjusted mean value (SMV) method to enhance the SMV method. Jung et al. (2020b) proposed an intelligent initial point for MPP search in RBDO to improve the efficiency of MPP search utilizing the sensitivity of MPP with respect to design point.

There has been a multiple MPP issue when the limit-state function is highly non-linear. To resolve this issue, Der Kiureghian and Dakessian (1998) firstly developed an algorithm to find multiple MPPs (i.e., multiple design points), and Au et al. (1999) tried to resolve the issue using asymptotic approximation and IS. However, research on multiple MPPs has been still limited so far.

4.1.4 Dimension reduction approaches

When the number of variables is large, DRM is a sought after technique. The underlying idea of most DRM techniques is to translate the function from regular variable space to reduced dimension variable space. The variables in the reduced dimension are usually a combination of the regular variables that contribute the most. Mean-based DRM (Rahman and Xu 2004; Xu and Rahman 2004; Lee et al. 2008a) and MPP-based DRM (Rahman and Wei 2006) were proposed and widely used. Lee et al. (2008b) use the inverse MPP-based approach to enable efficient sensitivity estimation. Eigenvector dimension reduction (EDR) method for probabilistic analysis without sensitivity information was proposed (Youn and Wang 2008; Youn and Xi 2009). In order to overcome the limitations of the univariate approximation (Lee et al. 2008b), several advances have been implemented (Bae and Alyanak 2016; Kang et al. 2017a; Jung et al. 2019a; Park et al. 2020). Researchers combine a DR technique and maximum entropy for efficient uncertainty analysis (Li and Zhang 2011; Chen et al. 2019b). Active subspace (AS) emerged as a popular DRM in the last decade which typically identifies a one-dimensional structure in the map from inputs, irrespective of the dimension of the problem. Constantine et al. (2015) reduce a computationally expensive (9500 CPU hours/run) 7 variable problem to a one variable problem. UQ of satellite conceptual design is performed in Hu et al. (2015). They employ bootstrap to identify confidence intervals of the AS and verify the results with an MCS estimate. Often, metamodels are built in the AS for analysis purposes (Hu et al. 2017; Jiang and Li 2017; Ji et al. 2019). Duong et al. (2019) solve a multi-objective formulation using AS and PCE. Hu et al. (2017) present an alliance algorithm to solve a multidisciplinary optimization where the in-loop UQ is achieved by using AS. Tripathy et al. (2016) proposed a probabilistic version of AS that does not require gradient information and works well in high dimension. Recent advances include a study that combines active learning, active subspace, GP, and MCS (Zhou and Peng 2020); a deep learning-based decomposition of high-dimensional input variables to low-dimensional latent space (Li and Wang 2020); and using it for reliability analysis, a deep learning-based high-dimensional UQ with AS (Tripathy and Bilionis 2018).

4.1.5 Conservative estimates, p-box models, and time-dependent reliability models

When the available sample or probabilistic information is less, conservative estimates are also used by researchers (Picheny et al. 2008; Cho et al. 2016a; Ito et al. 2018). Surrogates are also constructed and used in a conservative estimate perspective (Viana et al. 2010; Zhao et al. 2013). Iooss and Le Gratiet (2019) approximate the functional risk curve using a metamodel and provide the confidence due to the approximation. They use the perturbed-law based sensitivity indices to understand the effect of misjudgment on the sensitivity of the functional risk curve to the input parameter's PDF.

When the distribution type or moment information is known, several approaches are available to develop the distributional p-box model (Zhang et al. 2010a; Oberguggenberger and Fellin 2008; Lee et al. 2016; Wang et al. 2018). When the distribution or moment information are not available, Liu et al. (2019) construct the cumulative distribution function p-box based on the maximum entropy principle. The interval Monte Carlo is developed by combining the interval sampling and interval finite element method for uncertain analysis with p-boxes (Zhang et al. 2010a, b, 2011). Xiao et al. (2016) perform monotonicity analysis on probability transformations of the random variables. This allows capturing the relations between the interval distribution parameters and probability bounds of the structural response which is then used to develop parametrized p-boxes. Liu et al. (2018a) propose an optimized sparse grid numerical integration to calculate the bounds of the statistical moments of the response function and the cumulants which are then used with a saddlepoint approximation to obtain the whole range of probability bounds of the response function. Simon and Bicking (2017) proposed a hybrid approach to model and analyze reliability estimates. They use p-box models, acyclic graphs, and belief functions to account for different types and levels of uncertainty information available. Liu et al. (2018a) use an optimized univariate DRM to compute the bounds of statistical moments which is then utilized to identify the p-box from the family of Johnson distribution.

Analytical and sampling approaches are widely used to solve time-dependent reliability analysis (Hu and Du 2013a, b, 2015). Surrogates are also used extensively in reliability estimates that require time series data (das Chagas Moura et al. 2011; Wang and Wang 2012; Kaymaz and McMahon 2005; Zhang et al. 2017; Wang and Chen 2017; Hawchar et al. 2018; Wang and Matthies 2019; Wang et al. 2019c). Wu et al. (2018) propose to use an inverse UQ under the Bayesian framework which allows capturing the uncertainties in its estimates rather than merely determining the best-fit values. They project the time series data on to principal component subspace and perform the propagation in the reduced space. Recent developments include learning-based concepts such as transfer learning for time-dependent reliability prediction (Zafar and Wang 2020).

4.2 Non-probabilistic techniques

Difficulty in obtaining sufficient data from complex engineering systems leads to the development of non-probabilistic methods. Non-probabilistic approaches, unlike probabilistic techniques, can operate with small sample size to treat uncertainties. The non-probabilistic approaches can be attributed to stem from the argument of lack of credibility in probabilistic analysis results (Annis 2004) when the sample size is small. The choice of opting for a non-probabilistic approach depends on the degree of extraction of information from the available scarce description, need of non-probabilistic information, computational complexity, and expenses (Moens and Vandepitte 2005). The initial studies on non-probabilistic approaches were presented by Ben-Haim and Elishakoff (1995) illustrating the use of non-probabilistic convex models for reliability analysis in structural optimization. Recent years have witnessed a surge in the application of non-probabilistic approaches in the areas of UQ (Gersem et al. 2006; Wang and Matthies 2019), aerospace engineering (Gersem et al. 2006; Wang and Matthies 2019), (Zheng and Qiu 2018), controller design (Wang et al. 2019b), and structural reliability and analysis (Qiu and Wang 2003; Kang et al. 2011; Hao et al. 2017; Guo and Lu 2015; Meng et al. 2016; Jiang et al. 2013a; Meng et al. 2019; Cheng et al. 2020; Luo et al. 2021).

Non-probabilistic approaches can be broadly classified into two categories: (1) interval-based approach and (2) convex model-based approach. Recent advances in interval arithmetic led to the progress of interval-based non-probabilistic approaches. By definition, an interval scalar consists of a single continuous domain in the domain of real numbers $\mathbb{R}$. The range is bounded by a lower and an upper bound. Variables in which bounds can be used to represent uncertainties are modeled using interval analysis approach. Such uncertainties are referred to by the term uncertain-but-bounded variables. Whereas, in the convex model approach, all the possible values of the uncertain parameters are confined to a multidimensional convex set. Convex models are preferred in problems where the uncertain parameters are expected to possess a correlation, as the interval approach assumes that the parameters are uncorrelated. These non-probabilistic methods are predominantly used in the fields of reliability analysis and optimization and UQ. The review explores the different non-probabilistic techniques used to qualify the safety level of structures with inherent uncertainties.

