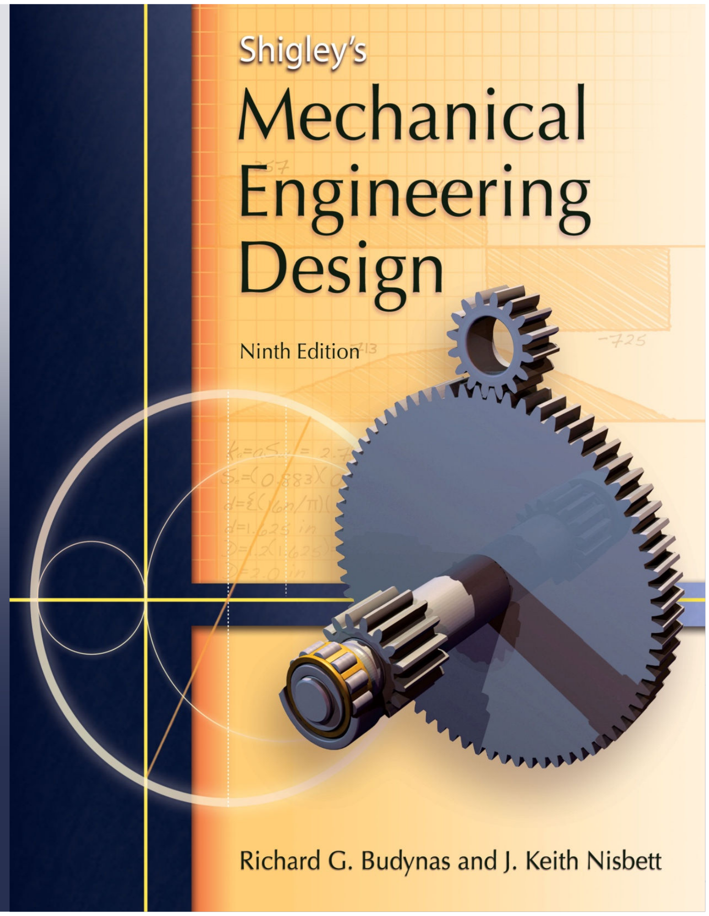
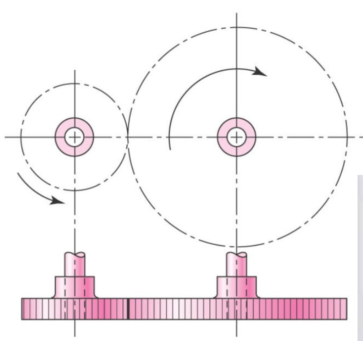
Lecture Slides

Chapter 13

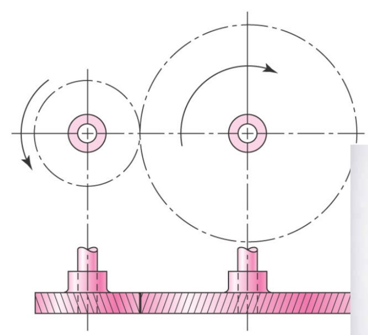
Gears – General



Types of Gears



Spur gear
(düz dişli)

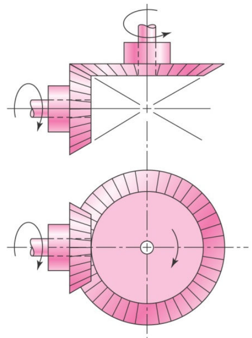


Helical gear
(helisel dişli)

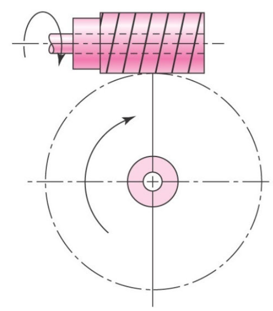
- The simplest type
- Teeth parallel to the axis of rotation
- Transmit motion between parallel shafts

- Teeth inclined to the axis of rotation
- Transmit motion between parallel or nonparallel shafts
- Quieter than spur gears because of the more gradual engagement of teeth during meshing

Types of Gears



Bevel gear
(konik dişli)



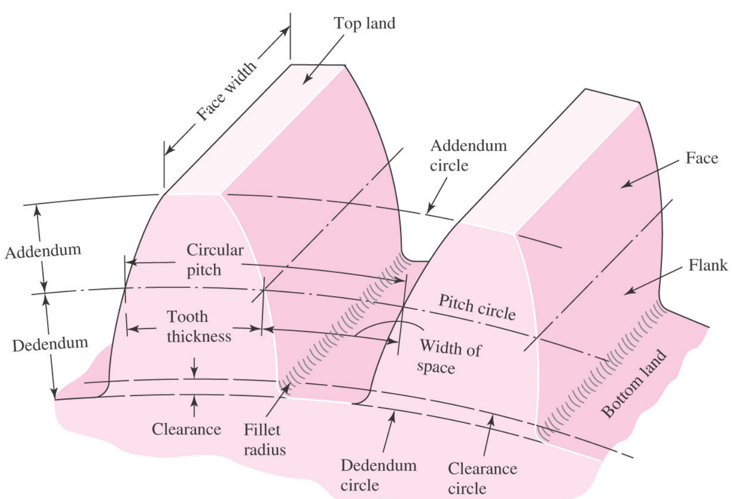
Worm gear
(sonsuz dişli)

Güç akımını 90° değiştirmek için kullanılır.

