Comparing Effectiveness of Measures That Improve Aircraft Structural Safety

E. Acar[1]; A. Kale[2]; and R. T. Haftka[3]

Abstract: This paper aims to discover how the measures that improve aircraft structural safety compare with each other in terms of effectiveness. The safety measures we include here are a load safety factor of 1.5, conservative material properties, redundancy, certification tests, error reduction, and variability reduction. We consider a static point stress design with a simple redundancy model. We model individual errors in calculation (loads, stresses, failure) and in geometry and variability in loading, material properties, and geometry. We use a probabilistic model based on assumed uniform distribution for errors as we often have only upper limits on errors. For variabilities we also use some lognormal distributions. We find that error reduction is more effective than certification testing, which is more effective than using an extra load safety factor. Variability reduction is found to be a very effective way of reducing the probability of failure (more effective than error reduction), but it should be accompanied with an increased B-basis value. In addition, certification testing is found to be effective when errors are large, whereas structural redundancy is found to be more effective when errors are low. We also find that as safety measures are added and the probability of failure is reduced, the uncertainty in that probability of failure increases.

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Introduction

Aircraft structures have traditionally been designed using a deterministic approach based on Federal Aviation Administration (FAA) regulations. In the deterministic approach, safety of aircraft structures has been achieved by combining a large number of measures including a safety factor of 1.5, conservative material properties (A-basis or B-basis values), tests, and redundancy. In addition, inspections and quality control along with improved accuracy of structural analysis and failure assessment are also amongst measures that improve aircraft structural safety. Even though the deterministic design leads to a remarkable level of safety for aircraft structures, there is a growing interest in replacing safety factors by reliability-based design (e.g., Lincoln 1980; Wirsching 1992; SAE 1997; Long and Narciso 1999) to attain the same level of safety with lighter structures. Moreover, probabilistic analysis can also be used to compare the relative effectiveness of various safety measures in improving the structural safety.

In previous work (Acar et al. 2006), we explored how safety measures compensate for errors and variability. The major finding of that paper was that certification tests are most effective when errors are large, variability is low, and the overall safety factor is low. That paper mainly focused on the effectiveness of certification testing, but the relative effectiveness of safety measures was not addressed. The present paper takes a further step and aims to discover how measures that improve aircraft structural safety compare with one another in terms of weight effectiveness. In addition, we modeled structural redundancy—another safety measure—and compared the effectiveness of error and variability reduction with other safety measures in this paper.

We consider a static point stress design and simplify the modeling of redundancy by assuming that the structure will fail only if two local failure events (possibly correlated) occur. Aircraft structures have more complex failure modes, such as fatigue and fracture, which require substantially different treatment and the consideration of the effects of inspections [see Kale and Haftka (2007)]. However, this simple model still serves to further our understanding of the interaction between various safety measures.

The next section of the paper discusses the safety measures taken during aircraft structural design. Then, we use a simple uncertainty classification that distinguishes uncertainties that affect an entire fleet of an aircraft model (errors) from uncertainties that vary from one aircraft to the next (variability). Next, we discuss our modeling of errors and variability. The effect of certification tests on error distribution is analyzed in a following section. Next, the details of the calculation of the probability of failure via separable Monte Carlo simulations (MCS) are given. Finally, the results and concluding remarks are given in the last two sections of the paper, respectively.

[1]Postdoctoral Associate, Center for Advanced Vehicular Systems, Mississippi State Univ., Mississippi State, MS 39762; formerly, Research Assistant, Dept. of Mechanical and Aerospace Engineering, Univ. of Florida, Gainesville, FL 32611-6250. E-mail: erdem@cavs.msstate.edu

[2]Research Engineer, General Electric, Research Circle K1-4B18B, Niskayuna, NY 12309; formerly, Research Assistant, Dept. of Mechanical and Aerospace Engineering, Univ. of Florida, Gainesville, FL 32611-6250. E-mail: kalea@research.ge.com

[3]Distinguished Professor, Dept. of Mechanical and Aerospace Engineering, Univ. of Florida, Gainesville, FL 32611-6250. E-mail: haftka@ufl.edu

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Table 1. Uncertainty Classification

Type of uncertainty	Spread	Cause	Remedies
Error (mostly epistemic)	Departure of the average fleet of an aircraft model (e.g., Boeing 737-400) from an ideal	Errors in predicting structural failure, construction errors, deliberate changes	Testing and simulation to improve the mathematical model and the solution
Variability (aleatory)	Departure of an individual aircraft from fleet level average	Variability in tooling, manufacturing process, and flying environment	Improvement of tooling and construction; quality control

Safety Measures

As noted earlier, aircraft structural design is still carried out by using code-based design, rather than probabilistic design. Safety is improved through conservative design practices that include the use of safety factors and conservative material properties. Safety is also improved by testing of components, redundancy, improved modeling to reduce errors, and improved manufacturing to reduce variability. The following gives a brief description of these safety measures.

Load safety factor: In transport aircraft design, FAA regulations state the use of a load safety factor of 1.5 (FAR-25.303). That is, aircraft structures are designed to withstand 1.5 times the limit load without failure.

Conservative material properties: In order to account for uncertainty in material properties, FAA regulations state the use of conservative material properties (FAR-25.613). The conservative material properties are characterized as A-basis and B-basis material property values, and the use of A-basis or B-basis values depends on the redundancy. If there is single failure path in the structure, A-basis values are used, whereas for the case of multiple failure paths (i.e., redundant structures), B-basis values are used. Detailed information on these values is provided in Chap. 8 of Vol. 1 of the *Composite Materials Handbook* (ASTM 2002). The basis values are determined by testing a number of coupons selected at random from a material batch. The A-basis value is determined by calculating the value of a material property exceeded by 99% of the population with 95% confidence, while the B-basis value is the value of a material property exceeded by 90% of the population with 95% confidence. In this paper, we take the redundancy of the structure into account, so we use B-basis values (see Appendix I for the B-basis value calculation). The number of coupon tests is assumed to be 40.

Tests: Tests of major structural components reduce stress and material uncertainties for given extreme loads due to inadequate structural models. These tests are conducted in a building block procedure [*Composite Materials Handbook* (ASTM 2002), Vol. 1, Chap. 2)]. First, individual coupons are tested, and then a subassembly is tested, followed by a full-scale test of the entire structure. In this paper, we only consider the final certification test for an aircraft. Other tests are assumed to be error reduction measures and their effect is analyzed indirectly by considering the effect of error reduction.

Redundancy: Transport airliners are designed with double and triple redundancy features in all major systems to minimize the failure probability. Redundancy is intended to ensure that a single component failure does not lead to catastrophic failure of the system. In the present work, we assume that an aircraft structure will fail if two local failures occur in the structure.

Error reduction: Improvements in the accuracy of structural analysis and failure prediction of aircraft structures reduce errors and enhance the level of safety of the structures (Acar et al. 2006). These improvements may be due to better modeling techniques developed by researchers, more detailed finite element models made possible by faster computers, or more accurate failure predictions due to extensive testing.

Variability reduction: Examples of mechanisms that reduce variability in material properties include quality control and improved manufacturing processes (Qu et al. 2003). Variability in damage and aging effects is accomplished through inspections and structural health monitoring. Variability in loads may be reduced by better pilot training and information that allows pilots to more effectively avoid regions of high turbulence. Here we investigate only the effect of reduced variability in material properties.

Structural Uncertainties

A good analysis of different sources of uncertainty in engineering simulations is provided by Oberkampf et al. (2000, 2002). To simplify the analysis, we use a classification that distinguishes between errors (uncertainties that apply equally to the entire fleet of an aircraft model) and variability (uncertainties that vary for the individual aircraft) as presented in Table 1. The distinction is important because safety measures usually target either errors or variability. Whereas variabilities are random uncertainties that can be readily modeled probabilistically, errors are fixed for a given aircraft model (e.g., Boeing 737-400) but they are largely unknown. As errors are epistemic, they are often modeled using fuzzy numbers or possibility analysis (Antonsson and Otto 1995; Nikolaidis et al. 2004; Vanegas and Labib 2005). Because information on errors is typically available only in terms of error bounds, we model errors probabilistically by using uniform distributions to maximize the entropy.

Errors are uncertain at the time of the design but they will not vary for a single structural component on a particular aircraft, whereas the variabilities vary for individual structural components. To model errors, we assume that we have a large number of nominally identical aircraft being designed (e.g., by Airbus, Boeing, Embraer, Bombardier, etc.), with the errors being fixed for each aircraft.

Errors, Variability, and Total Safety Factor

The following first discusses the errors in design and construction. Next, a total error and a total safety factor are introduced, finally, simulation of variability is discussed.

Errors in Design

We consider static point stress design for simplicity. Other types of failures such as fatigue, corrosion, or crack instability are not taken into account. We assume that an aircraft structure will fail only if two local failure events occur. For example, we assume that the wing will fail structurally if two local failures occur at the wing panels. The correlation coefficient between the limit state functions defining the probabilities of these two events is assumed to be 0.5.