4.2.1 Interval-based approaches

Qiu and Wang (2003) present a numerical interval analysis method for the dynamical response of structures with UBB external excitation and parameters. Taylor series was used to obtain the bounds of the interval dynamical response vector. The interval structure parameter vector was decomposed into the sum of the nominal vector and deviation vector from which the reliability of the dynamic response structure was calculated. In control theory, Wang et al. (2019b) proposed a closed-loop controller design based on the concept of non-probabilistic time-variant reliability-based optimization for structural vibration suppression problems. The boundary rules of the output response are deduced from the state-space transformation, and this transformation is converted to an interval function, and bounds are calculated using Taylor expansion. The optimal control parameters are obtained through the particle swarm algorithm. Luo et al. (2021) proposed the conversion of a probability-based reliability optimization problem into a possibility-based reliability optimization problem where the constraints on the ultimate bearing capacity are mapped to interval functions using a satisfaction degree and the optimization is solved using the method of Simulated Annealing. In line with this idea, Meng et al. (2019) proposed an active learning method with Kriging metamodel to rebuild the limit-state function only by considering the most concerned point for reliability analysis. The importance learning function is used to select the most concerned point, and the Kriging limit-state function is built around the neighborhood of most concerned point and demonstrated using a non-probabilistic approach. When mixed uncertainties are available (Jiang et al. 2012b), parameters with sufficient information were treated as random variables while others with less information were treated as interval variables. When both interval and random variables are present (Yoo and Lee 2014), through a sampling-based approach, the probability of failure can be directly used to obtain the worst combination of interval variables. Parameter intervals of the basic variables in conjunction with conventional reliability theory (Qiu et al. 2008) allow obtaining system failure probability interval. Based on the shape function and interval analysis, Liu et al. (2016) proposed an inverse approach to compute the dynamic load. Interval process model extends the interval method (Li et al. 2019a) for quantifying time varying parameters. Instead of using a probability distribution, an interval parameter is used to characterize the imprecision of the time varying parameter at arbitrary time. Shi and Lu (2019) analyze the safety of dynamic structure by a new dynamic reliability analysis while accounting for both random and interval input parameters by constructing a second-level limit-state function

4.2.2 Convex model-based approach

Convex model-based non-probabilistic approaches for reliability analysis are used under the circumstances where the uncertain parameters are correlated and when interval models result in an over-conservative design. Convexity approaches employ interval analysis (Moore et al. 2009) and several geometries such as ellipsoid, parallelepiped (Jiang et al. 2015), and super ellipsoid (Elishakoff and Bekel 2013) to bound the uncertain region. Ben-Haim and Elishakoff (2013) introduced ellipsoid models to applied mechanics field. Ben-Haim (1994) introduced the first non-probabilistic reliability concept which was based on convex models. Elishakoff and Zingales (2003) studied the differences between probabilistic and non-probabilistic, anti-optimization analyses of uncertainty and concluded that for near-unity reliability range these two approaches tend to each other. Wang et al. (2011) introduced a convex model for structural reliability where ellipse is used to capture the region of uncertainty. They concluded that both probabilistic approach and convex models are compatible, and the latter can be used where probabilistic information is not available. Ellipse-based convex model and interval analysis approach are compared by Qiu and Wang (2005). They observe that if one knows the form of the convex set, convex models should be used, else interval analysis is a more practical approach. Super-ellipsoid model for uncertainty analysis was introduced by Elishakoff and Bekel (2013). Jiang et al. (2015) introduced a non-probabilistic safety measure based on multidimensional parallelepiped model. Here, the uncertainty is characterized by the interval approach with closed bounds. The works of Kang et al. (2011) illustrate a multi-ellipsoid convex model-based non-probabilistic reliability index to treat boundary uncertainties. The proposed method only requires implicit forms of limit-state function in seeking the concerned performance point for solving the structural optimization problem.

4.3 Remarks on uncertainty analysis

Oftentimes, the choice of the approaches is dependent on the computational budget and details available. Based on these features, the approaches are compared in Table 1. This permits the user to choose appropriate approaches based on the type of data that is available.

A table comparing the various techniques under the different approaches discussed, based on features such as algorithmic complexity, ease of implementation, and computational time is discussed in Table 2, along with the remarks for the particular technique. Each shaded circle is a score. More the score, the method scores well in that feature.

5 Design optimization under uncertainties

Uncertainties which may affect performances of engineering systems are ubiquitous in the real world so that design optimization under uncertainties has been developed and exploited in various engineering applications. As shown in Fig. 3, the design optimization under uncertainties is classified in this paper according to the type of uncertainty that it deals with.

When only aleatory uncertainty exists, RBDO is utilized to find a reliable optimum design. Research on RBDO under aleatory uncertainty assumes that there is no epistemic uncertainty, which means that the input statistical model is perfectly known and the simulation model is accurate. Thus, it mainly focuses on efficient and accurate multidimensional integration for reliability analysis and sensitivity analysis for gradient-based design optimization since the simulation model is commonly computationally expensive. RBDO under aleatory uncertainty is classified into analytical approaches and sampling approaches according to how to evaluate the reliability. If a limit-state function is approximated, it mainly utilizes MPP as

Table 1 Remarks on uncertainty analysis approaches

Approaches	Sample/time required	Information needed for limit-state	Model multiple failure modes	Remarks
Sampling	More	Not required	Can be applied directly	Simple implementation but time consuming
Expansion	Less	Not required	Requires algorithm modification	Quick but need to watch out for approximation errors
Analytical	Very less	Required (can be approximated)	Requires complex algorithm and implementation	Distribution information required
Dimension Reduction	Very less	Required (can be approximated)	Requires complex algorithm and implementation	Distribution information required
Non-probabilistic	Less	Not Required	Can handle	In early stages of design, extremely useful to provide ball park numbers on probabilistic quantities

Table 2 Table of comparison between various available uncertainty analysis techniques

	Modelling Complexity	Implementation	Computational Time	Remarks
Sampling approaches				
MCS	●●●	●●●	●	– Accuracy depends upon the number of samples
Importance sampling	●●	●◐	●●◐	– Requires knowledge of variance of the distribution sample – Sampling density choice influences outcome
Adaptive/Sequence Sampling	●●	●◐	●●●	– Requires prior information of the acquired samples – Different strategies available for different class of problem
Expansion approaches				
Taylor series	●●●	●●●	●◐	– Requires explicit form of derivatives of the function – Prone to errors when higher order terms are truncated
Perturbation methods	●●	●◐	●●◐	– Ability to solve highly nonlinear equations – Selection of perturbation variable needs special attention
PCE	●●	●◐	●●●	– Not very efficient for non-normal random input distribution – Under white noise, performance diminishes
Analytical approaches				
FORM/SORM	●●◐	●●	●●◐	– When underlying variables are non-normal, errors are incurred during transformation and handling multiple modes of failure need modification to the base formulation – Cannot handle highly nonlinear LSFs
Dimension reduction approaches				
Eigen dimension reduction	●●	●●	●◐	– Does not require sensitivity of system responses – Permits statistically correlated and non-normally distributed random inputs. However, careful evaluation in case of higher order correlated terms is necessary
Active subspace	●●	●◐	●●	– Very attractive for prohibitively expensive computer models. – Requires gradient information and variation in the non active subspace is not preserved
Non-probabilistic approaches				
Interval analysis	●●●	●●●	●●●	– Suitable when no information other than bounds are available but the approximations are usually conservative or correspond to worst case – Can deal only with independent uncertain variables
Convexity approaches	●●	●●	●●◐	– Suitable when less data but more than the bounds are available – Can deal with dependent uncertain variables
Fuzzy, possibility theory	●●	●◐	●●	– Suitable when subjective information about the process needs to be accounted – Very attractive for exploring designs in the conceptual design stage