- Teeth formed on conical surfaces
- Transmit motion between intersecting shafts

- Transmit motion between nonparallel and nonintersecting shafts

Nomenclature of Spur-Gear Teeth



$$P = \frac{N}{d}$$

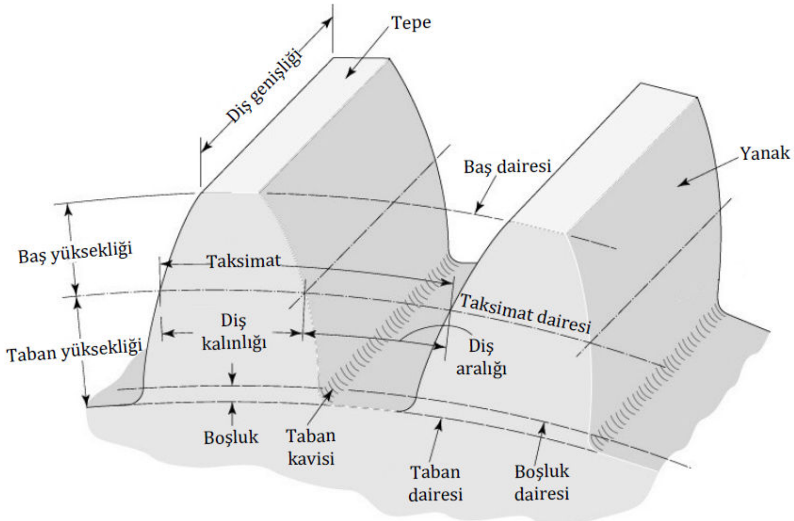
$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

Dişli Terminolojisi (Düz Dişli)



$$m = \frac{d}{N}$$

$$p = \frac{\pi\, d}{N} = \pi\, m$$

m : modül
d : taksimat dairesi çapı
N : diş sayısı
p : taksimat (hatve)

Tooth Action

- First point of contact at *a* where flank of pinion touches tip of gear
- Last point of contact at *b* where tip of pinion touches flank of gear
- Line *ab* is *line of action*
- *Angle of action* is sum of *angle of approach* and *angle of recess*

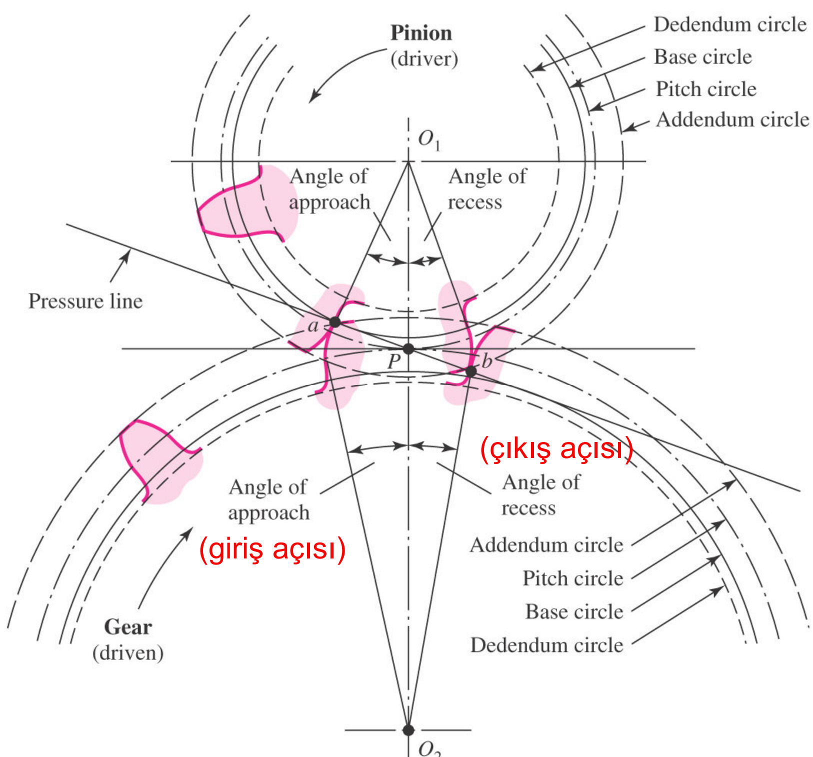


Fig. 13–12

Relation of Base Circle to Pressure Angle

$$r_b = r \cos \phi \qquad (13\text{–}6)$$

Hareketin iletilebilmesi için, dişlerin temel dairenin üst kısmında temas etmesi gerekir.