Before starting the structural design, aerodynamic analysis needs to be performed to determine the loads acting on the aircraft. However, the calculated design load value, P_{calc}, differs from the true design load P_d under conditions corresponding to FAA design specifications (e.g., gust-strength specifications). Since each company has different design practices, the error in load calculation, e_p, is different from one company to another. The calculated design load P_{calc} is expressed in terms of the true design load P_d as

$$P_{\text{calc}} = (1 + e_P)P_d \tag{1}$$

Besides the error in load calculation, an aircraft company may also make errors in stress calculation. We consider a small region in a structural part, characterized by a thickness t and width w, that resists the load in that region. The value of the stress in a structural part calculated by the stress analysis team, σ_{calc}, can be expressed in terms of the load values calculated by the load team P_{calc}, the design width w_{design}, and the thickness t of the structural part by introducing the term e_σ representing error in the stress analysis

$$\sigma_{\text{calc}} = (1 + e_\sigma)\frac{P_{\text{calc}}}{w_{\text{design}}t} \tag{2}$$

Eq. (3) is used by a structural designer to calculate the design thickness t_{design} required to carry the calculated design load times the safety factor S_F. That is

$$t_{\text{design}} = (1 + e_\sigma)\frac{S_{FL}P_{\text{calc}}}{w_{\text{design}}(\sigma_a)_{\text{calc}}} = (1 + e_\sigma)(1 + e_P)\frac{S_{FL}P_d}{w_{\text{design}}(\sigma_a)_{\text{calc}}} \tag{3}$$

where $(\sigma_a)_{\text{calc}}$=value of allowable stress for the structure used in the design, which is calculated based on coupon tests using failure models such as Tresca or von Mises. Since these failure theories are not exact, we have

$$(\sigma_a)_{\text{calc}} = (1 - e_f)(\sigma_a)_{\text{true}} \tag{4}$$

where e_f=error associated with failure prediction. Moreover, the errors due to the limited amount of coupon testing to determine the allowables, and the differences between the material properties used by the designer and the average true properties of the material used in production are included in this error. Note that the formulation of Eq. (4) is different to that of Eqs. (1) and (2) in that the sign in front of the error factor e_f is negative, because we consistently formulate the expressions such that positive error implies a conservative decision.

Combining Eqs. (3) and (4), we can express the design value of the load carrying area as

$$A_{\text{design}} = t_{\text{design}}w_{\text{design}} = \frac{(1 + e_\sigma)(1 + e_P)}{1 - e_f}\frac{S_{FL}P_d}{(\sigma_a)_{\text{true}}} \tag{5}$$

Errors in Construction

In addition to the above-mentioned errors, there will also be construction errors in the geometric parameters. These construction errors represent the difference between the values of these parameters in an average airplane (fleet-average) built by an aircraft company and the design values of these parameters. The error in width, e_w, represents the deviation of the design width of the structural part, w_{design}, from the average value of the width of the structural part built by the company prior to certification testing, w_{proto}. We use the term prototype and the subscript proto to denote the aircraft built for certification. We assume that the actual built airplane after certification will not be different from the prototype airplane, but the distribution of errors will change because some designs will not be certified. Thus

$$w_{\text{proto}} = (1 + e_w)w_{\text{design}} \tag{6}$$

Similarly, the average prototype thickness value, t_{proto}, will differ from its design value such that

$$t_{\text{proto}} = (1 + e_t)t_{\text{design}} \tag{7}$$

Then, the prototype load carrying area A_{proto} can be expressed using the first equality of Eq. (5) as

$$A_{\text{proto}} = (1 + e_t)(1 + e_w)A_{\text{design}} \tag{8}$$

Table 2. Distribution of Error Factors and Their Bounds

Error factors	Distribution type	Mean	Bounds (%)
Error in stress calculation, e_σ	Uniform	0.0	±5
Error in load calculation, e_P	Uniform	0.0	±10
Error in width, e_w	Uniform	0.0	±1
Error in thickness, e_t	Uniform	0.0	±2
Error in failure prediction, e_f	Uniform	0.0	±20

Table 2 presents nominal values for the errors assumed here. In the "Results" section of the paper we will vary these error bounds and investigate the effects of these changes on the probability of failure. As seen in Table 2, the error having the largest bound in its distribution is the error in failure prediction e_f, because we use it to also model the likelihood of unexpected failure modes.

The errors here are modeled by uniform distributions, following the principle of maximum entropy. For instance, the error in the average prototype thickness of a structural part (e_t) is defined in terms of the error bound $(b_t)_{\text{proto}}$ via

$$e_t = U[0,(b_t)_{\text{proto}}] \tag{9}$$

Here "U" indicates that the distribution is uniform and "0 (zero)" is the average value of e_t. Table 2 shows that $(b_t)_{\text{proto}}$=0.02. Hence, the lower bound for the thickness value is the average value minus 2% of the average and the upper bound for the thickness value is the average value plus 2% of the average. Commonly available random number generators provide random numbers uniformly distributed between 0 and 1. Then, the error in the average prototype thickness can be calculated from Eq. (10) using such random numbers r as

$$e_t = (2r - 1)(b_t)_{\text{proto}} \tag{10}$$

Total Error Factor, e_{total}

The expression for the prototype load carrying area, A_{proto}, of a structural part can be reformulated by combining Eqs. (5) and (8) as

$$A_{\text{proto}} = (1 + e_{\text{total}}) \frac{S_{FL} P_d}{(\sigma_a)_{\text{true}}} \tag{11}$$

where

$$e_{\text{total}} = \frac{(1 + e_\sigma)(1 + e_P)(1 + e_t)(1 + e_w)}{1 - e_f} - 1 \tag{12}$$

Here e_{total} represents the cumulative effect of the individual errors (e_σ, e_P, …) on the load carrying capacity of the structural part.

Total Safety Factor

The total safety factor, S_F, of a structural part represents the effects of all safety measures and errors on the prototype structural part. Without safety measures and errors, we would have a load carrying area, A_0, required to carry the design load P_d

$$A_0 = \frac{P_d}{\bar{\sigma}_f} \tag{13}$$

where $\bar{\sigma}_f$=average value of the failure stress. Then, the total safety factor of a prototype structural component can be defined as the ratio of A_{proto}/A_0

$$(S_F)_{\text{proto}} = \frac{A_{\text{proto}}}{A_0} = (1 + e_{\text{total}}) S_{FL} \frac{\bar{\sigma}_f}{(\sigma_a)_{\text{true}}} \tag{14}$$

Here we take S_{FL}=1.5 and conservative material properties are based on B-basis values. Certification tests add another layer of safety. Structures with large negative e_{total} (unconservative) fail certification, so the certification process adds safety by biasing the distribution of e_{total}. Denoting the area after certification (or certified area) by A_{cert}, the total safety factor of a certified structural part is

$$(S_F)_{\text{cert}} = \frac{A_{\text{cert}}}{A_0} \tag{15}$$

Variability

In the previous sections, we analyzed the different types of errors made in the design and construction stages, representing the differences between the prototype fleet-average values of geometry, material, and loading parameters and their corresponding design values. For a given design, these parameters vary from one aircraft to another in the fleet due to variabilities in tooling, construction, flying environment, etc. For instance, the thickness of an individual structural part, t_{ind}, is defined in terms of its prototype fleet-average value, t_{proto}, by

$$t_{\text{ind}} = (1 + v_t) t_{\text{proto}} \tag{16}$$

Since thickness variation is typically specified in terms of tolerance bounds, we assume that v_t has a uniform distribution with 3% bounds (see Table 3). Then, the load carrying area of an individual structural part A_{ind} can be defined as

$$A_{\text{ind}} = t_{\text{ind}} w_{\text{ind}} = (1 + v_t) t_{\text{proto}} (1 + v_w) w_{\text{proto}} = (1 + v_t)(1 + v_w) A_{\text{proto}} \tag{17}$$

where v_w represents effect of the variability on the prototype width.

Table 3 presents the assumed distributions for variabilities. Note that the thickness error in Table 2 is uniformly distributed with bounds of ±2%. Thus the difference between all thicknesses over the fleets of all companies is up to ±5%. However, the combined effect of the uniformly distributed error and variability is not uniformly distributed.

Table 3. Distribution of Random Variables Having Variability

Variables	Distribution type	Mean	Scatter
Load acting on individual structural parts under actual service conditions, P_{ind}	Lognormal	P_d=100	10% c.o.v.[a]
Width of an individual structural part, w_{ind}	Uniform	w_{proto}	1% bounds
Thickness of an individual structural part, t_{ind}	Uniform	t_{proto}	3% bounds
Failure stress, σ_f	Lognormal	150	8% c.o.v.[a]
v_w	Uniform	0	1% bounds
v_t	Uniform	0	3% bounds

[a]c.o.v.=coefficient of variation.

