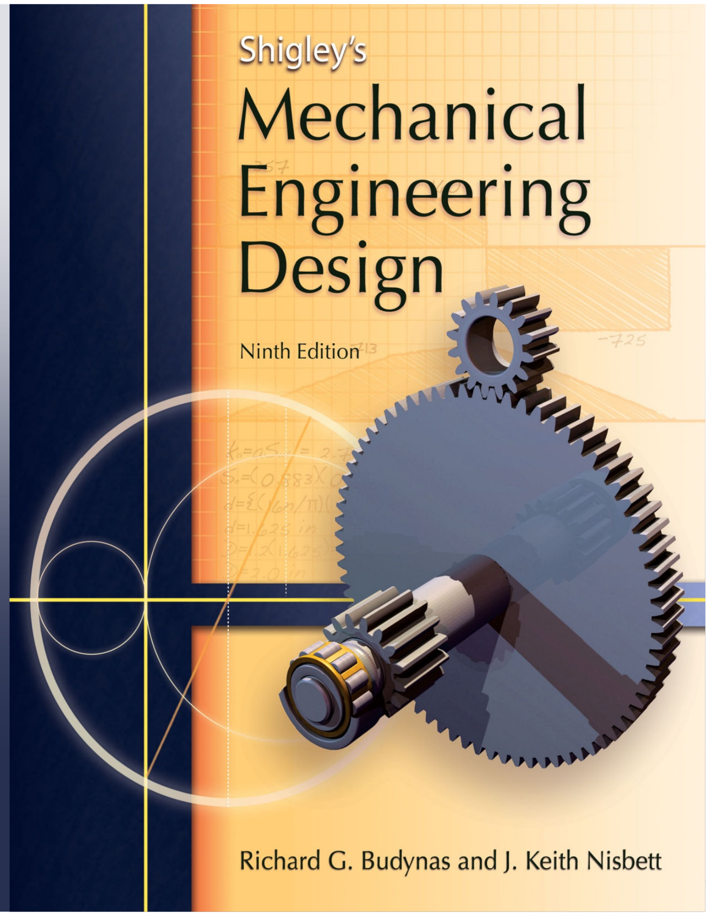
Lecture Slides



Chapter 6

Fatigue Failure Resulting from Variable Loading

Chapter Outline

6–1 Introduction to Fatigue in Metals 266
6–2 Approach to Fatigue Failure in Analysis and Design 272
6–3 Fatigue-Life Methods 273
6–4 The Stress-Life Method 273
6–5 The Strain-Life Method 276
6–6 The Linear-Elastic Fracture Mechanics Method 278
6–7 The Endurance Limit 282
6–8 Fatigue Strength 283
6–9 Endurance Limit Modifying Factors 286
6–10 Stress Concentration and Notch Sensitivity 295
6–11 Characterizing Fluctuating Stresses 300
6–12 Fatigue Failure Criteria for Fluctuating Stress 303
6–13 Torsional Fatigue Strength under Fluctuating Stresses 317
6–14 Combinations of Loading Modes 317
6–15 Varying, Fluctuating Stresses; Cumulative Fatigue Damage 321
6–16 Surface Fatigue Strength 327
6–17 Stochastic Analysis 330
6–18 Road Maps and Important Design Equations for the Stress-Life Method 344

Introduction to Fatigue in Metals

- Fatigue is a process which causes premature irreversible damage or failure of a component subjected to repeated loading.
- Maximum stresses well below yield strength
- Failure occurs after many stress cycles
- Failure is by sudden ultimate fracture
- No visible warning in advance of failure

Stages of Fatigue Failure

- *Stage I* – Initiation of micro-crack due to cyclic plastic deformation
- *Stage II* – Progresses to macro-crack that repeatedly opens and closes, creating bands called *beach marks*
- *Stage III* – Crack has propagated far enough that remaining material is insufficient to carry the load, and fails by simple ultimate failure