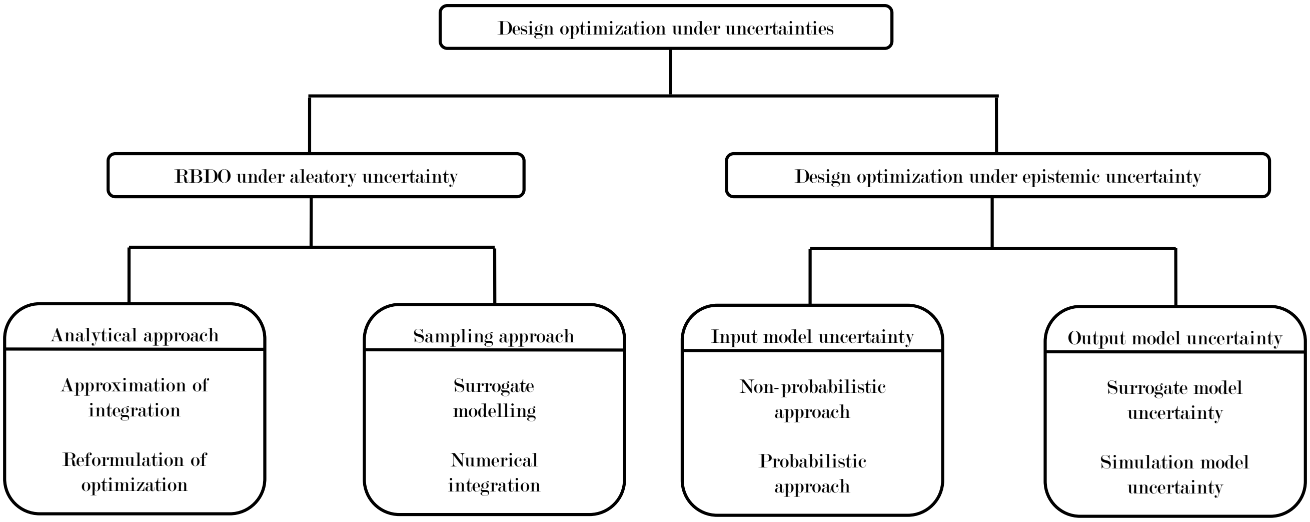

Fig. 3 Design under uncertainty tree structure

a reference point for the approximation. On the other hand, the sampling approach requires repeated simulations for reliability estimation so that a surrogate model is generally utilized to alleviate the significant computational burden. The analytical approaches can be further categorized into various methods depending on (1) how to approximate multidimensional integration, and (2) how to reformulate design optimization. The sampling approaches can be classified into two categories: (1) sampling approaches with surrogate modeling whose main focuses are efficient surrogate modeling and sequential sampling methods, and (2) sampling approaches with efficient numerical integration for reliability and its sensitivity analysis. In addition to RBDO under aleatory uncertainty which mainly focuses on component design, there have been approaches which deal with system design optimization under uncertainties. Hence, the system RBDO methods are briefly reviewed in this section as well.

Design optimization under both aleatory and epistemic uncertainties has concerned lack of knowledge, data, and information. Thus, the epistemic uncertainty is classified into input and output model uncertainty. Since reliability is the probability that response of interest is satisfied under input uncertainty, both input and output uncertainties can affect the reliability analysis and lead to an unreliable optimum. The input model uncertainty means that an accurate input statistical model is unknown but insufficient input data are available. Thus, UQ explained in Sect. 3 is required to handle or manage input uncertainties. There are two approaches to deal with the input model uncertainty: (1) non-probabilistic approaches such as interval analysis, fuzzy set theory, and evidence theory to describe the uncertainty, which are generally known to be effective when input data are extremely scarce, and (2) probabilistic approaches to model the input uncertainty through known PDF with its epistemic uncertainty existing in distribution parameters and types. The output model uncertainty where the epistemic uncertainty comes from inaccurate output responses can be also classified into two: (1) surrogate model uncertainty caused by the limited number of simulation samples for surrogate modeling, and (2) simulation model uncertainty represented as unknown model parameters, discrepancy between experimental results and simulation response, and measurement error.

5.1 Design optimization under aleatory uncertainty

This section explains RBDO research under aleatory uncertainty where input statistical models are assumed to be given. Thus, the multidimensional integration to estimate reliability is the main focus of research. Regardless of approaches, they aim to obtain an accurate optimum satisfying the constraints with the minimum number of computer simulations. For accuracy comparison, the optimum design obtained using MCS is usually used as a benchmark.

5.1.1 Analytical approaches

Analytical approaches are traditional ways to approximate the limit-state function. Motivation of the approaches can be classified into two ways. First, the improvement of integration has been developed. The multidimensional integration can be approximated through moment-based methods and MPP-based approximations such as FORM. Besides, many methods for accurate approximations on multidimensional integration are proposed. Second, reformulation of double-loop RBDO which requires heavy computations has been discussed. To this end, decoupled loop and single-loop RBDOs are proposed to reduce the computational burden.

Approximation of integration Approximation of a limit-state function and its multidimensional integration for reliability analysis affect efficiency and accuracy of RBDO. FORM has been widely used and developed in many different ways based on MPP such as PMA and reliability index approach as mentioned in Sect. 4 (Hasofer and Lind 1974; Tu et al. 1999; Chiralaksanakul and Mahadevan 2005). Lin et al. (2011) proposed a modified reliability index to prevent divergence of FORM using a new definition of reliability index since traditional reliability index fails to find the true MPP when the origin in the U-space is within the failure region. Du and Hu (2012) proposed FORM with truncated random variables. Zhang and Du (2010) tried to improve the accuracy of FORM while maintaining a similar level of efficiency. It can be achieved by univariate DRM with quadratic functions and saddlepoint approximation. Recently, Chen et al. (2019a) integrated SORA with identification of multiple MPPs so that it can be treated in decoupled RBDO.

SORM approximates the limit-state function by a quadratic polynomial function (Breitung 1984). Lee et al. (2012) and Park and Lee (2018) developed the novel SORM to improve the accuracy of SORM through orthogonal transformation and integration using general chi-square distribution, where errors due to approximating the quadratic function by parabolic surface and calculation of reliability are eliminated. Its sensitivity analysis for RBDO is also developed by Yoo et al. (2014). Mansour and Olsson (2014) developed a closed-form expression for reliability by eliminating the rotation of the Hessian matrix. On the other hand, Lim et al. (2014) exploited the symmetric rank-one update to approximate the Hessian matrix using the path of MPP search in approximated SORM to reduce the computations for the Hessian matrix calculation. Huang et al. (2018) estimated a cumulative generating function through quadratic approximation and saddlepoint approximation to compute the reliability analytically.