Certification Tests

After a structural part has been built with errors in stress, load, width, allowable stress, and thickness, we simulate certification testing for the structural part. Recall that the structural part will not be manufactured with complete fidelity to the design due to variability in the geometric properties. That is, the values of these parameters in individual structural parts w_{ind} and t_{ind} will be different from their prototype fleet-average values w_{proto} and t_{proto} due to variability. The structural part is then loaded with the design axial force of S_F times P_{calc}, and if the stress exceeds the failure stress of the structure σ_f, then the structure fails and the design is rejected; otherwise it is certified for use. That is, the structural part is certified if the following inequality is satisfied:

$$\sigma - \sigma_f = \frac{S_{FL} P_{\text{calc}}}{w_{\text{ind}} t_{\text{ind}}} - \sigma_f \leqslant 0 \tag{18}$$

The total safety factor [see Eq. (14)] depends on the load safety factor, the ratio of the failure stress to the B-basis allowable stress, and the total error factor. Using the FAA definition (see Appendix I), the B-basis properties are affected by the number of coupon tests. As the number of tests increases, the B-basis value also increases. This means that a lower total safety factor is used. Amongst the terms in the total safety factor expression, the error term is subject to the largest change due to certification testing. Certification tests reduce the probability of failure by mainly changing the distribution of the error factor e_{total}. Without certification testing, we assume uniform distributions for all the individual errors. As noted earlier, this reflects typical lack of information on error distributions and we use uniform distribution to maximize entropy (or randomness). However, since designs based on unconservative models are more likely to fail certification, the distribution of e_{total} becomes conservative for structures that pass certification. In order to quantify this effect, we calculated the updated distribution of the error factor e_{total} by Monte Carlo simulation (MCS) of a sample size of 1,000,000. Calculation of updated error distributions is based on the following procedure.

1. Generate individual errors based on their initial distribution (uniform);
2. Calculate the total error from Eq. (12);
3. Calculate the area before certification, A_{proto}, from Eq. (11);

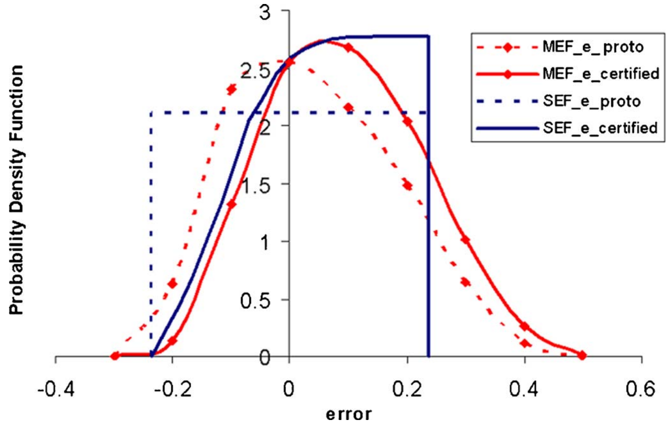

Fig. 1. Comparing distributions of prototype and certified total error e_{total} of SEF and MEF models. The distributions are obtained from simulation of 1,000,000 structural parts. The lower and upper bounds for the single error are taken as −22.3 and 25.0%, respectively, to match the mean and standard deviation of the total error factor in the MEF model (see Table 12).

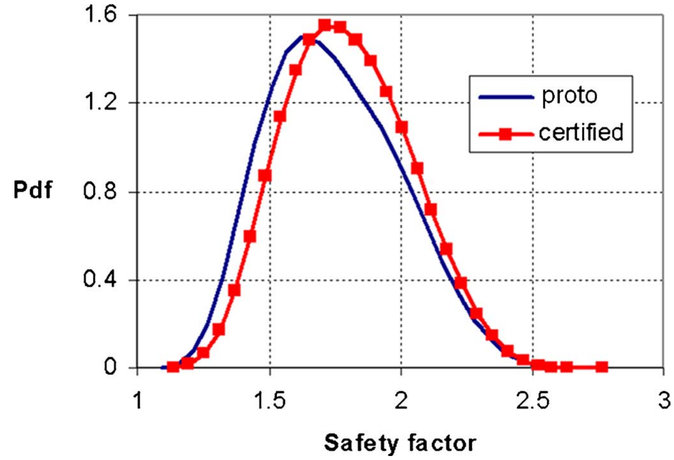

Fig. 2. Initial (precertification) and updated (after certification) distribution of the total safety factor S_F. The distributions are obtained via Monte Carlo simulations with 1,000,000 structural part models.

4. Calculate area for individual structural parts from Eq. (17); and
5. Simulate certification test [Eq. (18)];
   - Pass=store the errors to form the updated error distribution; and
   - Fail=do nothing.
6. Check if the number of MCS is reached;
   - Yes=stop; and
   - No=go to step 1.

Note here that the error updating procedure is performed in Stage 2 of the separable Monte Carlo simulations (see next section). However, for error updating we can use standard Monte Carlo because tails are not as important compared to probability of failure calculation where they are.

In a previous paper (Acar et al. 2006), we represented the overall error with a single error factor e, hereinafter termed the "single error factor model (SEF model)," and we used uniform distribution for the initial (i.e., prototype) distribution of this error. In the present work, we use a more complex representation of error with individual error factors, hereinafter termed the "multiple error factor model (MEF model)," and we represent the initial distribution of each individual error factor with uniform distribution. In this case, the distribution of the total error is no longer uniform.

Fig. 1 shows how certification tests update the distribution of the total error for the SEF and MEF models. For both models the initial distribution is updated such that the likelihood of conservative values of the total error is increased. This is due to the fact that structures designed with unconservative (negative) errors are likely to be rejected in certification tests. Notice that the SEF model exaggerates the effectiveness of certification testing. The reader is referred to Appendix II for a detailed comparison of the two error models.

Fig. 2 shows the distributions of the prototype and certified total safety factors of the MEF model. Notice that the structural parts designed with low total safety factors are likely to be rejected in the certification testing. The mean and standard deviations of prototype and certified distributions of the error factor and the total safety factor are listed in Table 4. Comparing the mean and standard deviation of the prototype and certified total error (and similarly the total safety factor), we see that the mean is increased and the standard deviation is reduced due to certification testing.

Probability of Failure Calculation

As noted earlier, we assume that structural failure requires the failure of two structural parts. In the following, we first describe the probability of failure calculations of a single structural part by using separable MCS. Then, we discuss the calculation of the system probability of failure.

Probability of Failure Calculation by Separable MCS

To calculate the probability of failure, we first incorporate the statistical distributions of errors and variability in a Monte Carlo simulation. Errors are uncertain at the time of design, but do not change for individual realizations (in actual service) of a particular design. On the other hand, all individual realizations of a particular design are different from one another due to variability. In a previous paper (Acar et al. 2006), we implemented this through a two-level Monte Carlo simulation. At the upper level we simulated different aircraft companies by assigning random errors to each, at the lower level we simulated variability in dimensions, material properties, and loads related to manufacturing variability and variability in service conditions. This provided not only the overall probability of failure, but also its variation from one company to another (which we measured by the standard

Table 4. Mean and Standard Deviations of the Prototype and Certified Distributions of the Error Factor e_{total} and the Total Safety Factor S_F Shown in Figs. 1 and 2

	Mean	Standard deviation
Prototype total error	0.0137	0.137
Certified total error	0.0429	0.130
Prototype safety factor	1.747	0.237
Certified safety factor	1.799	0.226

Note: The calculations are performed with 1,000,000 MCS.

deviation of the probability of failure). This variation is important because it is a measure of the confidence in the value of the probability of failure due to the epistemic uncertainty (lack of knowledge) in the errors. However, the process requires trillions of simulations for good accuracy.

In order to address the computational burden, we turned to the separable Monte Carlo procedure (e.g., Smarslok and Haftka 2006). This procedure applies when the failure condition can be expressed as $g_1(x_1) > g_2(x_2)$, where x_1 and x_2=two disjoint sets of random variables. To take advantage of this procedure, we need to formulate the failure condition in a separable form, so that g_1 will depend only on variabilities and g_2 only on errors. The common formulation of the structural failure condition is in the form of a stress exceeding the material limit. This form, however, is not separable. For example, the stress depends on variability in material properties as well as design area, which reflects errors in the analysis process. To bring the failure condition to the right form, we instead formulate it as the required cross-sectional area A'_{req} being larger than the prototype area A_{proto}, as given in

$$A_{proto} < \frac{A_{req}}{(1+v_t)(1+v_w)} \equiv A'_{req} \quad (19)$$

where A_{req}=crosssectional area required by individual realizations of a particular copy of an aircraft model to carry the loading in actual service conditions, P_{ind}; and A'_{req} is what the prototype area (fleet average) needs to be in order for the particular copy to have the required area after allowing for variability in width and thickness

$$A_{req} = P_{ind}/\sigma_f \quad (20)$$

The required area depends only on variability, while the prototype area depends only on errors. When certification testing is taken into account, the prototype area, A_{proto}, is replaced by the certified area, A_{cert}, which is the same as the prototype area for companies that pass certification. However, companies that fail are not included. That is, the failure condition is written as

$$\text{failure without certification tests:} \quad A'_{req} - A_{proto} > 0$$

$$\text{failure with certification tests:} \quad A'_{req} - A_{cert} > 0 \quad (21)$$

Eq. (21) can be normalized by dividing the terms with A_0 [load carrying area without errors or safety measures, Eq. (13)]. Since A_{proto}/A_0 or A_{cert}/A_0 are the total safety factors, Eq. (21) is equivalent to the requirement that failure occurs when the required safety factor is larger than the prototype one

$$\text{failure without certification tests:} \quad (S_F)_{req} - (S_F)_{proto} > 0 \quad (22a)$$

$$\text{failure with certification tests:} \quad (S_F)_{req} - (S_F)_{cert} > 0 \quad (22b)$$

where $(S_F)_{proto}$ and $(S_F)_{cert}$=prototype and certified total safety factors given in Eqs. (14) and (15); and the required total safety factor $(S_F)_{req}$ is calculated from

$$(S_F)_{req} = \frac{A'_{req}}{A_0} \quad (23)$$

For a given $(S_F)_{proto}$ we can calculate the probability of failure, Eq. (22*a*), by simulating all the variabilities with MCS.