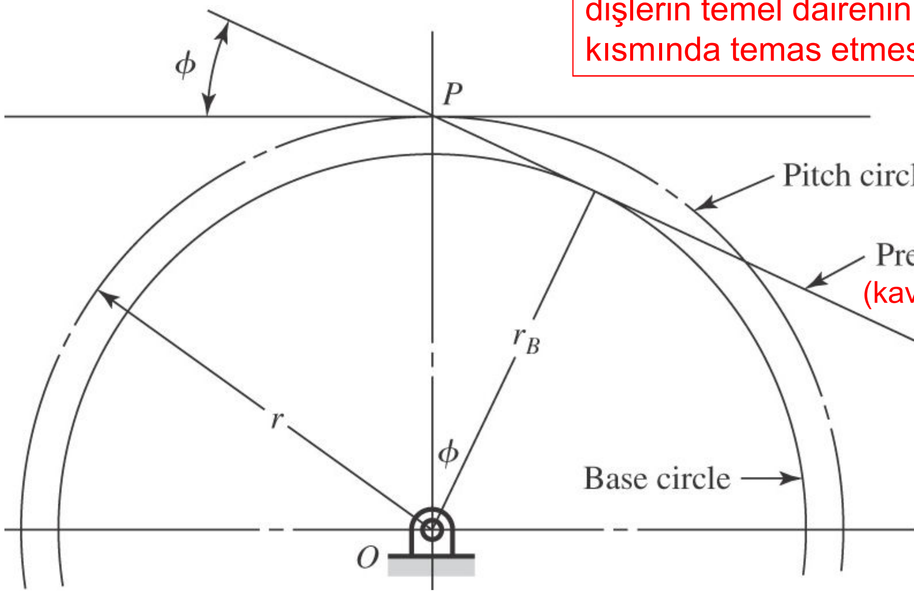


Fig. 13–10

Rack (Kremayer Dişli)

- A *rack* is a spur gear with an pitch diameter of infinity.
- The sides of the teeth are straight lines making an angle to the line of centers equal to the pressure angle.
- The *base pitch* and *circular pitch*, shown in Fig. 13–13, are related by

$$p_b = p_c \cos \phi$$

Dairesel hareketin doğrusal harekete çevrilmesi amacıyla kullanılır (örnek: iş tezgahlarındaki tablalarda)

