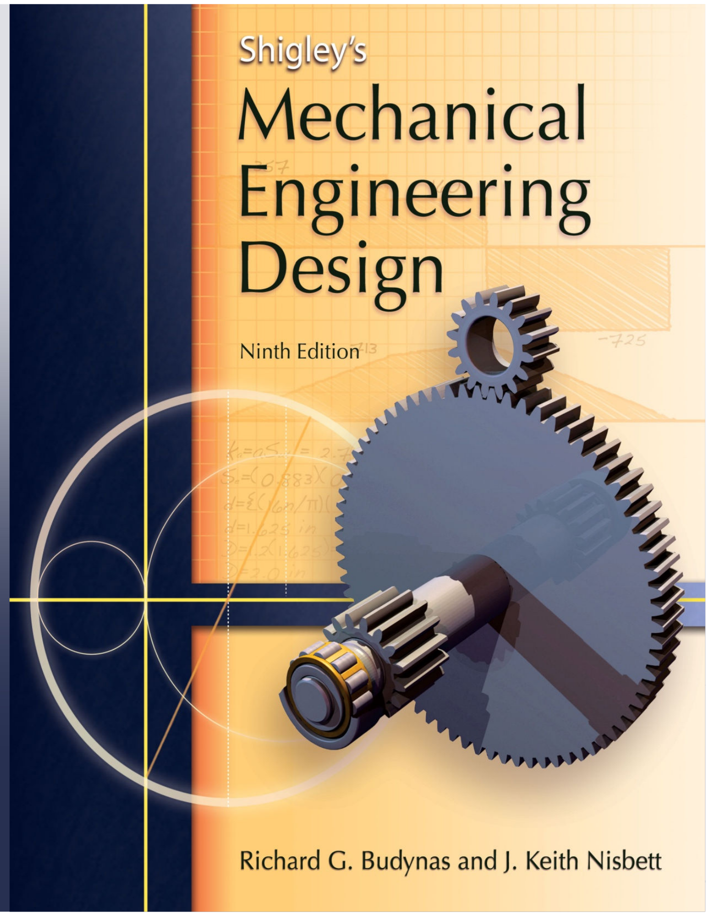
Lecture Slides



Chapter 7

Shafts and Shaft Components

Chapter Outline

Common Shaft Types

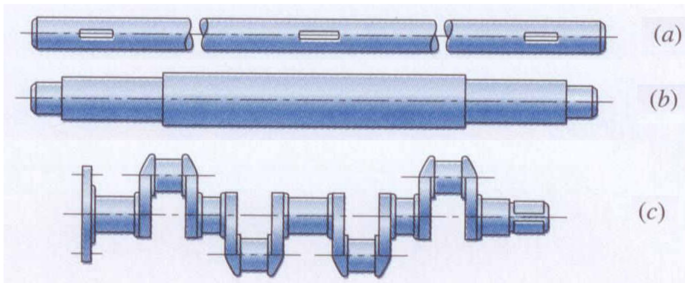


Figure 9.1 Common shaft types: (a) constant diameter; (b) stepped; (c) crank shaft.

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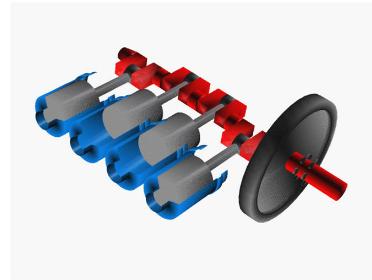
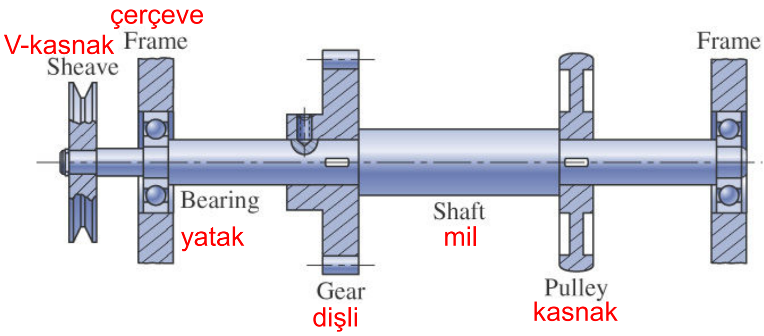