Fig. 3 shows the dependence of the probability of failure on the total safety factor using MCS with 1,000,000 variability samples. The zigzagging in Fig. 3, at high safety factor values is due to the limited MCS sample. Note that the probability of failure for a given total safety factor is one minus the cumulative distribution function (CDF) of the total required safety factor. This required safety factor depends on the four random variables P_{ind}, σ_f, v_t, and v_w. Among them, P_{ind} and σ_f have larger variabilities compared to v_t and v_w (see Table 3). We found that $(S_F)_{req}$ is accurately represented with a lognormal distribution, since P_{ind} and σ_f follow lognormal distributions. Fig. 3 also shows the probability of failure from the lognormal distribution with the same mean and standard deviation. Note that the nominal (load) safety factor of 1.5 is associated with a probability of failure of about 10^{-3}, while the probabilities of failure observed in practice (about 10^{-7}) correspond to a total safety factor of about 2.

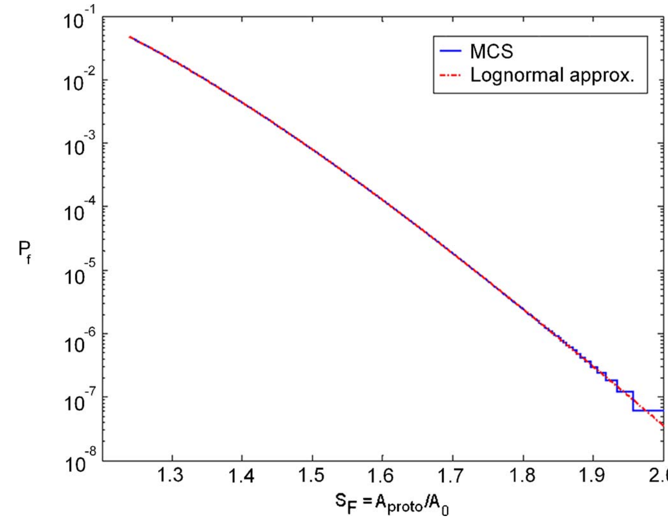

Fig. 3. The variation of the probability of failure with the prototype total safety factor. Note that P_f is one minus the cumulative distribution function of $(S_F)_{req}$.

Details of separable Monte Carlo simulation procedure can be found in Appendix III.

Additional Effect of Redundancy

The requirement of two failure events is modeled here as a parallel system. The large number of random variables contributing to the limit-state functions is likely to result in approximately normal distribution. Therefore, we assume that the limit-state functions of both failure events follow normal distribution to take advantage of known properties of the bivariate normal distribution. For a parallel system of two elements with equal failure probabilities, Eq. (24) is used to calculate the system probability of failure P_{FS} (see Appendix IV for details)

$$P_{FS} = P_f^2 + \frac{1}{2\pi}\int_0^{\rho} \frac{1}{\sqrt{1-z^2}} \exp\left(-\frac{\beta^2}{1+z}\right) dz \quad (24)$$

where P_f=probability of failure of a single structural part; ρ=correlation coefficient of the two limit states; and β=reliability index for a single structural part, which is related to P_f through

$$P_f = \Phi(-\beta) \quad (25)$$

where Φ=cumulative distribution function of the standard normal distribution.

Table 5. Average and Coefficient of Variation of the Probability of Failure for the Structural Parts Designed with B-Basis Properties and $S_F = 1.5$

K	CFR[a] (%)	$(S_F)_{proto}$[b]	$(S_F)_{cert}$[b]	P_{nc}[c]$/10^{-4}$	P_c[c]$/10^{-4}$	$\bar{P}_c/\bar{P}_{nc}$
0.25	6.4	1.725 (4.2%)	1.728 (4.1%)	0.244 (148%)	0.227 (148%)	0.930
0.50	9.3	1.730 (6.9%)	1.741 (6.7%)	0.763 (247%)	0.609 (257%)	0.798
0.75	13.4	1.737 (10.2%)	1.764 (9.7%)	2.70 (324%)	1.66 (357%)	0.616
0.82	14.7	1.740 (11.2%)	1.773 (10.6%)	3.79 (340%)	2.13 (384%)	0.561
1	18.0	1.747 (13.6%)	1.799 (12.5%)	8.83 (371%)	3.79 (450%)	0.430
1.5	26.0	1.779 (20.5%)	1.901 (17.8%)	60.0 (385%)	11.5 (583%)	0.191

Note: The numbers inside the parentheses represent the coefficient of variation of the relevant quantity.

[a]CFR=certification failure rate.

[b]$(S_F)_{proto}$ and $(S_F)_{cert}$=total safety factors before and after certification testing, respectively.

[c]P_{nc} and P_c=probabilities of failure before and after certification testing, respectively.

Results

In this section, the effectiveness of safety measures is investigated and the results are reported. First, we discuss the effects of error reduction. Then, the relative effectiveness of error reduction and certification is compared. Next, the effectiveness of redundancy is explored. Finally, the effectiveness of variability reduction is investigated.

Effect of Errors

We first investigate the effect of errors on the probability of failure of a single *structural part*. For the sake of simplicity, we scale all error components with a single multiplier, k, replacing Eq. (12) by

$$e_{\text{total}} = \frac{(1 + ke_\sigma)(1 + ke_P)(1 + ke_t)(1 + ke_w)}{1 - ke_f} - 1 \qquad (26)$$

and explore the effect of k on the probability of failure.

Table 5 presents the average and coefficient of variation of the probability of failure of a single structural part. The coefficient of variation of the failure probability is computed to explore our confidence in the probability of failure estimate, as it reflects the effect of the unknown errors. Columns 5 and 6 of Table 5 show a very high coefficient of variation for the failure probabilities (variability in the probability of failure for different aircraft models). We see that as the error grows (i.e., k increases), the coefficient of variation of failure probabilities after certification also grows. This is due to the fact that as the error bounds increase, the difference between companies also increases. Comparing the failure probabilities before certification (Column 5) and after certification (Column 6), we notice that even though certification tests reduce the mean failure probability, they increase the variability in failure probability. This effect of safety measures will be observed repeatedly and it is easiest to explain for the effect of redundancy. So discussion of this effect is given at the end of the next section.

Table 5 shows that for nominal error (i.e., $k=1$) the total safety factor before certification is 1.747, which is translated into a probability of failure of 8.83×10^{-4}. When the certification testing is included, the safety factor is increased to 1.799, which reduces the probability of failure to 3.79×10^{-4}. Notice also that the coefficient of variation of the safety factor is reduced from 13.6 to 12.5%, which is an indication that the certification testing is more effective than simply increasing the safety factor with an increased prototype area. A detailed analysis of the effectiveness of certification testing is given throughout this "Results" section of the paper.

Column 2 of Table 5 shows a rapid increase in the certification failure rate with increasing error. This is reflected in a rapid increase in the average safety factor of certified designs in Column 4, $(S_F)_{cert}$. This increased safety factor manifests itself in the last column of Table 5 that presents the effect of certification tests on failure probabilities. As we can see from that column, when the error increases, the ratio of the two failure probabilities decreases, demonstrating that the certification tests become more effective. This trend of the increase of the design areas and the probability ratios is similar to the one observed in Acar et al. (2006). Note, however, that even the average safety factor before certification $[(S_F)_{proto}$ in Column 3] increases with the error due to the asymmetry of the initial total error distribution (see Fig. 1).

Table 5 shows the huge waste of weight due to errors. For instance, for the nominal error (i.e., $k=1.0$), an *average* prototype total safety factor of 1.747 corresponds to a probability of failure of 8.83×10^{-4} according to Table 5, but we see from Fig. 3 that a safety factor of 1.747 approximately corresponds to a probability of failure of 7×10^{-6}, two orders of magnitude lower. This discrepancy is due to the high value of the coefficient of variation of the safety factor. For the nominal error, the coefficient of variation of the total safety factor is 14%. Two standard deviations below the mean safety factor is 1.272, and two standard deviations above the mean safety factor is 2.222. The probability of failure corresponding to the safety factor of 1.272 (from Fig. 3) is about 2.98×10^{-2}, while with the safety factor of 1.985 the probability of failure is essentially zero. So even though only about 0.8% of the designs have a safety factor below 1.272 (Fig. 2), these designs have a huge impact on the probability of failure. Reducing the error by half (i.e., $k=0.50$), reduces the weight by 1%, while at the same time the probability of failure is reduced by a factor of 3.