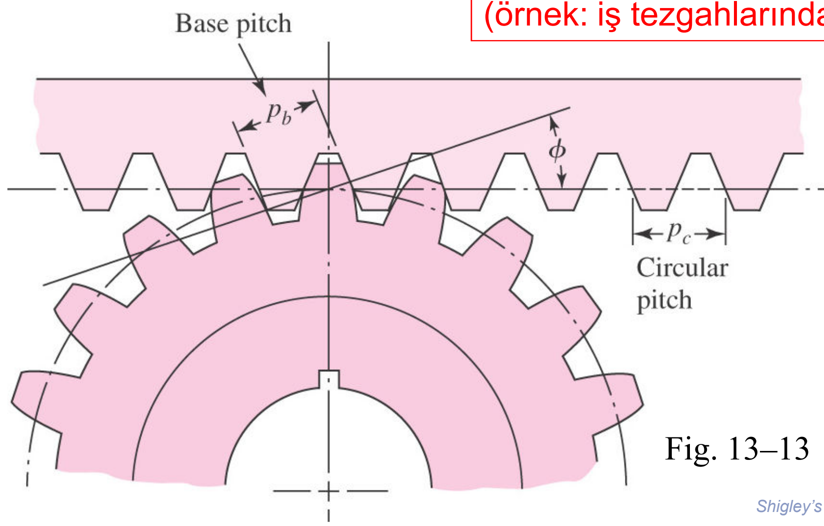


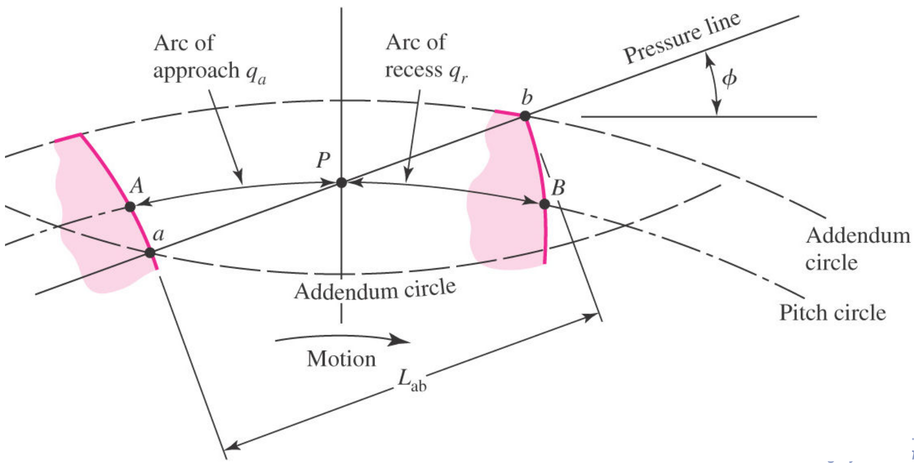
Fig. 13–13

Contact Ratio (Kavrama Oranı)

- Arc of action q_t is the sum of the arc of approach q_a and the arc of recess q_r, that is $q_t = q_a + q_r$
- The *contact ratio* m_c is the ratio of the arc of action and the circular pitch.

$$m_c = \frac{q_t}{p} \qquad m_c = \frac{L_{ab}}{p\cos\phi} \qquad (13\text{–}9)$$

- The contact ratio is the average number of pairs of teeth in contact. Contact ratio should be at least 1.2.



Interference (interferans)

- *Interference* occurs when contact occurs below the base circle
- Should be avoided.

Aksi takdirde, alt kesilme (undercutting) meydana gelir.

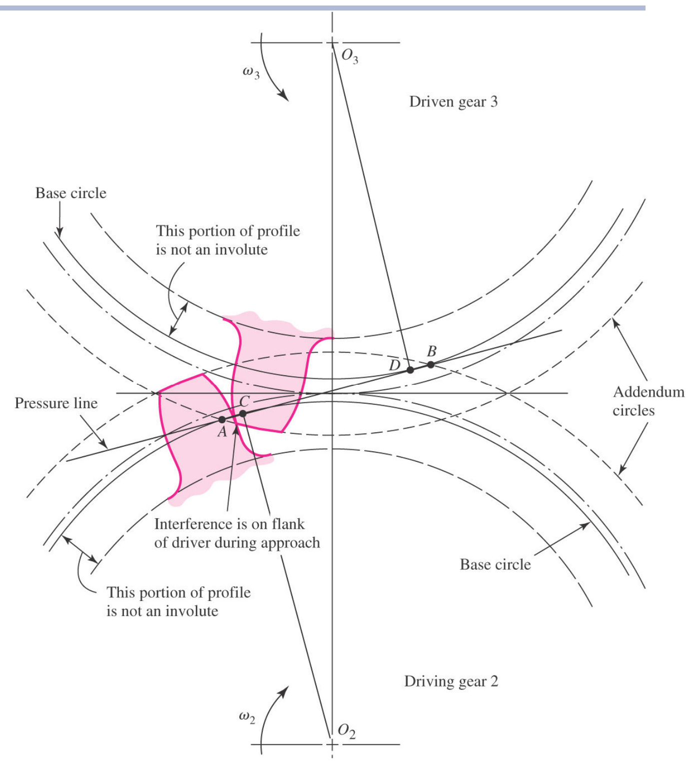


Fig. 13–16

Interference

- Interference can be eliminated by using more teeth on the pinion.
- However, if tooth size (that is diametral pitch *P*) is to be maintained, then an increase in teeth means **an increase in diameter**, since $P = N/d$.

- Interference can also be eliminated by using a larger pressure angle. This results in a smaller base circle, so more of the tooth profile is involute.
- This is the primary reason for larger pressure angle.
- Note that the disadvantage of a larger pressure angle is **an increase in radial force for the same amount of transmitted force**.

Interference of Spur Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- k =1 for full depth teeth. k = 0.8 for stub teeth (çok yüksek tork aktarıldığında meydana gelecek olan eğilme gerilmesini azaltmak için kullanılır).
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the **smallest pinion** which will not have interference is

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

- **Largest gear** with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

Interference

- For 20º pressure angle, the most useful values from Eqs. (13–11) and (13–12) are calculated and shown in the table below.

Minimum N_P	Max N_G	Integer Max N_G	Max Gear Ratio $m_G = N_G/N_P$
13	16.45	16	1.23
14	26.12	26	1.86
15	45.49	45	3
16	101.07	101	6.31
17	1309.86	1309	77

- Increasing the pressure angle to 25º allows smaller numbers of teeth

Minimum N_P	Max N_G	Integer Max N_G	Max Gear Ratio $m_G = N_G/N_P$
9	13.33	13	1.44
10	32.39	32	3.2
11	249.23	249	22.64

Forming of Gear Teeth

- **Common ways of forming gear teeth**
  - Sand casting
  - Shell molding
  - Investment casting
  - Permanent-mold casting
  - Die casting
  - Centrifugal casting
  - Powder-metallurgy
  - Extrusion
  - Injection molding (for thermoplastics)
  - Cold forming
- **Common ways of cutting gear teeth**
  - Milling
  - Shaping
  - Hobbing