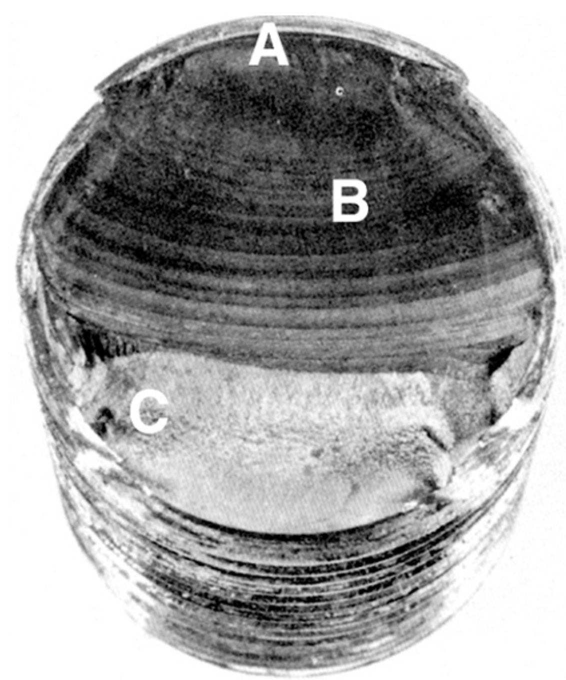


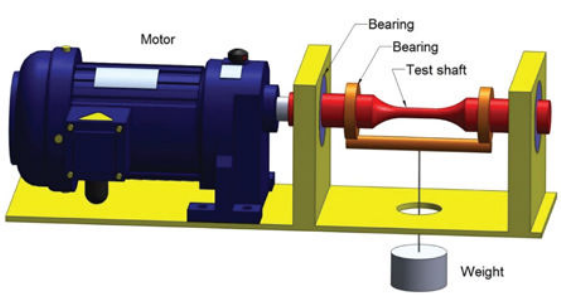
Fig. 6–1

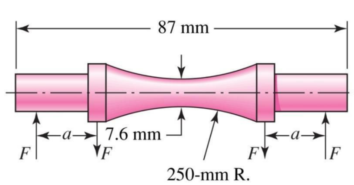
Fatigue-Life Methods

- Three major fatigue life models
  - to predict number of cycles to failure, N, for a specific level of loading
- 1- Stress-life method
  - Least accurate, particularly for low cycle applications
  - Most traditional, easiest to implement
- 2- Strain-life method
  - Detailed analysis of plastic deformation at localized regions
  - Several idealizations are compounded, leading to uncertainties in results
- 3- Linear-elastic fracture mechanics (LEFM) method
  - Assumes crack exists
  - Predicts crack growth with respect to stress intensity

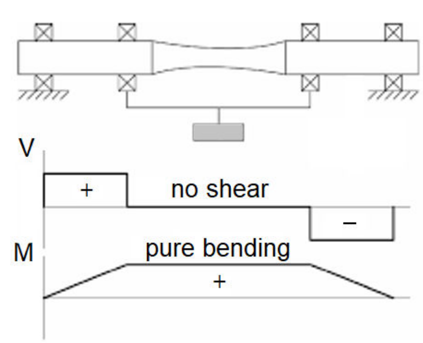
1 - Stress-Life Method

- Test specimens are subjected to repeated stress while counting cycles to failure
- Most common test machine is R. R. Moore high-speed rotating-beam machine
- Subjects specimen to pure bending with no transverse shear
- As specimen rotates, stress fluctuates between equal magnitudes of tension and compression, known as *completely reversed* stress cycling





Test shaft dimensions



S-N Diagram for Steel

- Number of cycles to failure at varying stress levels is plotted on **log-log scale**
- For steels, a knee occurs near 10^6 cycles
- Strength corresponding to the knee is called *endurance limit* S_e