Weight Saving due to Certification Testing and Error Reduction

We have seen in Table 5 that as structures built with unconservative errors are mostly eliminated by certification testing; the tests increase the average safety factor of the designs and therefore reduce the average probability of failure. As certification testing is expensive, it is useful to check if the same decrease in the probability of failure can be achieved by simply increasing the load carrying area by the same amount (i.e., by increasing the safety

Table 6. Reduction of the Weight of Structural Parts by Certification Testing for a Given Probability of Failure

k	$A_{r,nc}/A_0$ [a]	A_{cert}/A_0	P_{nc} [b]$/10^{-4}$	P_c [b]$/10^{-4}$	$\%\Delta A$ [c]
0.25	1.7285 (4.2%)	1.7283 (4.1%)	0.227 (148%)	0.227 (148%)	−0.01
0.50	1.743 (6.9%)	1.741 (6.7%)	0.609 (252%)	0.609 (257%)	−0.14
0.75	1.770 (10.3%)	1.764 (9.7%)	1.66 (342%)	1.66 (357%)	−0.36
1	1.815 (13.7%)	1.799 (12.5%)	3.79 (416%)	3.79 (450%)	−0.87
1.5	1.961 (20.7%)	1.901 (17.8%)	11.5 (530%)	11.5 (583%)	−3.09

Note: The numbers inside the parentheses represent the coefficient of variation of the relevant quantity.

[a]$A_{r,nc}$=required area with no certification testing, the area required to achieve the same probability of failure as certification.

[b]P_{nc} and P_c=probabilities of failure before and after certification testing, respectively.

[c]$\Delta A=(A_{cert}-A_{r,nc})/A_{r,nc}$ indicates weight saving due to testing while keeping the same level of safety.

factor) without certification testing. Column 2 of Table 6 shows that the required area with no certification testing, $A_{r,nc}$, is greater than the certified area, A_{cert}, (i.e., area after certification testing) shown in Column 3. The last column shows the weight saving by using certification test instead of a mere increase of the safety factor. We notice that weight saving increases rapidly as the error increases. For instance, when $k=0.25$ the weight saving is very small. Columns 4 and 5 show that even though we match the average probability of failure, there are small differences in the coefficients of variation.

To compare the effectiveness of certification testing and error reduction, we examine the case of the nominal error (i.e., $k=1.0$). For that case, certification testing allows us to use a normalized weight of 1.799, whereas to achieve the same probability of failure (3.79×10^{-4}) would require a normalized weight of 1.815. However, as shown in Table 5, we can achieve the same probability of failure without certification by reducing the error bounds by 18%, that is by reducing k from 1.0 to 0.82, accompanied by a normalized weight of 1.740 (see Table 5). So while certification testing reduces the weight by 0.87%, reducing errors by 18% would reduce the weight by 4.13%. So error reduction is much more effective than certification testing in reducing weight.

Effect of Redundancy

To explore the effect of redundancy, we first compare the failure probability of a single structural part to that of a structural system that fails due to failure of two structural parts. Certification testing is simulated by modeling the testing of one structural part and certifying the structural system based on this test. Table 7 shows that while the average failure probability is reduced through structural redundancy, the coefficients of variation of the failure probabilities are increased. That is, even though the safety is improved, our confidence in the failure probability estimation is reduced. This behavior is similar to the effect of certification (Table 5). In addition, we also notice that as the error grows, the benefit of redundancy also diminishes. This result reflects the fact that high errors result in high probabilities of failure, and redundancy is more effective for smaller probabilities of failure. This behavior, however, is opposite to that resulting from certification testing. We notice that even though one safety measure—certification testing—is more effective when errors are high, another safety measure—redundancy—is more effective when errors are low. So the level of uncertainty in the problem may decide on the efficient use of safety measures.

Comparing the reduction probabilities of failure before and after certification listed in Columns 4 and 7 of Table 7, we notice that the effect of redundancy is enhanced through certification testing.

Next, we investigate the interaction of two safety measures: redundancy and certification testing. Comparing the probability ratios in Table 8, we see that including redundancy improves the effectiveness of certification testing. Mathematically, this can be explained with the following example. For a nominal error, $k=1.0$, the probabilities of failure before and after certification of a structural part are 8.83 and 3.79×10^{-4}, respectively. The system probabilities of failure before and after certification are calculated by using Eq. (24) as 1.31 and 0.39×10^{-4}, respectively. Notice that the system failure probability ratio is smaller than the component probability ratio, because redundancy is more effective for small probabilities of failure. Physically, the reason for the increase in the effectiveness certification is that in the certification test, failure of a single part leads to rejection of the design of structural system, whereas under actual service conditions, two failure events are needed for the failure of the structure. Thus, modeling

Table 7. Effect of Redundancy on the Probabilities of Failure

	Before certification			After certification		
K	Part P_{nc} [a]$/10^{-4}$	System P_{nc} [a]$/10^{-4}$	Reduction [b]	Part P_c [a]$/10^{-4}$	System P_c [a]$/10^{-4}$	Reduction [b]
0.25	0.244 (148%)	0.005 (230%)	52.1	0.227 (148%)	0.004 (230%)	53.5
0.50	0.763 (247%)	0.029 (388%)	26.3	0.609 (257%)	0.022 (408%)	28.0
0.75	2.70 (324%)	0.195 (503%)	13.8	1.66 (357%)	0.106 (568%)	15.6
1	8.83 (371%)	1.11 (563%)	7.9	3.79 (450%)	0.390 (718%)	9.7
1.5	60.0 (385%)	17.2 (549%)	3.5	11.5 (583%)	2.21 (945%)	5.2

Note: The numbers inside the parentheses represent the coefficient of variation of the relevant quantity. The coefficient correlation between failures of structural parts is taken as 0.5.

[a]P_{nc} and P_c=probabilities of failure before and after certification testing, respectively.

[b]The ratio of P_f's of the structural part and the system of two parts.

Table 8. Effect of Redundancy on the Effectiveness of Certification Testing

k	$\bar{P}_c/\bar{P}_{nc}$ (part)	$\bar{P}_c/\bar{P}_{nc}$ (system)
0.25	0.930	0.905
0.50	0.798	0.749
0.75	0.616	0.543
1	0.430	0.350
1.5	0.191	0.129

Note: The coefficient correlation between failures of structural parts is taken as 0.5. $\bar{P}_{nc}$ and $\bar{P}_c$=mean values of probabilities of failure before and after certification testing, respectively.

redundancy is equivalent to modeling a relatively more severe certification testing. This result is similar to the finding of Kale and Haftka (2005), who explored the effect of safety measures on aircraft structures designed for fatigue. They found that certification testing of an aircraft structure with a large machined crack of B-basis initial size was more effective than testing the structure with a random (natural) crack.

Effect of the Correlation Coefficient

Recall that the correlation coefficient of the probabilities of failure of the two structural parts was assumed to be 0.5. Table 9 shows that as the correlation coefficient decreases, the probability of failure of the system also decreases, but at the same time our confidence in the probability estimation also reduces. The last column of Table 9 shows that as the correlation coefficient decreases, certification testing becomes more effective, which can be explained as follows. As the coefficient of correlation decreases, the structural parts behave more independently. Applying certification testing based on the failure of a single structural part means using more severe certification testing. This reminds us that as with any redundant system it pays to reduce the correlation coefficient of duplicate hardware (e.g., to use a back up part made by a different company). It is intriguing to speculate on the possible application to structural design. Is it feasible, for example, to buy structural materials from different vendors for skin and stiffeners?

Increased Uncertainty in Probability of Failure

We have observed increased variability in the probability of failure as it is reduced by various safety measures. It is easiest to explain for the case of redundancy with zero correlation coefficient. Consider, for example, two companies, one with a single-component probability of failure of 0.2 and the other of 0.1. The average probability of failure is 0.15, the standard deviation is 0.05, and the coefficient of variation is 33%. With redundancy, the two probabilities will be reduced to 0.04 and 0.01. Now the average is 0.025, the standard deviation is 0.015, and the coefficient of variation is 60%.

As more safety measures are applied, failure requires a higher and higher number of simultaneous detrimental errors and variabilities. This means that the above-presented example for redundancy is also likely to explain the increased variability in the probability of failure as these measures are added.

Table 9. Effect of Correlation Coefficient ρ on System Failure Probabilities and Effectiveness of Certification Testing

ρ	P_{nc}[a]$/10^{-4}$	P_c[a]$/10^{-4}$	$\bar{P}_c/\bar{P}_{nc}$
0.3	0.506 (678%)	0.161 (885%)	0.319
0.4	0.761 (615%)	0.255 (794%)	0.335
0.5	1.11 (563%)	0.390 (718%)	0.350
0.6	1.60 (519%)	0.583 (655%)	0.365
0.7	2.27 (480%)	0.859 (516%)	0.378

Note: The numbers inside the parentheses represent the coefficient of variation of the relevant quantity. The error multiplier k is taken as 1.0.
[a]P_{nc} and P_c=probabilities of failure before and after certification testing, respectively.

Table 10. Additional Safety Factor due to Redundancy

k	$(S_{F-\text{add}})_{nc}$	$(S_{F-\text{add}})_c$	% increase due certification
0.25	1.120	1.120	0.0
0.50	1.111	1.112	0.1
0.75	1.101	1.103	0.2
1	1.093	1.096	0.3
1.5	1.078	1.085	0.7

Additional Safety Factor due to Redundancy

Recall that the results given in Table 7 show how redundancy reduces the probability of failure. For instance, for $k=1.0$ the average probability of failure before certification, $\bar{P}_{nc}$, is reduced from 8.83×10^{-4} to 1.11×10^{-4}. This reduction in probability of failure leads to an increase in the total safety factor. For each error multiplier k value, we calculate the additional safety factor required to reduce the probability of failure of a structural part to that of the structural system. The second and third columns of Table 10 show two opposing effects on the additional safety factor. As the error grows, the probabilities of failure before and after certification increase, so the effect of redundancy decreases because the redundancy is more effective for lower failure probabilities. Hence, the additional safety factor due to redundancy decreases with increased error (see also Fig. 4). However, as indicated in the last column of Table 10 the ratio of safety factors after and before certification testing increases with increased error because the certification is more effective for high errors.