Parallel Helical Gears

- Similar to spur gears, but with teeth making a *helix angle* with respect to the gear centerline
- Adds axial force component to shaft and bearings
- Smoother transition of force between mating teeth due to gradual engagement and disengagement

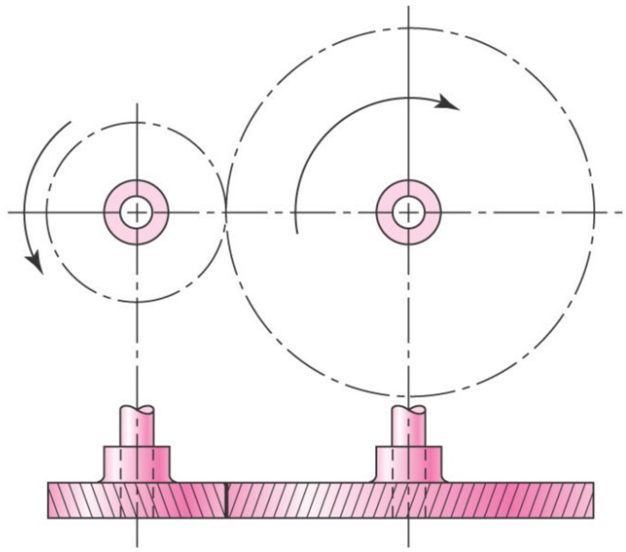


Fig. 13–2

Parallel Helical Gears

- *Transverse circular pitch* p_t is in the plane of rotation
- *Normal circular pitch* p_n is in the plane perpendicular to the teeth

$$p_n = p_t \cos\psi \qquad (13\text{–}16)$$

- *Axial pitch* p_x is along the direction of the shaft axis

$$p_x = \frac{p_t}{\tan\psi} \qquad (13\text{–}17)$$

- *Normal diametral pitch*

$$P_n = \frac{P_t}{\cos\psi} \qquad (13\text{–}18)$$

$$p_n P_n = \pi$$

- *Relationship between angles*

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \qquad (13\text{–}19)$$

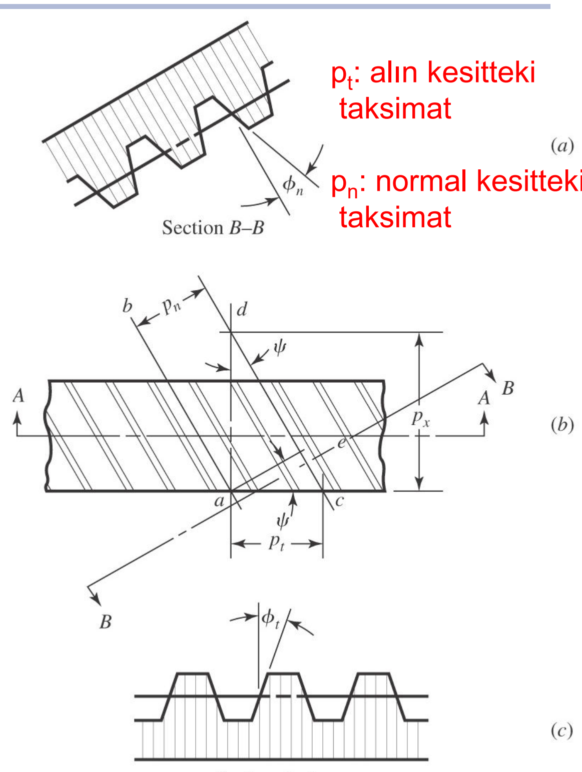


Fig. 13–22

Interference with Helical Gears

- When meshed with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right) \qquad (13\text{–}21)$$

- k =1 for full depth teeth. k = 0.8 for stub teeth

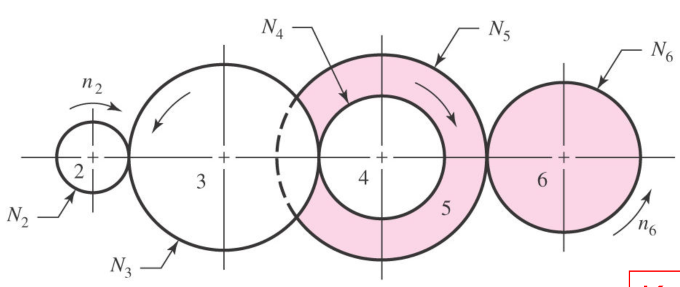
- When meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the **smallest pinion** which will not have interference is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m+\sqrt{m^2+(1+2m)\sin^2\phi_t}\right] \qquad (13\text{–}22)$$