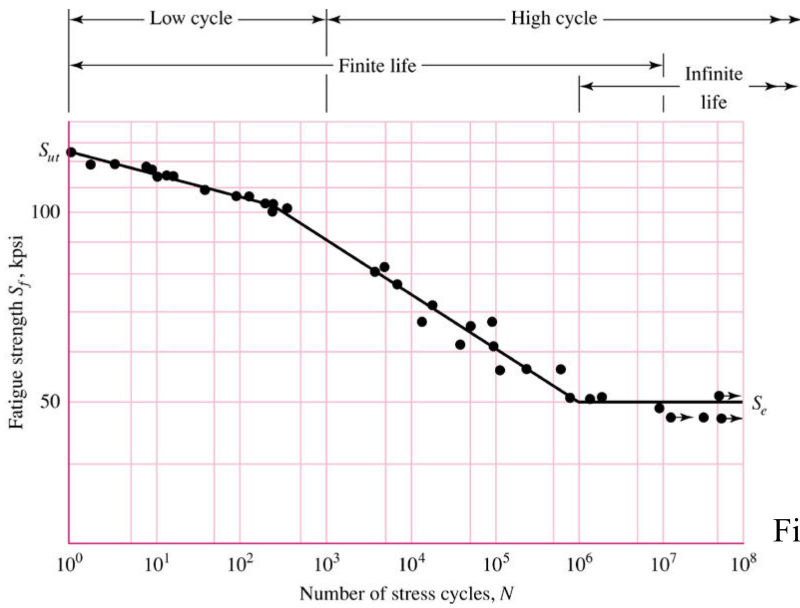


Fig. 6–10

S-N Diagram for Nonferrous Metals

- Nonferrous metals often do not have an endurance limit.
- Fatigue strength S_f is reported at a specific number of cycles
- Figure 6–11 shows typical *S-N* diagram for aluminum alloys

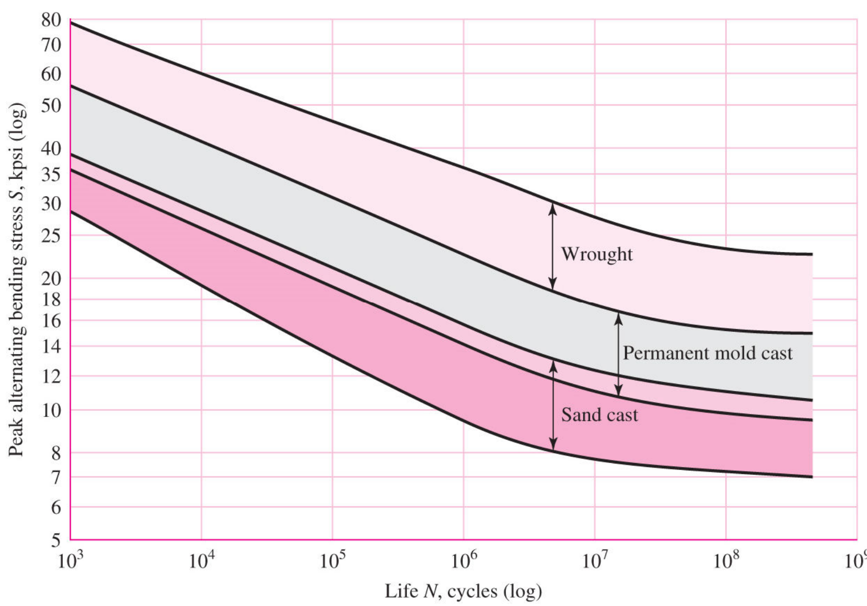


Fig. 6–11

2 - Strain-Life Method

- Total strain amplitude is half the total strain range

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

- For plastic strain amplitude $\frac{\Delta\varepsilon_p}{2} = \varepsilon'_F (2N)^c$
- For elastic strain amplitude $\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_F}{E} (2N)^b$
- Coffin-Manson relationship $\frac{\Delta\varepsilon}{2} = \frac{\sigma'_F}{E} (2N)^b + \varepsilon'_F (2N)^c$
- After performing a detailed analysis of plastic deformation, the numbers to failure can be calculated via Coffin-Manson relation.
- ***We will not cover this method in this course.***

3 - Linear-Elastic Fracture Mechanics Method

- Assumes Stage I fatigue (crack initiation) has occurred
- Predicts crack growth in Stage II with respect to stress intensity
- Stage III ultimate fracture occurs when the stress intensity factor K_I reaches some critical level K_{IC}
- ***We will not cover LEFM in this course.***

Application of the Stress Life Method

- Write equation for *S-N* line from 10^3 to 10^6 cycles
- **Two known points:**
- At $N = 10^3$ cycles, $S_f = f\,S_{ut}$
- At $N = 10^6$ cycles, $S_f = S_e$
- Equations for line:

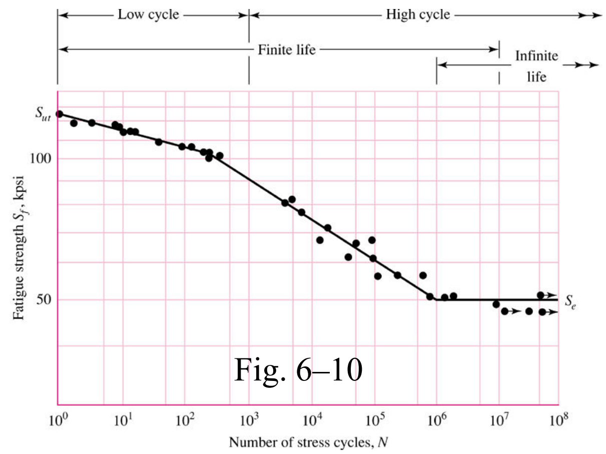


Fig. 6–10

$$S_f = a\,N^b \qquad (6\text{–}13)$$

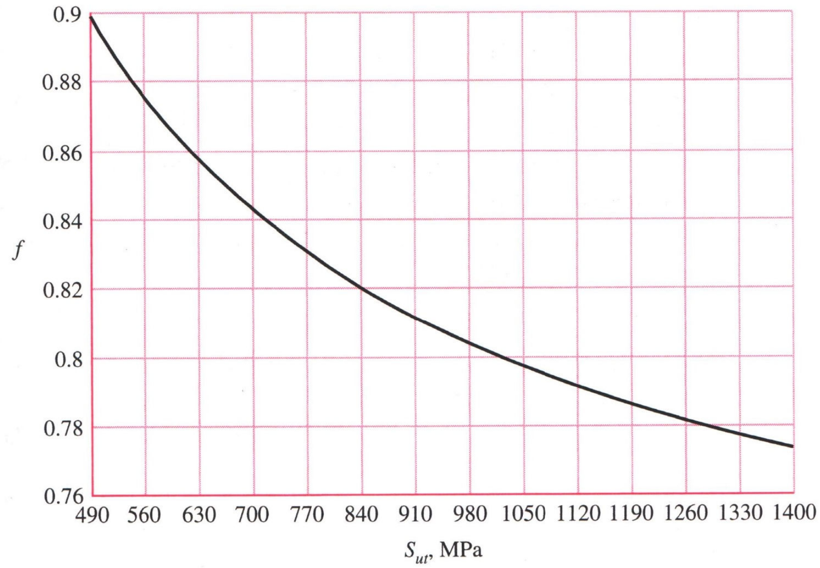
$$a = \frac{(f\,S_{ut})^2}{S_e} \qquad (6\text{–}14)$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) \qquad (6\text{–}15)$$

Continued..

Fatigue Strength Fraction f

f : ratio of the fatigue strength at N=10^3 cycles to the ultimate tensile strength (i.e., fatigue strength at N=1 cycle)



$$f = \frac{\left(S_f\right)_{N=10^3}}{\left(S_f\right)_{N=1}}$$

That is,

$$f = \frac{\left(S_f\right)_{N=10^3}}{S_{ut}}$$

Fig 6-18: Fatigue strength fraction, f, of S_{ut} at 10^3 cycles

The Endurance Limit

- Simplified estimate of endurance limit for steels for the rotating-beam specimen, S'_e

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$

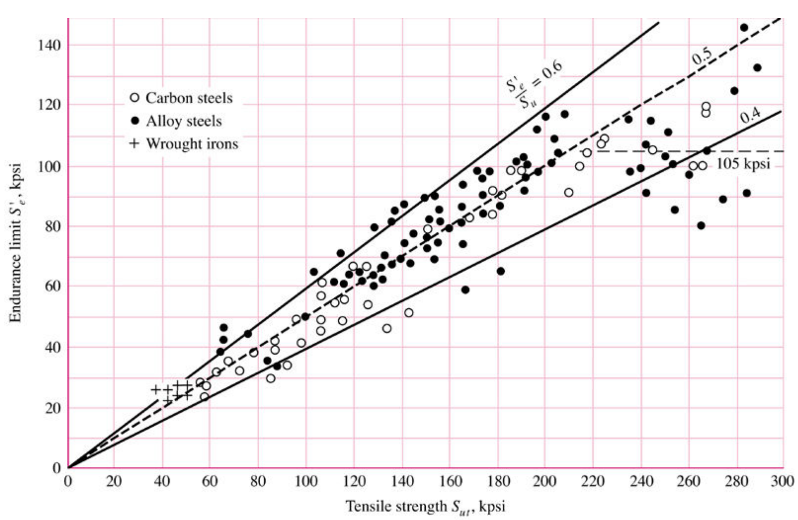


Fig. 6–17

Equations for *S-N* Diagram

- If a completely reversed stress σ_{rev} is given, setting $S_f = \sigma_{rev}$ in Eq. (6–13) and solving for N gives,