Effect of Variability Reduction

Finally, we investigate the effect of variability reduction on the average safety factor, design area, and system probability of failure. We observe from Table 11 that the average safety factor and design area increase with the increase of variability in failure

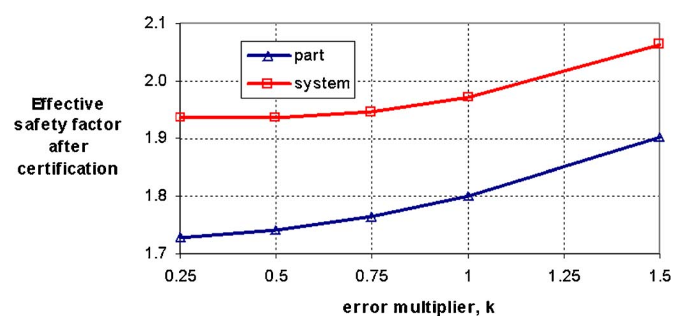

Fig. 4. Total safety factors for MEF model for the structural part and system after certification

Table 11. Comparison of System Failure Probabilities Corresponding to Different Variability in Failure Stress σ_f

c.o.v. (σ_f)	CFR[a] (%)	Average A_{proto}/A_0[b]	Average A_{cert}/A_0[b]	$\bar{P}_{nc}/10^{-4}$	$\bar{P}_c/10^{-4}$	P_f ratio
0	50.0	1.521	1.676	9.27	0.001	0.001
4%	32.3	1.629	1.727	2.00	0.008	0.040
8%	18.0	1.747	1.799	1.11	0.390	0.350
12%	11.6	1.878	1.910	1.19	0.737	0.619

[a]CFR=certification failure rate.

[b]A_{proto}/A_0 and A_{cert}/A_0=total safety factors before and after certification testing, respectively.

stress. In addition, we observe from the P_f ratio given in the last column of Table 11 that certification testing becomes less effective as variability increases. Fig. 5 also shows the reduced efficiency of testing with increased variability. The second column of Table 11 shows that the certification testing failure rate (CFR) reduces with increased variability. As variability is increased, the prototype load carrying area is also increased (Column 5), so CFR is reduced accordingly.

Table 11 shows two opposing effects of variability on the two failure probabilities (before certification, $\bar{P}_{nc}$, and after certification, $\bar{P}_c$, see Columns 7 and 8). When the coefficient of variation in the failure stress is increased from 0 to 8%, the safety factor before certification (Column 3) increases from 1.521 to 1.676, because a smaller B-basis value is used for the allowable failure stress. Note that the initial safety factor for no variability would be 1.5 if the error distribution (hence the safety factor distribution) was symmetric, but as the distributions are skewed (see Figs. 1 and 2) the safety factor is 1.521. The increase in the safety factor with increased error leads to a reduction in the probability of failure before certification (Column 5). However, for higher coefficients of variation, the probability of failure before certification increases again, because the increased safety factor is not enough to compensate for the large variation in airplanes. However, once certification is included, the picture is different. For no variability, even though the safety factor is increased by only 10% (from 1.521 to 1.676, see Columns 3 and 4), the probability of failure reduces four order of magnitudes (Columns 5 and 6) due to the high effectiveness of certification testing at low variability. As variability increases, the effectiveness of certification testing reduces (Column 7), so the probability of failure after certification is still high.

Table 11 also indicates the advantage of reducing variability. Reducing variability from 8 to 4% reduces the weight by 4%, while at the same time reducing the probability of failure by a factor of 50. However, the certification failure rate is unacceptably increased from 18 to 32%. To compensate for this, however,

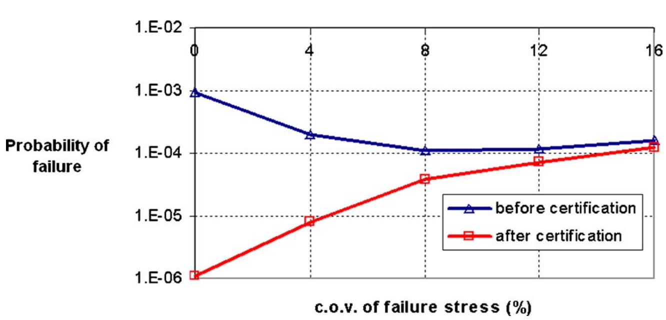

Fig. 5. Effect of variability on failure probability

a company may fargo the weight gain and have an additional safety factor of 1.747/1.629=1.072. This will give a reduced system probability of failure of 3.64×10^{-6} (compared to 3.90×10^{-5}) and a reduced certification failure rate of 14.7% (compared to 18.0%). A more efficient way, however, is to use the additional safety factors during the building block testing, which is not simulated in this paper.

In addition, Table 11 reveals that variability reduction is more effective than error reduction. For example, reducing all errors by half (i.e., reducing k from 1 to 0.5) leads to reducing the prototype safety factor from 1.747 to 1.730 (Table 5), along with reducing the system probability of failure from 3.90×10^{-5} to 2.20×10^{-6} (Table 7). On the other hand, reducing variability by half (that is, reducing c.o.v. of the failure stress from 8 to 4%) leads to reducing the prototype safety factor from 1.747 to 1.629, along with reducing the system probability of failure from 3.90×10^{-5} to 8.0×10^{-7} (Table 11). That is, variability reduction leads to more weight saving and probability of failure reduction than error reduction.

Concluding Remarks

The relative effectiveness of safety measures taken during aircraft structural design is demonstrated in this paper. The safety factor, conservative material properties, certification testing, redundancy, error and variability reduction were included in this study and the following was observed.

1. Although certification testing is more effective for improving safety rather than increased safety factors, it cannot compete with even a small reduction in errors.
2. Variability reduction is even more effective than error reduction, but it needs to be accompanied by increased internal safety factor to compensate for the increase in the B-basis value.
3. Our probabilities of failure are still high compared with the historical record (probability of failure of 10^{-7}). This is probably due to the effect of building block tests, which we will address in a future work.
4. One safety measure, certification testing, is more effective when errors are large, whereas another safety measure, redundancy, is more effective when errors are low. Certification testing is more effective when the variability is low. At a low variability level, redundancy accompanied with certification testing is effective.
5. For this specific example problem, adding redundancy by defining the system failure as probability of two simultaneous failure events is equivalent to using an additional safety factor of about 1.1.

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Appendix I. B-Basis Value Calculation

The B-basis value is the value exceeded by 90% of the population with 95% confidence. This is given by

$$B = \bar{X} - k_B s \tag{27}$$

where B=B-basis value; $\bar{X}$=sample mean; s=sample standard deviation; and k_B=tolerance coefficient for normal distribution given by

$$k_B = \frac{z_{1-p} + \sqrt{z_{1-p}^2 - ab}}{a}$$

$$a = 1 - \frac{z_{1-\gamma}^2}{2(N-1)}, \quad b = z_{1-p}^2 - \frac{z_{1-\gamma}}{N} \tag{28}$$

where N=sample size, γ=0.95, and z_{1-p}=critical value of normal distribution that is exceeded with a probability of $1-p$=0.1. The tolerance coefficient k_B for a lognormal distribution is obtained by first transforming the lognormally distributed variable to a normally distributed variable. Eqs. (27) and (28) can be used to obtain an intermediate value. This value is then converted back to the lognormally distributed variable using inverse transformation.

In order to obtain the B-basis values, we assume that 40 structural parts are randomly selected from a batch. The mean and standard deviation of 40 random values of allowable stress is calculated and used in determining the B-basis value of the allowable stress.

Appendix II. Comparison of the Single Error Model and the Multiple Error Model

In a previous paper (Acar et al. 2006), we used a single error factor model (SEF model), where the overall error is represented with a single error factor e, and uniform distribution is used for the initial distribution of this error. On the other hand, the present paper utilizes a multiple error factor model (MEF model), which uses a more complex representation of error with individual error factors and where initial distributions of each individual error factor are represented with uniform distribution. In this case, the distribution of the total error is no longer uniform. We find that the SEF model exaggerates the effectiveness of certification testing (see Fig. 1). This is due to the fact that the SEF model does not consider the fact that errors in load calculation affect the load used in certification testing. In the SEF model (Acar et al. 2006), the certification testing is assumed to be performed with the average value of the true design load (P_d), whereas in the MEF model certification testing is performed with the calculated load (P_{calc}). Therefore, one component of the error cannot lead to failure in certification testing and this reduces the effectiveness of certification testing.