- **Largest gear** with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \qquad (13\text{–}23)$$

Gear Trains



For a pinion 2 driving a gear 3, the speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3}n_2\right| = \left|\frac{d_2}{d_3}n_2\right|$$

Kaymadan yuvarlanma:
Çizgisel hızlar eşittir.
$r_2\,\omega_2 = r_3\,\omega_3$
=> $N_2\,n_2 = N_3\,n_3$

TRAIN VALUE:

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \qquad (13\text{–}30)$$

$$n_L = e n_F \qquad (13\text{–}31)$$

Compound Gear Train

- A **practical limit** on train value for one pair of gears is **10 to 1**
- To obtain more, compound two gears onto the same shaft

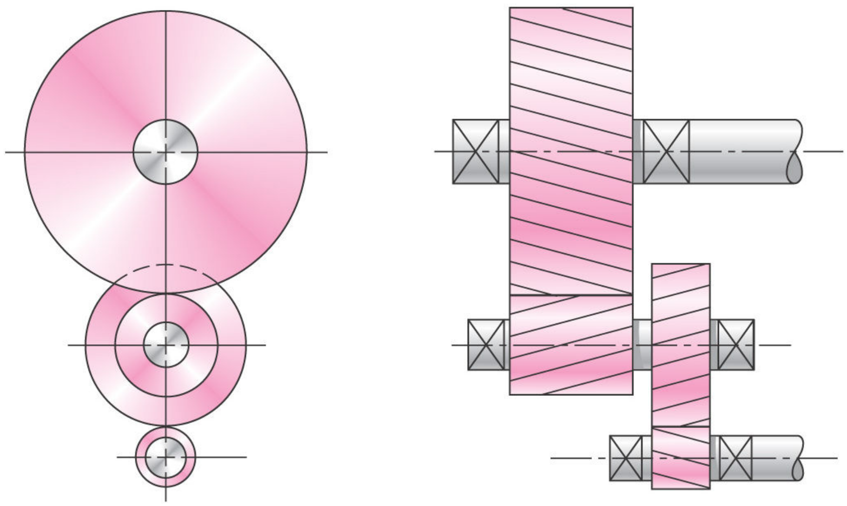


Fig. 13–28

Example 13–3

A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Example 13–4

A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Compound Reverted Gear Train

- A compound gear train with input and output shafts in-line
- Geometry condition must be satisfied

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$
$$P = N/d$$
$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$
$$N_2 + N_3 = N_4 + N_5$$

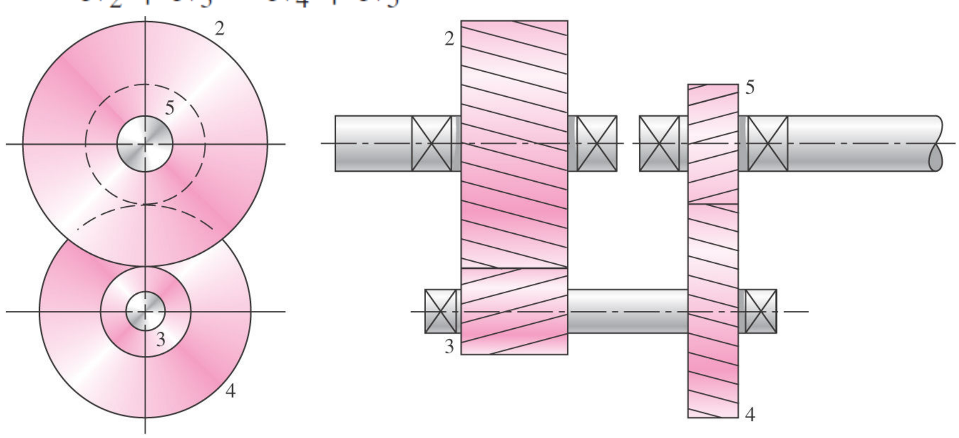


Fig. 13–29

Example 13–5

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Force Analysis – Spur Gearing