DRM is developed to approximate the multidimensional integration of a limit-state function as a summation of functions with reduced dimension. The univariate DRM is most widely used in RBDO which shows more accuracy than FORM and efficiency than SORM (Rahman and Xu 2004; Lee et al. 2008b). Youn and Xi (2009) proposed EDR to improve the accuracy of DRM by choosing samples along the eigenvectors to incorporate the statistical correlation. Kang et al. (2017a) developed the so-called HeDRM to reduce the effect of cross-terms of univariate DRM through rotational transformation. Jung et al. (2019a) further tried to reduce necessary number of function evaluations for MPP-based DRM utilizing the history of MPP search similar to approximated SORM, and it shows the same efficiency as FORM while maintaining the accuracy of DRM. Park et al. (2020) developed selective DRM which allocates the integration points using a statistical mode selection method such as Akaike information criterion.

On the other hand, there have been studies on moment-based RBDO which have advantages of not having difficulties of MPP search and multiple MPP problems in reliability analysis (Li and Zhang 2011). Huang and Du (2006) directly estimated cumulative generating function of the response through moments from the dimension reduction and saddlepoint approximation. Kang and Kwak (2009) exploited the maximum entropy principle for PDF modeling and RBDO. Ju and Lee (2008) combined the moment-based RBDO and surrogate model, and Rajan et al. (2020) also developed RBDO using higher-order moments of responses using local surrogate modeling to overcome the limitations in MPP-based methods.

Reformulation of optimization To reduce the computational burden of conventional double-loop RBDO, two approaches have been studied: decoupled loop and single-loop RBDOs. In the decoupled loop RBDO, reliability analysis and design optimization are sequentially performed until convergence. On the other hand, design optimization only is performed in the single-loop RBDO, and reliability analysis is perfectly approximated without any optimization loop.

For the decoupled loop RBDO, Du and Chen (2004) proposed SORA where a double loop is decoupled into reliability analysis and deterministic optimization using equivalent deterministic constraints which are shifted to a feasible direction according to reliability analysis. Hence, its computational efficiency is greatly improved. Zou and Mahadevan (2006) proposed a direct decoupling approach which also decoupled the double loop, but reliability analysis is performed through a sampling method instead of MPP-based approximation. Cho and Lee (2011) proposed the improved SORA where the shifted constraint in the deterministic optimization is replaced by a convex linearized function using the gradient and function value obtained from reliability analysis. Thus, no additional function evaluation is necessary, and it is shown that its convergence is better than SORA. On the other hand, Chen et al. (2013) proposed adaptive decoupling approach to enhance the efficiency of SORA using novel update angle strategy and feasibility-checking method. The update angle strategy can reduce the necessary function evaluations, and the feasibility-checking method enables to assess only violated and active constraints. Huang et al. (2016) proposed the incremental shifting vector utilizing the information of previous shifting vector. Chen et al. (2018) also proposed a probabilistic feasible region approach for RBDO to further enhance the efficiency of SORA by selectively assessing the reliability of each limit-state function. Wang et al. (2020) combined the SORA with moment method through univariate DRM and PDF estimation such as maximum entropy method.

On the other hand, the single-loop approach has been also exploited widely due to its high efficiency compared to conventional RBDOs. Various single-loop RBDOs to eliminate the reliability analysis such as single-loop single vector (Chen et al. 1997; Yang and Gu 2004) and single-loop approach (Liang et al. 2008) have been proposed, where the Karush–Kuhn–Tucker optimality condition is used to replace probabilistic constraints (Mohsine et al. 2006; Agarwal et al. 2007). Shan and Wang (2008) proposed reliable design space to further improve the efficiency of single-loop approach by approximating the gradient vector at MPP. Jeong and Park (2017) proposed single-loop single vector using the conjugate gradient where the convergence of single-loop single vector is improved by MPP estimation through conjugate gradient. Jiang et al. (2017) proposed adaptive hybrid single-loop method that is more suitable for non-linear RBDO sacrificing little efficiency by checking the feasibility of the approximate MPP. Similarly, Li et al. (2019d) proposed a new oscillating judgment criterion and adaptive modified chaos control method to select the MPP search formula and integrate with single-loop approach.

5.1.2 Sampling approaches

Sampling approaches are divided in this section into two: strategies for more accurate and efficient surrogate modeling during RBDO such as a sequential sampling, and various numerical integration methods. The sequential sampling attempts to find the best sampling point to be added in the current sample set to best improve accuracy of a surrogate model. The numerical integration method attempts to compute the reliability during optimization with less number of samples than MCS.

Strategies for accurate and efficient surrogate modeling-Dubourg et al. (2011) built a Kriging model on augmented reliability space and a strategy to sequentially refine the Kriging model, where the reliability is estimated through subset simulation. Bichon et al. (2008, 2013) integrate efficient global reliability analysis into RBDO. Chen et al. (2014, 2015) proposed a local adaptive sampling for RBDO to improve the efficiency of constructing a Kriging model based on constraint boundary sampling (Lee and Jung 2008), and important boundary sampling accounting for objective function, which is integrated with SORA. Similarly, Meng et al. (2018) exploited the adaptive directional boundary sampling accounting for objective function. Li and Cao (2016) proposed a local approximation method using MPP to check the feasibility of constraints and locally refine the surrogate model around MPP. On the other hand, Liu et al. (2017) developed local range RBDO including two phases which are to find the local range based on SVM and construct an accurate Kriging model in the local range. Li et al. (2019c) developed a sequential surrogate model method for RBDO using RBF by sequentially refining the surrogate model in the vicinity of the current design. PCE has been also popularly investigated to be used in RBDO. Hu and Youn (2011) proposed the adaptive-sparse PCE to overcome the curse of dimensionality by automatically detecting significant polynomials and adjusting the PCE order. Zhou and Lu (2019) developed the Bayesian compressed sensing for PCE surrogate model and integrated it with the active learning strategy for RBDO. Zhu and Du (2016) proposed the dependent Kriging prediction to consider the correlations between prediction at realizations of input random variables for MCS, so that a new learning function accounting for variation of reliability is used for sequential sampling. Wang and Wang (2014) developed cumulative confidence level to quantify the accuracy of reliability estimation using a surrogate model, and then a sequential sampling approach for RBDO is adopted based on cumulative confidence level.

Numerical integration methods Au and Beck (2001) and Au (2005) developed the subset simulation for reliability analysis and its sensitivity analysis, and Li and Cao (2016) developed Matlab code for the subset simulation and structural optimization. Lee et al. (2011) proposed stochastic sensitivity analysis of reliability and statistical moments when the input random variables are correlated. The score function is used to derive sensitivities and the input statistical model is described with parametric marginal distribution and copula function. Cho et al. (2016b) developed the sampling-based RBDO when standard deviations vary. Dubourg et al. (2013) proposed the metamodel-based IS using quasi-optimal instrumental PDF using probabilistic classification functions defined by a Kriging model. Zhu et al. (2015) proposed a new sampling-based RBDO via score function with a reweighting scheme so that its computational efficiency is improved. Recently, Chaudhuri et al. (2020) proposed IS reusing information accumulated from past iterations, which is exploited in RBDO to reduce the computational burden.