Note that the single error of the SEF model is symmetric. On the other hand, even though the individual errors of MEF model are symmetric, the total error has a bell-shaped distribution with a positive, hence conservative, mean. One of the interesting differences between the SEF and MEF models is that we have a built-in safety factor due to asymmetric error distribution. This asymmetry is due mostly to the term $1/(1-e_f)$ in Eq. (12). Whereas e_f is symmetrically distributed (−0.2, 0.2), $1/(1-e_f)$ varies in (0.833, 1.25). The conservative tilt of the total error may be serendipitous because it will partially account for designer bias response to the building block tests used to reduce e_f. Tests that show that the failure model is slightly conservative typically do not lead to updating of the model. In contrast, tests showing even small unconservative bias typically lead to correction of the failure model.

In order to compare the effect of the two models on the probability of failure calculations, we match the mean and standard deviation values of the total error distribution (MEF model) and those of a uniform distribution (SEF model). Then, the upper and lower bounds (lb and ub) for the uniformly distributed error factor can be calculated via Eq. (29), where μ_e and σ_e=mean and standard deviation of the total error, respectively

$$\text{lb} = \mu_e - \sqrt{3}\sigma_e, \quad \text{ub} = \mu_e + \sqrt{3}\sigma_e \tag{29}$$

Using the equivalent error bounds of the SEF model given on the right-hand side of Table 12 we calculate the probabilities of failure before and after certification testing for the SEF model and we compare them in Table 13 with corresponding failure probabilities of the MEF model from Table 8. In addition, the comparison of the probability of failures after certification for the two models is presented in Fig. 6.

The total safety factor for the SEF model is defined as

$$(S_F)_{\text{design}} = \frac{A_{\text{design}}}{A_0} = (1+e) S_{FL} \frac{\bar{\sigma}_f}{\sigma_a} \tag{30}$$

Similarly, the design area for the SEF model is expressed as

$$A_{\text{design}} = (1+e) \frac{S_{FL} P_d}{\sigma_a} \tag{31}$$

Using the SEF model, we repeat the calculation of the probabilities of failures. The comparison of the SEF and MEF models' probability of failure calculations are given in Table 13.

When we compare the probability of failure before certification, the mean values of the failure probabilities are higher for the SEF model than those for the MEF model at high errors (see Columns 2 and 5, Table 13) due to the use of uniform distribution for the total error factor. Comparing the failure probabilities after certification, we notice that the MEF model leads to higher probability of failure values, and higher $\bar{P}_c/\bar{P}_{nc}$ ratios (less effective

Table 12. Equivalent Error Bounds for the SEF Model Corresponding to the Same Standard Deviation in the MEF Model

k	Average $e_{\text{total}}^{\text{ini}}$	Standard deviation of $e_{\text{total}}^{\text{ini}}$	From MEF model → to SEF model	Lower bound for $e_{\text{total}}^{\text{ini}}$	Upper bound for $e_{\text{total}}^{\text{ini}}$
0.25	0.0009	0.033		−0.057	0.059
0.50	0.0034	0.067		−0.113	0.119
0.75	0.0076	0.101		−0.168	0.183
1.0	0.0137	0.137		−0.223	0.250
1.5	0.0317	0.212		−0.336	0.400

Note: The average and standard deviation is calculated via 1,000,000 MCS.

Table 13. Comparison of System Failure Probabilities for the SEF and MEF Models

k	$\bar{P}_{nc}^{\text{MEF}}/10^{-4}$	$\bar{P}_{c}^{\text{MEF}}/10^{-4}$	P_f[a] ratio*	$\bar{P}_{nc}^{\text{SEF}}/10^{-4}$	$\bar{P}_{c}^{\text{SEF}}/10^{-4}$	P_f ratio[a]
0.25	0.0	0.0	0	0	0	0
0.50	0.029	0.022	0.749	0.026	0.018	0.689
0.75	0.195	0.106	0.543	0.165	0.069	0.419
1	1.11	0.390	0.350	1.03	0.186	0.181
1.5	17.2	2.21	0.129	27.7	0.311	0.011

Note: The coefficient correlation between failures of structural parts is taken as 0.5.

[a]P_f=ratio of the average failure probabilities before and after certification testing; $\bar{P}_c/\bar{P}_{nc}$.

Table 14. Comparison of the Total Safety Factor S_F^* Used in the Design of Structural Parts for the SEF and MEF Models

k	$(S_F)_{\text{proto}}^{\text{MEF}}$	$(S_F)_{\text{cert}}^{\text{MEF}}$	S_F ratio[a]	$(S_F)_{\text{proto}}^{\text{SEF}}$	$(S_F)_{\text{cert}}^{\text{SEF}}$	S_F ratio[a]
0.25	1.725	1.728	1.002	1.725	1.729	1.002
0.5	1.730	1.741	1.007	1.730	1.745	1.009
0.75	1.737	1.764	1.016	1.737	1.776	1.023
1	1.747	1.799	1.030	1.747	1.825	1.044
1.5	1.779	1.901	1.069	1.779	1.954	1.099

[a]S_F=ratio of total safety factors before and after certification.

certification testing). Recall that this is due to the fact that in the MEF model error in load calculation is also included in the certification testing. This effect is also apparent when we compare the total safety factor values for these two models in Table 14 and in Fig. 7.

The single error factor after certification of failure probabilities in Table 13 also indicates that the effect of the error bound on the probability of failure after certification is not monotonic. One possible explanation for this behavior is the competing effects of error and the total safety factor. For the highest error bound, the total safety factor is increased to 2.108 (see Table 14), which overcomes the effect of high error on the probability of failure.

Comparing the total safety factors, S_F, after certification corresponding to the MEF and SEF models (Columns 3 and 6, Table 14), we see that the total safety factor corresponding to the SEF model is larger, which will in turn lead to a smaller probability of failure (see Table 13). Columns 4 and 7 of Table 14 exhibit the expected trend of an increase in the total safety factor ratio with increasing error bounds, reflecting more effective certification testing.

In short, the effect of using a more detailed error model can be summarized as follows:

1. The uniformly distributed individual error components add up to a bell-shaped representative total error. This total error has an asymmetric distribution and this asymmetry results in a built-in safety factor.
2. The single error model exaggerates the effectiveness of certification testing, because it does not include the fact that error in load calculation is also present in the certification process. The single error model inflates the design area after certification, thereby leading to underestimation of probabilities of failures.

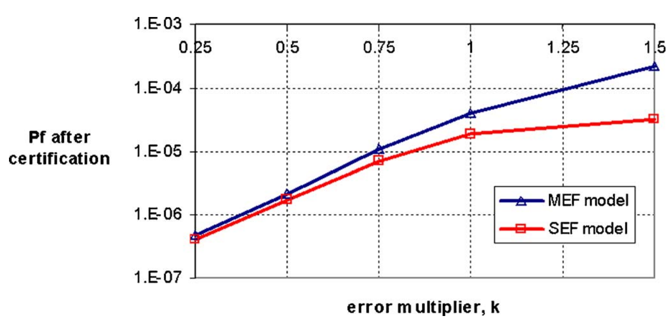

Fig. 6. System failure probabilities for the SEF and MEF models after certification

Appendix III. Details of the Separable Monte Carlo Simulation Procedure

The separable MCS procedure applies when the failure condition can be expressed as $g_1(x_1) > g_2(x_2)$, where x_1 and x_2=two disjoint sets of random variables. For that case, the probability of failure can be written as

$$P_f = \int f_2(t)[1 - F_1(t)]dt \tag{32}$$

where f_2=probability density function of g_2 and F_1=cumulative distribution function of g_1. Since the two sets of random variables are disjoint, we can perform one Monte Carlo simulation with x_1 to calculate F_1 and then perform a second Monte Carlo simulation on x_2 to calculate P_f from Eq. (32). Note that $1 - F_1$ in Eq. (32) is the probability of failure if g_2 takes the value t, and the second Monte Carlo simulation calculates the average of this probability over all possible values of g_2.

For our problem, f_2=probability density function of the prototype safety factor, A_{proto}/A_0, and F_1=cumulative distribution function of the required safety factor, A'_{req}/A_0. Since $f_1(\,)$ and $F_2(\,)$ depend on different sets of random variables, we separate the MCS into two stages.

In the first stage, the cumulative distribution function of the required safety factor, A'_{req}/A_0, is assessed. We use 1,000,000 MCS for this purpose. It is possible to assess CDF numerically by dividing the range of A'_{req}/A_0 into a number of bins (for instance, 1,000 bins) and calculating the CDF for each bin. Then, in the second stage, the CDF value can be obtained by interpolation.