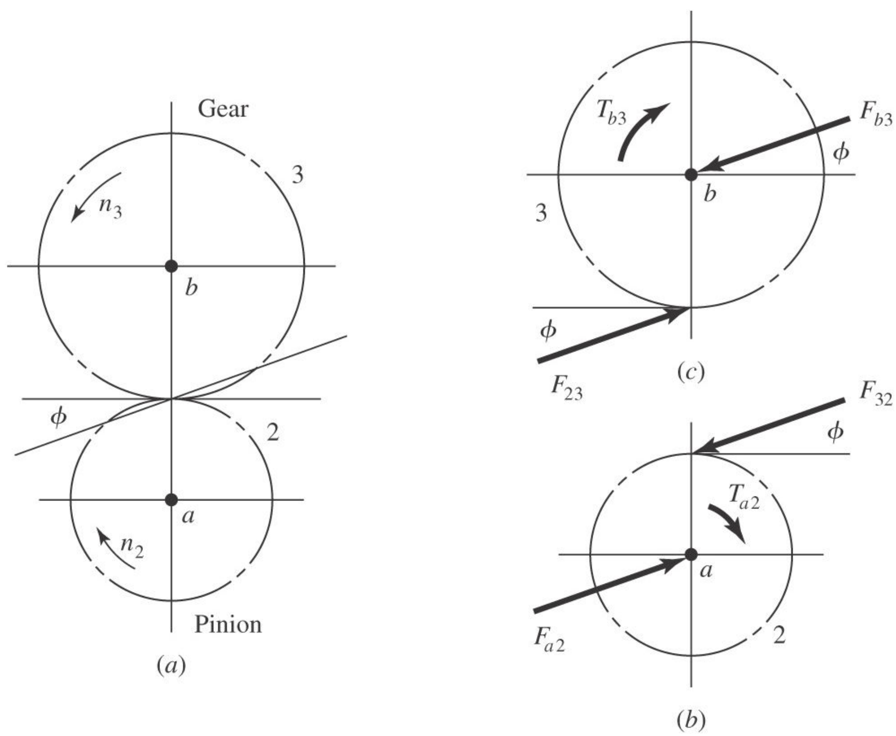


Fig. 13–32

Force Analysis – Spur Gearing

- *Transmitted load W_t* is the tangential load

$$W_t = F^t_{32}$$

- It is the useful component of force, transmitting the torque

$$T = \frac{d}{2} W_t$$

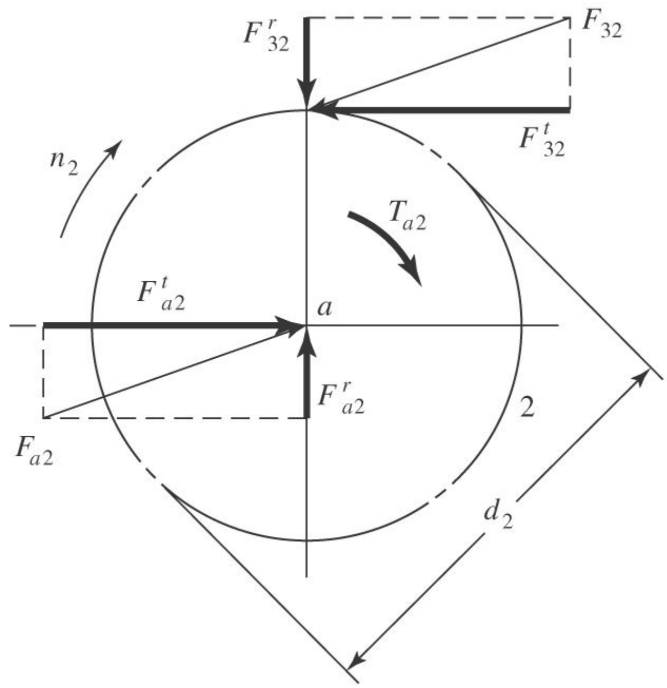


Fig. 13–33

Power in Spur Gearing

- *Transmitted power H*

$$H = T\omega = (W_t d/2)\,\omega \qquad (13\text{–}33)$$

- *Pitch-line velocity* is the linear velocity of a point on the gear at the radius of the pitch circle. It is a common term in tabulating gear data.

$$V = \pi d n/12 \qquad (13\text{–}34)$$

where V = pitch-line velocity, ft/min
d = gear diameter, in
n = gear speed, rev/min

Power in Spur Gearing

- Useful power relation in **US units,**

$$W_t = 33\,000\frac{H}{V} \qquad (13\text{–}35)$$

where W_t = transmitted load, lbf
H = power, hp
V = pitch-line velocity, ft/min

- In SI units,

$$W_t = \frac{60\,000H}{\pi d n} \qquad (13\text{–}36)$$

where W_t = transmitted load, kN
H = power, kW
d = gear diameter, mm
n = speed, rev/min

Example 13–7

Pinion 2 in Fig. 13–34*a* runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

idler gear: avare dişli (şafta tork aktarmaz)