5.1.3 System RBDO

Component RBDOs discussed so far find an optimum where multiple constraints are independently satisfied, whereas system RBDO finds an optimum with a single system constraint aggregating all probabilistic constraints. Thus, an effective approach to obtain the system reliability is the key idea of system RBDO studies. Ba-Abbad et al. (2006) proposed a modified approach for RBDO of series systems adapting SORA where the optimizer distributes the reliability of the system over its components while constraining the system reliability only. Liang et al. (2007) proposed a single-loop system RBDO that enables to distribute each target reliability of failure mode, so that only system reliability has to be assigned. McDonald and Mahadevan (2008) also developed the equivalent formulation including both component and system reliability constraints as single-loop RBDO. Song and Kang (2009) and Nguyen et al. (2010) proposed matrix-based system reliability method and system RBDO using matrix-based system reliability, so that it can account for statistical dependence between component events. Lee et al. (2010) exploited the MPP-based DRM to calculate the component reliability and integrate with Ditlevsen's second-order upper bound according to convexity of limit-state functions. Wang and Wang (2015b) developed the integrated PMA exploiting GP model to accurately estimate system reliability. Xiao et al. (2020) developed a system active learning Kriging for system RBDO based on expiration risk function to refine the Kriging model iteratively.

5.1.4 Robust design optimization (RDO)

Robust design optimization (RDO) attempts to improve the product quality by minimizing the variability of an output response propagated from input uncertainty. In other words, the optimal design gives a high degree of robustness that is relatively insensitive to input uncertainties, and several studies have widely investigated RDO frameworks to systematically organize the existing studies (Zang et al. 2005; Beyer and Sendhoff 2007; Schuëller and Jensen 2008; Chatterjee et al. 2019). Specifically, RDO concentrates on estimating the first two statistical moments of the output response and their sensitivities (Sandgren and Cameron 2002). Thus, there exist conceptual differences between RBDO and RDO. RBDO usually treats constraints of catastrophic failure in rare extreme events where the cost function to be minimized is generally deterministic, while RDO emphasizes the response sensitivity with respect to the input variations or allows for the maximum possible system variability. The efficient and accurate estimation of the statistical moments is the key to RDO research. Numerical integration such as MCS is the most intuitive way but requires a heavy computational burden, and thus the surrogate model to replace the simulation model can resolve it. Chakraborty et al. (2017) proposed RDO using polynomial correlated function expansion called high-dimensional model representation. Coppitters et al. (2019) used PCE to emulate the physical model for RDO of the photovoltaic–electrolyzer system. Recently, Chatterjee et al. (2019) presented an extensive survey to illustrate the performance of surrogate models in RDO, Keane and Voutchkov (2020) proposed a combined Co-kriging model for RDO. On the other hand, many approximation methods on integration have been proposed to reduce the number of function evaluations. For instance, the simple Taylor expansion (Lee and Park 2001; McAllister and Simpson 2003) and perturbation methods (Doltsinis and Kang 2004) are used for the moment estimation, Xu and Rahman (2004) proposed the univariate DRM for multidimensional integration, and Youn et al. (2005b) proposed the performance moment integration. In practical, reliability-based robust design optimization (RBRDO) rather than RDO is much effective in considering two objectives, which are reliability of constraints and robustness of product quality (Lee et al. 2008a; Youn and Xi 2009; Motta and Afonso 2016). The multi-objective cost function is one of the key issues to treat combined robustness measures. Yadav et al. (2010) proposed a multi-objective framework by addressing various quality losses simultaneously, Sun et al. (2011) showed multi-objective RDO on crashworthiness design of the vehicle to generate Pareto solutions, and Shahraki and Noorossana (2014) presented RBRDO using an evolutionary multi-objective genetic algorithm. Meanwhile, epistemic uncertainty induced by lack of knowledge also has been treated in RDO. Tang et al. (2012) developed RBRDO accounting for both reliability and robustness indices under epistemic uncertainty represented by info-gap theory. Kang and Bai (2013) proposed a new robustness measure and RDO based on a convex model for uncertain-but-bounded parameters. Zaman and Mahadevan (2013) developed RDO for multidisciplinary systems accounting for both aleatory and epistemic uncertainties, and Li et al. (2020) recently treated both parameter and model uncertainty in multidisciplinary RDO. In engineering application, Ghisu et al. (2011) presented the RDO of the gas turbine system, and Fang et al. (2015) performed RDO for fatigue life to design truck cap, and Lee et al. (2020b) exploited RDO to the thermoelectric generator system. Especially in the composite structure, das Neves Carneiro and António (2019) presented the RBRDO of angle-ply composite laminate structure accounting for both weight and determinant of covariance of response. Zhou et al. (2019a) exploited RDO to variable angle tow composite structures under the material and applied load uncertainties, and several studies on RDO of composite structures can be found (António and Hoffbauer 2009; Bacarreza et al. 2015).

5.2 Design optimization under both aleatory and epistemic uncertainties

All RBDO studies introduced in the previous section assume that statistical models of input random variables are given, and a simulation model gives exact responses. However, it is very difficult to know the statistical models of all input random variables in practical engineering applications, and the simulation model always shows discrepancy compared with the experimental results. Thus, estimation of the input statistical model needs to be performed using collected data as discussed in Sect. 3, and the estimated statistical model could be inaccurate when insufficient data are used. Also, inaccuracy of a surrogate model emulating the simulation model can be epistemic uncertainty in RBDO. In this section, studies to find an RBDO optimum accounting for epistemic uncertainties such as input model uncertainty and output model uncertainty including surrogate and simulation model uncertainty are discussed.

5.2.1 Input model uncertainty

Non-probabilistic approach This section introduces various design optimizations where input model uncertainty is described in non-probabilistic ways. Interval analysis is carried out when only nominal value and its lower and upper bounds are available through scarce data or engineer's experience. Du and Chen (2004) and Du et al. (2005) proposed a design optimization with the mixture of random and interval variables. Since identification of the worst case interval variables requires heavy computations, the sequential single loop is employed to improve the efficiency. Yoo and Lee (2014) developed sampling-based design optimization in the presence of interval variables. Cho et al. (2020) developed an MPP search method for mixture of random and interval variables and its sensitivity analysis where both types of variables can be iteratively updated to find a correct MPP. Fuzzy variables described using membership functions are employed in possibility-based design optimization. Du et al. (2006) integrated PMA with possibility-based design optimization improving the efficiency of the optimization by using the maximal possibility search method. Youn et al. (2007) considered product quality loss analogous with robustness of quality and integrated it with possibility-based design optimization. Lee et al. (2013) compared possibility-based design optimization with RBDO using confidence level and measured conservativeness of an optimum for each approach through mathematical and engineering examples. The evidence theory uses two measures which are belief and plausibility to quantify the bounds of the precise probability. Mourelatos and Zhou (2006) used the evidence theory to assess reliability with incomplete information, and a computationally efficient optimization is proposed. Alyanak et al. (2008) developed a design optimization method using the evidence theory based on a gradient projection technique. Jiang et al. (2013b) developed the most probable focal element corresponding to MPP in RBDO, and then reliability can be obtained efficiently through most probable focal element. Kang et al. (2011) proposed a non-probabilistic RBDO exhibiting uncertain-but-bounded parameters. The non-probabilistic reliability index based on a multi-ellipsoid convex model is used to quantify the reliability in RBDO. Guo and Lu (2015) presented a mixed interval-convex model non-probabilistic RBDO methodology for structures with uncertain-but-bounded parameters, where the input uncertain parameters were treated as interval variables. Meng et al. (2016) developed a decouple approach for the non-probabilistic RBDO by shifting constraints of deterministic optimization, and Hao et al. (2017) proposed an efficient adaptive-loop method for the non-probabilistic RBDO aiming at improving efficiency, where the convex model is used to describe the input uncertainty. Recently, Kanno (2019) developed a data-driven non-parametric RBDO accounting for confidence level of reliability.