Figures from A.C.Ugural Mechanical Design - An Integrated Approach



Shaft Layout

- **Issues to consider:**
  - Axial layout of components
  - Supporting axial loads
  - Providing for torque transmission
  - Assembly and Disassembly

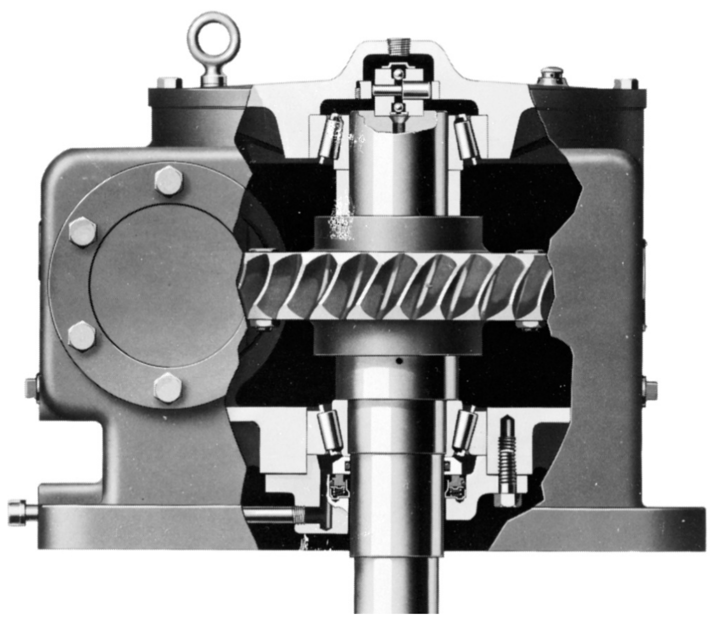
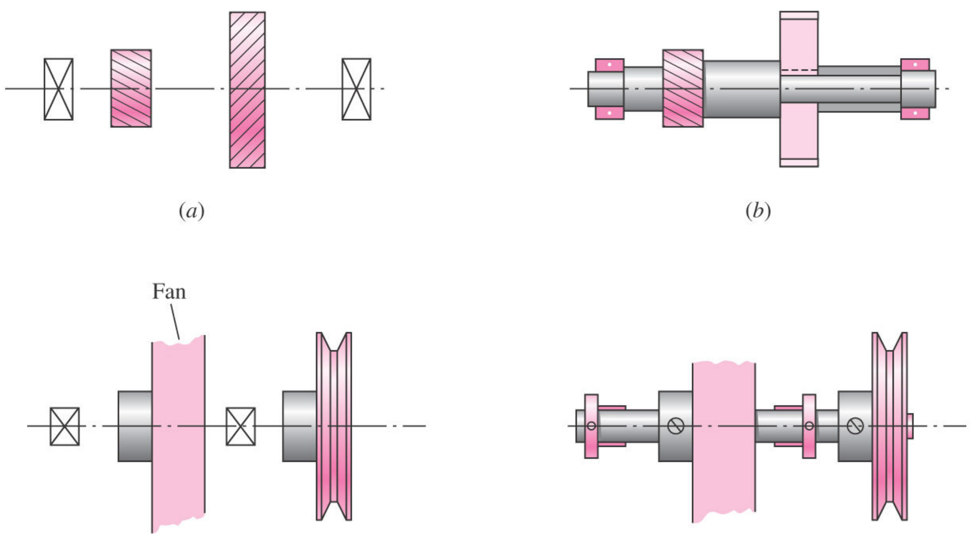


Fig. 7-1

Axial Layout of Components

- It is best to support load-carrying components between bearings (see Fig.a,b), rather than cantilevered outboard of the bearings (see Fig.c,d).
- Only two bearings are used for the most cases (except for very long shafts).