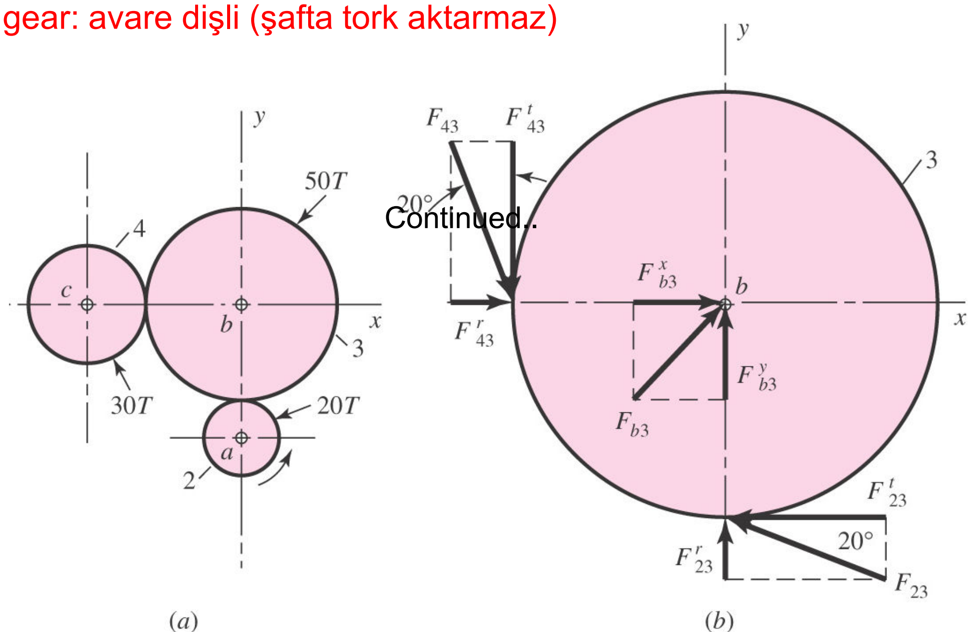


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Fig. 13–34

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Force Analysis – Helical Gearing

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi \quad (13\text{–}39)$$

$$W_a = W \cos \phi_n \sin \psi$$

Usually, we know W_t, so

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi \quad (13\text{–}40)$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

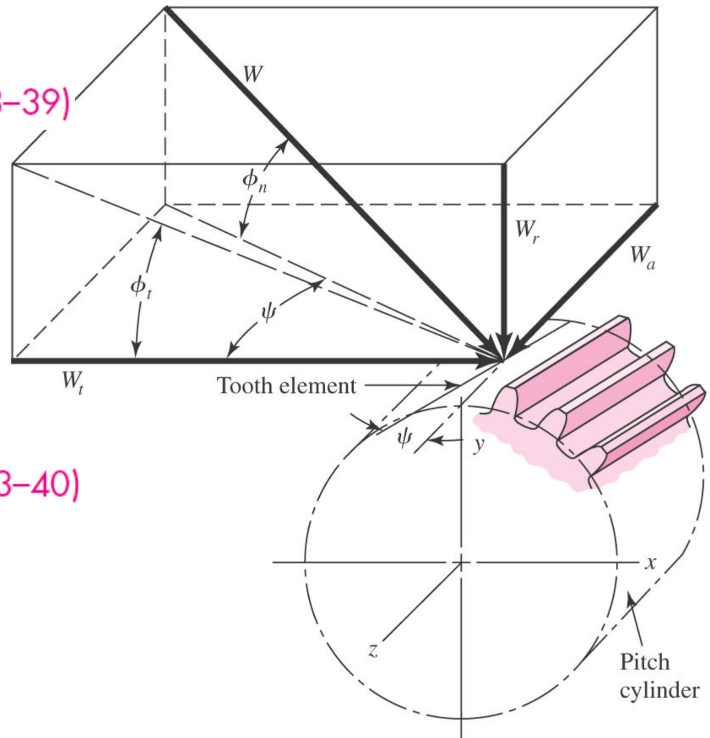

Fig. 13–37

Example 13–9

In Fig. 13–38 a 1-hp electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

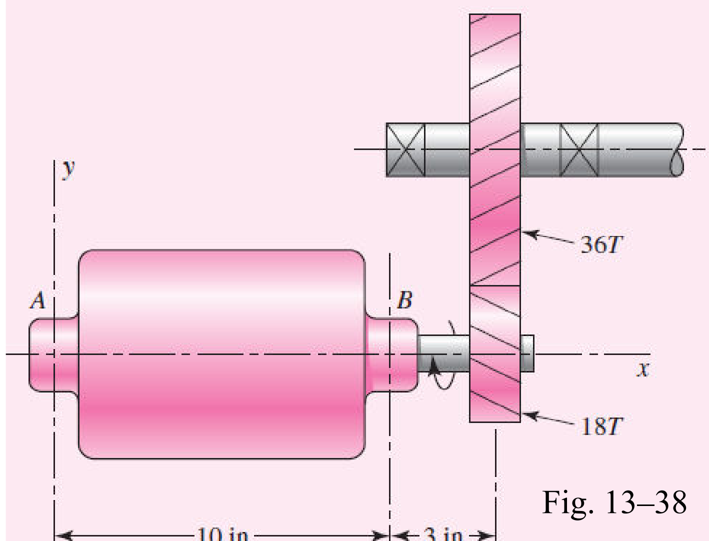

Fig. 13–38

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