Probabilistic approach This section introduces developments in RBDO where input statistical model is described by a probabilistic way such as the Bayes' theorem where the input distribution parameters can be adjusted or follow a posterior distribution obtained from input data. Youn and Wang (2008) proposed a Bayesian RBDO combined with EDR when the input statistical model is unknown. Noh et al. (2011a, b) developed RBDO with confidence level by adjusting input standard deviations and correlation coefficients when input data are not sufficient. Cho et al. (2016a) proposed a conservative RBDO using conservativeness of reliability where input distribution parameters and types are quantified through the Bayesian approach. Following the previous frameworks, Jung et al. (2019b, 2020a) tried to reduce the computational cost for the conservative RBDO exploiting the MPP approach in the space of distribution parameters and developed the bi-objective confidence-based design optimization to determine the optimal number of input data. Zaman and Mahadevan (2017) exploited a four-parameter flexible Johnson family of distribution to describe the input statistical model. Moon et al. (2018) extend the previous approach to biased simulation models so that both input and output test data are used to estimate the distribution of reliability. Ito et al. (2018) proposed the conservative reliability index that can be decomposed into target reliability index and epistemic reliability index, so that aleatory and epistemic uncertainties of input random variables can be considered simultaneously. Moon et al. (2019) exploited a bootstrapping method for bandwidth to obtain the distribution of reliability using KDE as the input statistical model.

5.2.2 Output model uncertainty

The output model uncertainty means that output responses of interest would be inaccurate due to insufficient simulation samples for surrogate modeling and fundamental inability of a simulation model to numerically emulate the real physical model. Thus, various researches can be included in this section such as model calibration and validation induced by biased simulation models with unknown parameters and quantification of surrogate model uncertainty.

Surrogate model uncertainty Picheny et al. (2008) proposed margin to the response predicted by a surrogate model using biased fitting. Viana et al. (2010) exploited the cross-validation method to determine safety margin of a surrogate model. Zhao et al. (2013) developed weighted Kriging variance for sampling-based RBDO using Akaike information criterion. An and Choi (2012) proposed the Bayesian framework incorporating the input model and surrogate model uncertainties quantified as hyper-parameters of Kriging to perform integrated reliability analysis. The surrogate model uncertainty in reliability analysis quantified as correlated prediction in Kriging is also taken into account in the research of Nannapaneni et al. (2016). Li and Wang (2018) proposed confidence-driven design optimization to avoid underestimation of reliability in RBDO due to insufficient simulation data. Li and Wang (2019) also developed RBDO accounting for surrogate model UQ using equivalent reliability index exploiting the Gaussian mixture model. Jung et al. (2021) recently proposed the confidence-based design optimization accounting for distribution of reliability induced by surrogate model uncertainty.

Simulation model uncertainty Simulation models such as finite element analysis numerically solve a partial differential equation under various assumptions yielding discrepancy with experimental results. The goal of model calibration of simulation model is to accurately emulate the experiment by estimating unknown model parameters and model discrepancy. Xiong et al. (2009) proposed the maximum likelihood estimation-based approach to estimate the distribution parameters of unknown parameters instead of the Bayesian approach. Arendt et al. (2012) proposed an overall framework for model updating, a modular Bayesian approach, so that GP for the simulation model and bias function and posterior distribution of calibration parameters can be obtained. Jiang et al. (2013b) proposed RBDO under model and parameter uncertainties using GP modeling exploiting the multi-fidelity structure. Pan et al. (2016) developed a copula-based approach for bias modeling and unknown parameter calibration, and then model bias can be expressed as conditional PDF. Shi and Lin (2016) showed a new RBDO exploiting adaptive response surface using the Bayesian metric to prevent inaccurate response surface and GP for bias modeling. Moon et al. (2017) proposed RBDO using confidence-based model validation, which means that the distribution of reliability is taken into account using adaptive KDE of insufficient experimental data. Xi (2019) developed confidence-based reliability analysis considering the epistemic uncertainty induced by simulation model uncertainty for three representative scenarios. Recently, Lee et al. (2019a) and Jiang et al. (2020) investigated studies on various statistical model calibration and validation strategies, and categorized it for clarity.

5.3 Remarks (Discussion, Consideration) on design and optimization under uncertainties

In this section, several remarks on design optimization under uncertainties are given to arrange pros and cons of each approach, potential uses, and promising perspectives. Design optimization under aleatory uncertainty mainly treats a multidimensional integration on joint PDF of random variables in this paper. The analytical approach focuses on the approximation of a limit-state function at MPP, so that it is very efficient but cannot ensure accuracy since non-linearity of the limit-state function is unknown. In particular, it can be much erroneous when decoupled and single loops requiring additional approximations are used instead of double loop. The alternatives such as the moment-based method can alleviate the difficulty induced by approximation at MPP, but it still requires estimation of higher moments and the approximation of parametric PDF. Thus, it can be addressed that an analytical approach based on approximation is suitable for a high-dimensional problem or a problem where simulation is extremely expensive so that surrogate modeling is not available.

On the other hand, the sampling approach is a more practical method in the real world since it is more reliable than the analytical approach, and its convergence is guaranteed as the number of samples increases. In fact, the accuracy of the sampling approach is highly dependent on the accuracy of the surrogate model since the direct numerical integration using the simulation model is extremely difficult. Each surrogate model has different characteristics, and it is very challenging to determine which surrogate model is suitable in such a situation. To resolve the difficulty of DoE in surrogate modeling, various strategies to update the surrogate model, also called active learning, have been widely investigated for RBDO. Especially, the method of effectively combining active learning and numerical integration could be a good option in RBDO. Thus, it is addressed that if surrogate modeling is available, active learning Kriging combining efficient integration can be recommended since its accuracy can be quantified and improved through additional simulations. Epistemic uncertainty induced by insufficient data, knowledge, and information always exists in the real word, but conventional RBDOs assumed that prior information and the sufficient number of data are available to estimate all models, and thus it has focused on efficient and accurate numerical integrations for reliability analysis, surrogate modeling, and simulation model calibrations. However, we addressed that epistemic uncertainty in each process has to be properly quantified and taken into account in reliability analysis and design optimization for practical uses.

Design optimization under input model uncertainty can be categorized into non-probabilistic and probabilistic methods. Although it depends on the characteristics of the input uncertainty, non-probabilistic methods such as the convex model, which has been vastly studied recently, are known to be effective when the input data are extremely limited and bounded. On the other hand, the probabilistic method using joint PDF and Bayes' theorem can be erroneous when uncertainty quantification of epistemic uncertainty is inaccurate due to a very small number of data and wrong assumptions on PDF, but it can show the convergence as more data are available and has a perfectly theoretical background. To the best of the authors' knowledge, however, no studies have yet to thoroughly compare the effectiveness of these two approaches quantitatively or theoretically according to the number of given data. Design optimization under output model uncertainty, such as surrogate model uncertainty and simulation model uncertainty, has been limited even though it frequently occurs in the real world. Specifically, efforts have been made to improve the surrogate model and calibrate the simulation model to estimate unknown model parameters and model bias. However, epistemic uncertainty cannot be perfectly reduced, and the available data are always limited. Therefore, it should be taken into account to cope with various situations in the real world. Unfortunately, the studies on surrogate modeling and simulation model calibration are organized in the literature, but the quantification of output model uncertainty and conservative design optimization accounting for it have been limited and should be developed in the future.