On the other hand, we notice for our problem that the dominant terms in A'_{req}/A_0 are P_{ind} and σ_f, as they have much larger variabilities than v_t and v_w (see Table 3). Since P_{ind} and σ_f follow the lognormal distribution, it is possible to represent A'_{req}/A_0 with

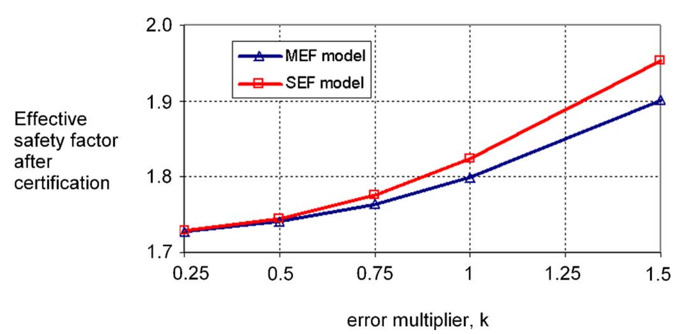

Fig. 7. Total safety factors for the SEF and MEF model after certification

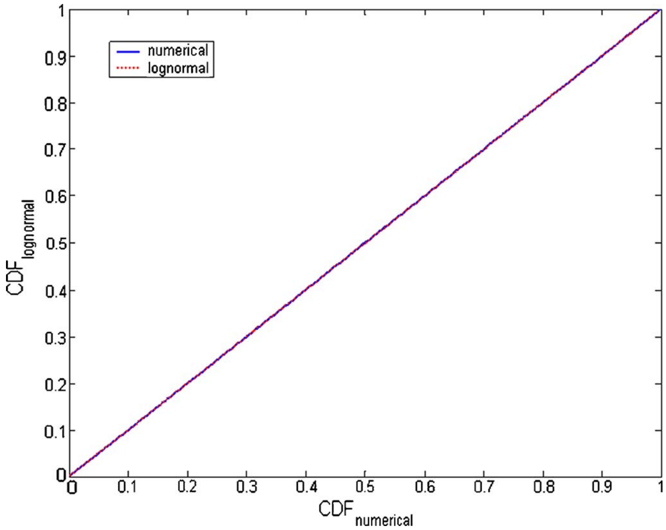

Fig. 8. Comparison of numerical CDF with the assumed lognormal for the distribution of the required safety factor

lognormal distribution. We indeed found that numerical CDF is in good agreement with the assumed lognormal as shown in Fig. 8.

To ensure that the assumed lognormal distribution leads to an accurate probability of failure estimations, we performed the following study. Five different sets of A'_{req}/A_0 values are obtained from five different MCS with 1,000,000 sample size. Then, the probabilities of failure are calculated using the same second-stage random numbers for both numerical CDF and assumed lognormal CDF. Table 15 shows that the probability of failure estimation using assumed lognormal CDF is accurate to the third digit and also has a smaller standard deviation indicating that the numerical noise is reduced.

Fig. 9 represents flowchart of a separable MCS procedure. Stage 1 represents the simulation of variabilities in the actual service conditions to generate the probability of failure as shown in Fig. 3. This probability of failure is one minus the cumulative distribution function (CDF) of the required safety factor $(S_F)_{\mathrm{req}}$. In Stage 1, $M=1{,}000{,}000$ simulations are performed and CDF of $(S_F)_{\mathrm{req}}$ is assessed. A detailed discussion on CDF assessment for $(S_F)_{\mathrm{req}}$ is given in Appendix III.

In Stage 2, $N=1{,}000{,}000$ designs are generated for N different aircraft companies. For each new design, different random error factors e_σ, e_P, e_w, e_t, and e_m are picked from their corresponding distributions to generate the prototype safety factor, $(S_F)_{\mathrm{proto}}$. Then, each design is subjected to certification testing. If it passes, we obtain the probability of failure from the distribution obtained in Stage 1 (Fig. 3). We calculate the average and coefficient of

Table 15. Comparison of the Probability of Failure Estimations

	P_f estimation using numerical CDF ($\times 10^{-4}$)	P_f estimation using assumed lognormal CDF ($\times 10^{-4}$)
MCS 1	8.961	8.855
MCS 2	8.902	8.807
MCS 3	8.901	8.825
MCS 4	8.734	8.856
MCS 5	8.859	8.816
Average	8.871	8.832
s.d.	0.085	0.023

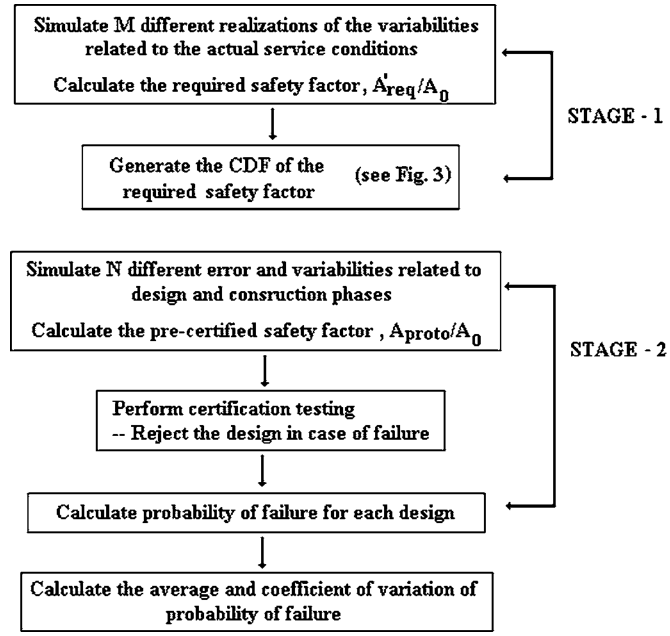

Fig. 9. Flowchart for MCS of component design and failure

variation (c.o.v.) of the failure probability over all designs and explore the effects of error, variability, and safety measures on these values in the "Results" section.

The separable Monte Carlo procedure reduces the computational burden greatly. For instance, if the probability of failure is 2.5×10^{-5}, a million simulations varying both errors and variability simultaneously estimate this probability with 20% error. We found for our problem that the use of the separable Monte Carlo procedure requires only 20,000 simulations (10,000 simulations for Stage 1 and 10,000 for Stage 2) for the same level of accuracy.

Appendix IV. Calculation of the System Failure Probability Using Bivariate Normal Distribution

Bivariate normal distribution describes the joint behavior of two random variables X_1 and X_2, for which the marginal distributions are normally distributed and correlated through the correlation coefficient ρ. The probability density function is defined as [see Melchers (1999)]

$$f_{X_1X_2}(x_1,x_2,\rho)=\frac{1}{2\pi\sigma_{X_1}\sigma_{X_2}}\exp\left(-\frac{1}{2}\frac{h^2+k^2-2\rho hk}{1-\rho^2}\right) \quad (33)$$

where $h=(x_1-\mu_1)/\sigma_1$ and $k=(x_2-\mu_2)/\sigma_2$; μ_1 and σ_1=mean and standard deviation of variable X_1; and μ_2 and σ_2=mean and standard deviation of variable X_2.

The joint cumulative distribution is defined as

$$F_{X_1X_2}(x_1,x_2,\rho)\equiv \Pr\left[\bigcap_{i=1}^{2}(X_i\leqslant x_i)\right] = \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} f_{X_1X_2}(u,v,\rho)\,du\,dv=\Phi_2(x_1,x_2,\rho) \quad (34)$$

In addition, $\Phi_2(\ )$ can be reduced to a single integral (Owen 1956)

$$\Phi_2(h,k,\rho) = \frac{1}{2\pi}\int_0^{\rho} \frac{1}{\sqrt{1-z^2}} \exp\left(-\frac{1}{2}\frac{h^2+k^2-2\rho hk}{1-z^2}\right)dz + \Phi(h)\Phi(k) \quad (35)$$

where Φ=standard normal cumulative distribution function.

The two local failure events requirement of our problem is modeled as a parallel system. Thus we aim at computing the probability of failure of a parallel system composed of two elements having equal failure probabilities. We assume that the limit-state functions for these two elements follow normal distribution. Thus we can use the bivariate normal distribution to calculate the system probability of failure. Since the failure probabilities are identical, the reliability indices are also identical (i.e., $h=k=\beta$). Then Eq. (35) can further be simplified into Eq. (36). Thus, given the probability of failure of a single element and the correlation coefficient ρ, Eq. (36) can be used to calculate system failure probability P_{FS}

$$P_{FS} = \Phi_2(-\beta,-\beta,\rho) = P_f^2 + \frac{1}{2\pi}\int_0^{\rho} \frac{1}{\sqrt{1-z^2}} \exp\left(-\frac{\beta^2}{1+z}\right)dz \quad (36)$$

where P_f and β=probability of failure and the reliability index for a single element, respectively, which are related to each other through Eq. (37)

$$P_f = \Phi(-\beta) \quad (37)$$

Notation

The following symbols are used in this paper:

A_{ind}, A_{proto}, A_{cert} = individual, prototype, and after certification values of load carrying area, respectively;

A'_{req} = minimum required load carrying area for an individual structural part to carry the loading under actual service conditions without failure;

A_0 = load carrying area if no safety measures applied;

e_m, e_P, e_σ, e_t, e_w = error factor for material failure stress, load, stress, thickness, and width;

e_{total} = cumulative effect of various errors;

k = error multiplier;

M = number of simulations in the first stage of MCS;

N = number of simulations in the second stage of MCS;

P_{ind}, P_{calc}, P_d = individual, calculated, and design load, respectively;

$\bar{P}_{nc}$, $\bar{P}_c$ = average value of probability of failure before and after certification;

S_{FL} = load safety factor (FAA code requirement);

$(S_F)_{proto}$, $(S_F)_{cert}$, $(S_F)_{req}$ = prototype, certified, and required total safety factor, respectively;

t_{ind} = thickness of an individual structural part;

t_{proto} = average value of component thickness built by an aircraft company prior to certification test;

w_{ind} = width of an individual structural part;

w_{proto} = average value of component width built by an aircraft company prior to certification test; and

σ_f = failure stress.

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