(a) (b) (c) (d)

Shaft Design for Stress

- Stresses are only evaluated at critical locations
- Critical locations are usually
  - On the outer surface
  - Where the bending moment is large
  - Where the torque is present
  - Where stress concentrations exist

Shaft Stresses

- Standard stress equations can be customized for shafts for convenience
- Axial loads are generally small and constant, so will be ignored in this section
- Standard alternating and midrange stresses

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I} \tag{7-1}$$

$$\tau_a = K_{fs} \frac{T_a c}{J} \qquad \tau_m = K_{fs} \frac{T_m c}{J} \tag{7-2}$$

- Customized for round shafts

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3} \tag{7-3}$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3} \tag{7-4}$$

Shaft Stresses

- Combine stresses into von Mises stresses

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{7-5}$$

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{7-6}$$

Shaft Stresses

- Substitute von Mises stresses into failure criteria equation. For example, using modified Goodman line,

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs}T_m)^2\right]^{1/2}\right\} \quad (7\text{–}7)$$

- Solving for d is convenient for design purposes

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs}T_m)^2\right]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}8)$$

Shaft Stresses

- Similar approach can be taken with any of the fatigue failure criteria
- Equations are referred to by referencing both the Distortion Energy method of combining stresses and the fatigue failure locus name. For example, *DE-Goodman*, *DE-Gerber,* etc.
- ***DE-Gerber:***

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}10)$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs}T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs}T_m)^2}$$

Shaft Stresses for Rotating Shaft

- For rotating shaft with steady bending and torsion
  - **Bending stress is completely reversed**, since a stress element on the surface cycles from equal tension to compression during each rotation
  - **Torsional stress is steady**
  - Previous equations simplify with M_m and T_a equal to 0

$$d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right\} \right)^{1/3} \quad (7\text{–}8)$$

$$d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} (2 K_f M_a) + \frac{1}{S_{ut}} (\sqrt{3} K_{fs} T_m) \right\} \right)^{1/3}$$

Checking for Yielding in Shafts

- **Always necessary to consider static failure, even in fatigue situation**
- Use von Mises maximum stress to check for yielding,

$$\sigma'_{max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2 \right]^{1/2} = \left[\left(\frac{32 K_f (M_m + M_a)}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} (T_m + T_a)}{\pi d^3} \right)^2 \right]^{1/2} \quad (7\text{–}15)$$

$$n_y = \frac{S_y}{\sigma'_{max}} \quad (7\text{–}16)$$

- Alternate simple check is to obtain **conservative estimate** of σ'_{max} by summing σ'_a and σ'_m

$$\sigma'_{max} \doteq \sigma'_a + \sigma'_m$$

Example 7-1

At a machined shaft shoulder the small diameter d is 1.100 in, the large diameter D is 1.65 in, and the fillet radius is 0.11 in. The bending moment is 1260 lbf · in and the steady torsion moment is 1100 lbf · in. The heat-treated steel shaft has an ultimate strength of $S_{ut} = 105$ kpsi and a yield strength of $S_y = 82$ kpsi. The reliability goal is 0.99.

(a) Determine the fatigue factor of safety of the design using each of the fatigue failure criteria described in this section.

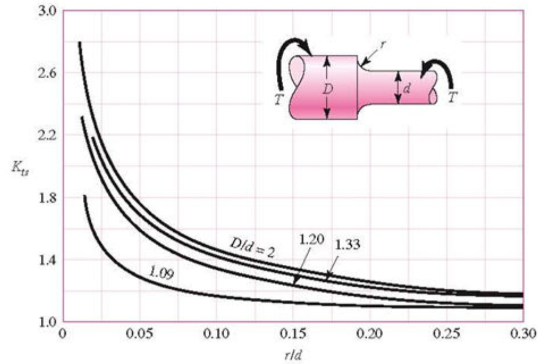
(b) Determine the yielding factor of safety.

Continued..