6 Applications, software, and benchmark problems

6.1 Applications

Though plenty of methods were introduced and developed to treat uncertainties, only in the last decade, the methods were liberally used in industrial applications across different domains. In the following, we list the various applications addressed under the different classes.

6.1.1 Applications of uncertainty modeling

Wunsch et al. (2015) presented quantification of combined operational and geometrical uncertainties in turbo-machinery design. Allen et al. (2015) discussed uncertainty management in the integrated realization of engineered materials and components. Azevedo et al. (2015) presented the calibration of traffic microscopic simulation models for safety analysis analyzed considering four different key uncertainty sources: the input data, the calibration methodology, the model structure and its parameters, and the output data. Hu et al. (2017) discussed uncertainty quantification and management in additive manufacturing, and presented current status, needs, and opportunities. Håkansson (2019) used a general uncertainty management framework to analyze the standard uncertainty resulting in the heat transfer coefficients obtained with sensor experiments. Kumar et al. (2020) demonstrated an efficient uncertainty quantification and management in the early-stage design of composite applications. Son et al. (2020) addressed statistical model improvement consisting of model calibration, validation, and refinement techniques using a case study of an automobile steering column and then applied the uncertainty modeling results to RBDO.

6.1.2 Applications of uncertainty analysis

Cadini et al. (2015) demonstrate sampling-based approach in radioactive waste repository and Radaideh and Kozlowski (2020) provide a nuclear reactor example. Booker et al. (1999) and Hosder et al. (2001) discuss applications in helicopter rotor blade and high-speed civil aircraft, respectively. Iooss and Le Gratiet (2019) develop a functional risk curve in non-destructive testing in aeronautics and steam generator tubes application. A Krylov space dimension reduction approach is demonstrated on a 64-pin radio frequency-integrated circuit-linear time-invariant system by Freund (2003). A high-speed delft catamaran example is demonstrated on using RBF in Volpi et al. (2015). Bichon et al. (2008) demonstrate efficient global optimization on bistable MEMS. Taflanidis and Beck (2008a, 2008b), Jensen et al. (2009), and Li et al. (2016) discuss examples on x-storey structures subject to different types of base loads.

In the functional expansion approaches, Ishigami, Sobol, and Morris functions are discussed in Blatman and Sudret (2010a, 2010b) and Zhou and Lu (2019). An urban drainage simulation is demonstrated in Nagel et al. (2020). Dodson and Parks (2015) and Wang and Matthies (2019) present approaches to perform airfoil-shaped optimization while robust design of airfoil is also discussed (Dodson and Parks 2015). Wu et al. (2017) discuss a nuclear reactor system design and Kumar et al. (2020) present a rotor blade design and associated dynamic studies. Chan et al. (2007) and Du and Chen (2004) use analytical approaches to propagate uncertainties in Passive vehicle suspension design and a vehicle crashworthiness of side impact, respectively. Mukhopadhyay et al. (2016) discuss uncertainty propagation in composite structures. When the dimensions are large, dimension reduction techniques are employed to reduce the dimension in which the propagation or analysis can be performed. Several applications in automotives such as vehicle side impact (Lee et al. 2008a), lower control arm of a vehicular system (Youn and Wang 2008), and aerospace applications such as conceptual sizing (Bae and Alyanak 2016), rotating disk (Park et al. 2020), hypersonic scramjet (Constantine et al. 2015), and satellite design (Hu et al. 2015, 2017). Non-probabilistic approaches are used in design of composite laminated panels in supersonic flow (Zheng and Qiu 2018) and ultra-precision high-speed press (Cheng et al. 2020). Interval approaches are used to demonstrate an 8 degrees of freedom vehicle vibration problem (Li et al. 2019a) and design of an automobile front axle (Shi and Lu 2019).

6.1.3 Applications of RBDO

Allen and Maute (2004) allowed the design of aeroelastic structures where the reliability of structural and aerodynamic criteria should be satisfied, and thus the optimized structure yields significantly improved results. In addition, there have been many efforts on aerodynamic field applying RBDO framework (Paiva et al. 2014; Nikbay and Kuru 2013). Sun et al. (2017) applied multi-objective RBDO to tailor rolled blank hat-shaped structure, one of the key lightweight technology for vehicle crashworthiness. Youn et al. (2004) and Acar and Solanki (2009) also utilized RBDO in vehicle design for crashworthiness. Grujicic et al. (2010) exploited RBDO for durability of suspension in the presence of the uncertainty of material properties and shape parameters. Lee et al. (2019b, 2020a) applied RBDO to electric vehicle design in reliability-based design for market systems and to shared autonomous electric vehicle design and operation under uncertainties. Fan et al. (2019) used the Kriging model to perform RBDO of crane bridges. Hassan and Crossley (2008) proposed RBDO of spacecraft, and Shin and Lee (2014) exploited RBDO to determine the optimal radius and speed limit in windy environments under uncertainty. RBDO framework for composite structure has also been widely investigated. Gomes et al. (2011) and Lopez et al. (2011) proposed the RBDO of laminated composites structure, and Sohouli et al. (2018) and Duan et al. (2020) exploited the efficient RBDO for a composite structure such as decoupled and single loops. Li et al. (2017) developed the framework for RBDO of wind turbine drivetrains under wind and manufacturing uncertainties. Azarkish and Rashki (2019) performed reliability analysis and sensitivity analysis of shell and tube heat exchangers. Makhloufi et al. (2016) performed RBDO or wire bonding in power microelectronic devices. Li et al. (2019b) introduced the multidisciplinary RBDO of a cooling turbine blade along with heat transfer analysis and strength analysis. Ronold and Larsen (2000), Toft and Sørensen (2011), and Hu et al. (2016) conducted RBDO of wind turbine blades.

A two-dimensional mathematical example is one of the most widely used mathematical formulations in RBDO since it can be visualized and has narrow safe region, and modified formulation with much non-linear second constraint is also popularly used (Youn and Choi 2004; Youn et al. 2005a). RBDO for crashworthiness of vehicle side impact has nine design variables and two design parameters, and the limit-state functions are approximated as polynomials using response surface method, so that it is proper to verify RBDO as a problem with moderately high dimension (Youn et al. 2004). On the other hand, the ten-bar truss example has maximum stress constraints with knockdown factor, and the two-story steel frame example has ultimate and serviceability limit-states involving bending moment, axial force, and displacement. These examples are also widely recommended to demonstrate the RBDO for a simple engineering application framework since it has a quite large number of random variables, multiple limit-state functions, engineering aspects related to simple static analysis, and it is easy to implement. Hosseinzadeh et al. (2018) and Jo et al. (2021) exploited the bracket's FEA model for deflection and modal analysis using Matlab partial differential equation toolbox. Jung et al. (2020b) solved the heat transfer problem for the cooling flange using COMSOL to calculate the cooling power. Park et al. (2020) and Jung et al. (2021) also use the FEA model for crank arm optimization in Hyperstudy, analyzing the maximum von-Mises stress under static loading.

6.2 Software

Several commercial software packages provide capabilities to treat uncertainty. In the following, we provide some insights into few software. (All material are from the respective software's website. These are by no means comprehensive in terms of the list of software and the capabilities in the listed software). Altair offers uncertainty analysis capability through RAMDO (2021) which includes generation of random numbers to simulate input variability and perform RBDO and RDO. It also has inbuilt dynamic kriging and variance window approach. Ansys, in its OptiSLang (2021) product, has reduced order modeling, Robust design, and Model calibration modules. Dassault Systemes Simulia, in its Isight (2021) product, has capabilities on different metamodels, MCS with a variety of distributions, Mean value method, FORM/SORM, and Taguchi techniques for six sigma robustness. Esteco, in its modeFRONTIER (2021) module, can generate samples from a variety of probabilistic distributions including capabilities such as MCS, PCE, Adaptive PCE (Least Angle Regression method is used to find, rank, and reduce the most significant polynomial terms), and Sobol indices for sensitivity studies. In addition, frameworks for RDO, RBDO, and Taguchi techniques are available. OmniQuest (2021), as part of their Iliad module, offers capabilities in RBDO, RDO, n-sigma Design, MCS, approximate MCS, and approximate Latin hypercube sampling on a variety of distributions. SmartUQ (2021) has capabilities on statistical calibration, quick emulator building with large data, propagation of uncertainties using metamodels, or PCE. Softwares such as Mathematica and Matlab permit programming all the techniques discussed in this manuscript. UQLab (Marelli and Sudret 2014) is an open source (academic use) Matlab add on. Several open source packages in python such as UQpy (Olivier and Shields 2020) and UQ-Pyl (Wang et al. 2016) are also available. A list of such software packages is provided in UQWorld (2021).

6.3 Benchmark problems

This section covers several widely used example problems used in structural reliability estimation and RBDO. Whenever a new reliability estimation or RBDO method was proposed, these example problems were used to show that the proposed method was capable of handling problems with the following properties: (1) large number of random variables, (2) non-linear limit-state function(s), (3) noisy limit-state function(s), (4) multiple failure regions

Table 3 Benchmark problems for RBDO

Problem	Reference paper(s)	Number of random variables	Used probability distributions	Number of limit-state functions
Burst margin of a disk	Wang et al. (2004)	6	Weibull, Normal, Uniform	1
Duffing type oscillator	Schueller and Pradlwarter (2007)	4000	All normal	1
Embankment dam	Schueller and Pradlwarter (2007)	2160	Normal, Lognormal	1
Hyperbola with multiple MPPs	Engelund and Rackwitz (1993)	2	All normal	1
I-beam problem	Wang et al. (2004)	10	All normal	1
N-dimensional hyperplane	Engelund and Rackwitz (1993)	2, 10, 50	All normal	1
Noisy limit-state	Liu and Der Kiureghian (1991)	6	All lognormal	1
Parallel system	Melchers (1989)	5	All normal	4
Shear beam model	Schueller and Pradlwarter (2007)	225	All normal	1
Series system	Melchers (1989)	6	All normal	3
Tuned mass damper	Chen et al. (1999)	2	All normal	1
Limit-states: Linear, Non-linear, Quadratic, Numerical	Papaioannou et al. (2015)	20, 100, 100, 200	Standard Normal, Normal	1, 2, 1, 1
Non-linear oscillator	Echard et al. (2013)	6	Normal	1

Table 4 Benchmark problems for structural reliability estimation

Problem	Reference paper(s)	Number of random/design variables	Used probability distributions	Number of limit-state functions
Cantilever beam	Wu et al. (2001)	4/2	All normal	2
Composite panel	Qu et al. (2003)	12/4	All normal	1
Mathematical example-1	Youn and Choi (2004)	2/2	Normal, Gumbel	3
Mathematical example-2	Aoues and Chateauneuf (2010)	2/2	All normal	1
Short column design	Aoues and Chateauneuf (2010)	6/2	Gumbel, Weibull, Lognormal	1
Ten-bar truss	Kumar et al. (2009)	27/10	Uniform, Extreme type I, Lognormal	1
Torque arm problem-1	Kim et al. (2006)	8/8	All normal	1
Torque arm problem-2	Acar (2016)	10/7	Normal, Lognormal	1
Two-bar truss	Ramakrishnan and Rao (1996)	5/2	Normal, Beta, Gumbel, Lognormal	2
Two story steel frame	Aoues and Chateauneuf (2010)	6/2	Normal, Lognormal	5
Vehicle side crash	Youn and Choi (2004)	11/9	All normal	6
Fortini's clutch	Lee and Chen (2009)	4	Beta, Normal, Rayleigh	1
Piecewise linear function Metaball function	Breitung (2019)	2	Standard Normal	2,1

(multiple MPPs), (5) multiple failure modes, and (6) variables having different probability distributions. Tables 3 and 4 provide information about the benchmark problems used in structural reliability estimation and RBDO, respectively.

7 Concluding remarks

Despite the fact that design optimization of structural and multidisciplinary systems under uncertainties has been a topic of growing interest over the past decades, the current literature lacks a thorough review of uncertainty treatment practices including uncertainty modeling, uncertainty analysis, and design under uncertainty. This article provides a comprehensive review of the uncertainty treatment practices, and complement existing reviews on similar subjects (e.g., reliability analysis, reliability-based optimization, uncertainty representation, sensitivity analysis under uncertainty, and uncertainty handling theories). From this review, we can draw the following conclusions and provide recommendations for future studies:

– Probability theory is the oldest and still the most widely used uncertainty handling theory, whereas the combined use of various uncertainty handling theories can take advantage of the ability of each theory for better uncertainty modeling. Hybrid approaches that focus on combined use of various uncertainty handling theories is an active area of research.
– In probabilistic handling, uncertainty is characterized by using a probability distribution function when a parametric approach is used, whereas it is characterized by KDE when a non-parametric approach is used. Multivariate modeling that considers correlation between uncertain variables is an active area of research.
– When moderate samples are available, uncertainty modeling and propagation are typically through metamodels and probabilistic approaches. Though metamodel construction and usage is widely adopted, models such as polynomial response surface, Kriging, SVR, and RBF are mostly used. Usage of stochastic approaches and machine learning algorithms are to be tested and adopted. Propagation of uncertainties through metamodels is an active area with potential avenues of improvement in using multi-fidelity metamodels for propagation.
– In the event of large dimensions, dimension reduction techniques are widely used. However, propagation of uncertainties in the reduced dimension and mapping the errors in the reduced space to the original space along with handling mixed variables are open research area.
– When available samples are scarce, approaches based on interval theory and convex models are used. Though different approaches are used for modeling convex regions, choice of convex models for data without prior information on their characteristics is not available. Hence, an ensemble of geometries and respective error measures along with a continuation to uncertainty design is a potential research area.
– Both aleatory and epistemic uncertainties are handled in RBDO to find an optimum design to satisfy a given target reliability or confidence. Various methods reviewed in uncertainty modeling and analysis are combined with RBDO methods under various uncertainty situations. Research in RBDO mainly focuses on how to improve its accuracy or efficiency or both.
– RBDO under aleatory uncertainty is classified into MPP-based approach and sampling-based approach. The MPP-based approach can be further classified into various methods depending on MPP search methods, optimization formulation, and limit-state function approximation. The sampling-based approach is usually combined with surrogate models to alleviate its computational burden. To further improve computational efficiency of surrogate modeling, efficient surrogate modeling combined with sequential sampling strategies have been studied.
– Both input statistical model and output model uncertainties are considered in RBDO under epistemic uncertainty. Its main strategy is to guarantee a reliable or confident optimum design even when insufficient information is available to model the input or output. In addition to active research topics that this article covers, various machine learning techniques are currently being attempted to further improve accuracy and efficiency of RBDO.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Replication of results In this review paper, we do not provide any results to replicate.

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