Determine Kt and Kts from figures

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

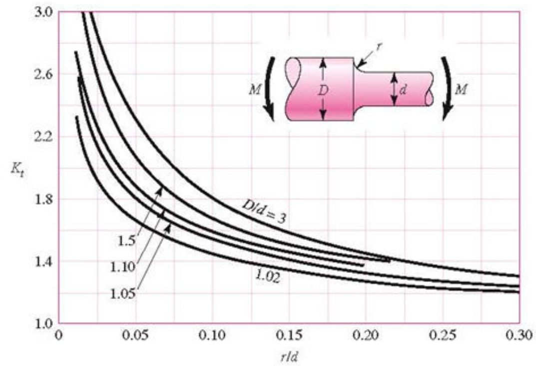


D/d=1.50
r/d=01.0

Kts=1.42

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Kt=1.68

Determine q and qs from figures

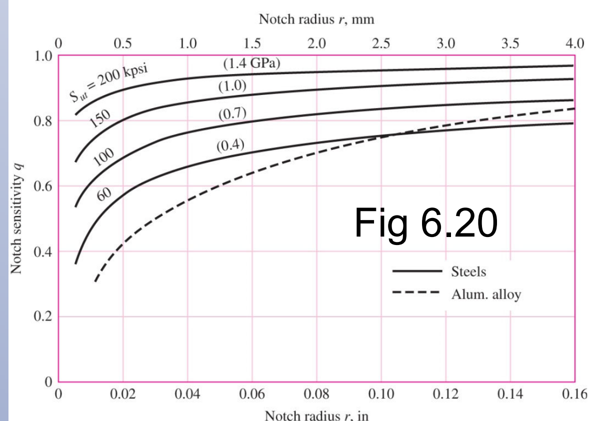


Fig 6.20

r=0.11 in
Sut=105 ksi

q=0.85

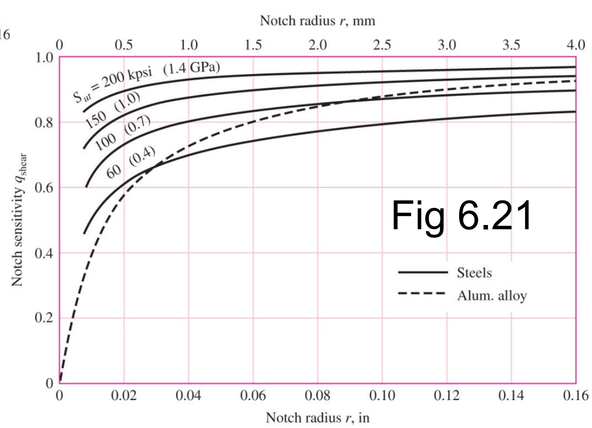


Fig 6.21

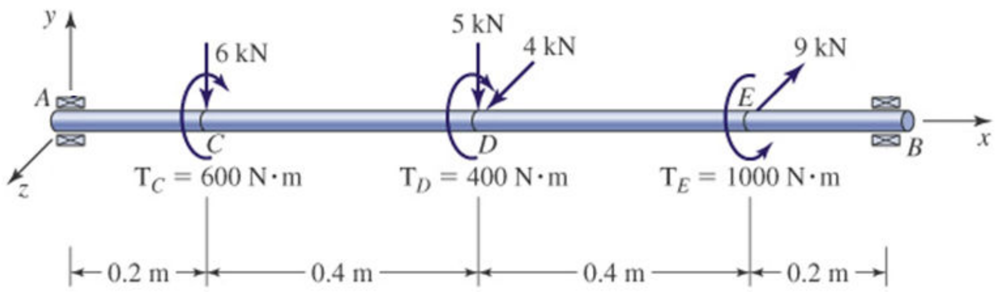
qs=0.88

Additional Example (from Ugural's book)

EXAMPLE 9.2

Shaft Design for Repeated Torsion and Bending

Power is transmitted from a motor through a gear at E to pulleys at D and C of a revolving solid shaft AB with ground surface. Figure 9.3a shows the corresponding load diagram of the shaft. The shaft is mounted on bearings at the ends A and B. Determine the required diameter of the shaft by employing the maximum energy of distortion theory of failure incorporating the Soderberg fatigue relation.



Given: The shaft is made of steel with an ultimate strength of 810 MPa and a yield strength of 605 MPa. Torque fluctuates 10% each way from the mean value. The fatigue stress-concentration factor for bending and torsion is equal to 1.4. The operating temperature is 500°C maximum.

Design Assumptions: Bearings act as simple supports. A factor of safety of $n = 2$ is used. The survival rate is taken to be 50%.

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