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} \qquad (6\text{–}16)$$

- Note that the typical *S-N* diagram is only applicable for completely reversed stresses
- For other stress situations, a completely reversed stress with the same life expectancy must be used on the *S-N* diagram

Example 6-2

Given a 1050 HR steel, *estimate* $S_{ut} = 630$ MPa

(*a*) the rotating-beam endurance limit at 10^6 cycles.

(*b*) the endurance strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure

(*c*) the expected life of a polished rotating-beam specimen under a completely reversed stress of 385 MPa

Solution

Endurance Limit Modifying Factors

- Endurance limit S'_e is for carefully prepared and tested specimen
- If warranted, S_e is obtained from testing of actual parts
- When testing of actual parts is not practical, a set of ***Marin factors*** are used to adjust the endurance limit

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

k_a = surface condition modification factor
k_b = size modification factor
k_c = load modification factor
k_d = temperature modification factor
k_e = reliability factor[13]
k_f = miscellaneous-effects modification factor
S'_e = rotary-beam test specimen endurance limit
S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

Surface Factor k_a

- Stresses tend to be high at the surface
- Surface finish has an impact on initiation of cracks at localized stress concentrations
- Surface factor is a function of ultimate strength. Higher strengths are more sensitive to rough surfaces.

$$k_a = a S_{ut}^b \qquad (6\text{–}19)$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground (taşlama)	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled (sıcak çekme)	14.4	57.7	−0.718
As-forged (dövme)	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis,* vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook,* McGraw-Hill, New York.

Size Factor k_b

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- **For bending and torsion loads**, the trend of the size factor data is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

- Applies only for round, rotating diameter
- **For axial load, there is no size effect, so $k_b = 1$**

Size Factor k_b

- For parts that are not round and rotating, an equivalent round rotating diameter is obtained.
- Equate the volume of material stressed at and above 95% of the maximum stress to the same volume in the rotating-beam specimen.
- Lengths cancel, so equate the areas.
- For a rotating round section, the 95% stress area is the area of a ring,

$$A_{0.95\sigma} = \frac{\pi}{4}[d^2 - (0.95d)^2] = 0.0766d^2 \qquad (6\text{–}22)$$

- Equate 95% stress area for other conditions to Eq. (6–22) and solve for d as the equivalent round rotating diameter

Size Factor k_b

- For non-rotating round,

$$A_{0.95\sigma} = 0.01046d^2 \qquad (6\text{–}23)$$

- Equating to Eq. (6-22) and solving for equivalent diameter,

$$d_e = 0.370d \qquad (6\text{–}24)$$

- Similarly, for rectangular section *h x b*, $A_{95\sigma} = 0.05\, hb$. Equating to Eq. (6–22),

$$d_e = 0.808(hb)^{1/2} \qquad (6\text{–}25)$$

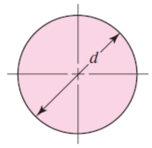
- Other common cross sections are given in Table 6–3

Continued..

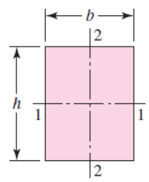
Size Factor k_b

Table 6–3

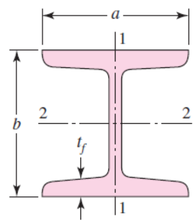
$A_{95\sigma}$ for common non-rotating structural shapes



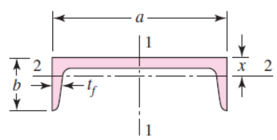
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

Loading Factor k_c

- Accounts for changes in endurance limit for different types of fatigue loading.
- Only to be used for single load types. Use Combination Loading method (Sec. 6–14) when more than one load type is present.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \qquad (6\text{–}26)$$

Temperature Factor k_d

- Temperature factor can be obtained from table or polynomial fit.

$$k_d = \frac{S_T}{S_{RT}}$$

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Temperature Factor k_d

- A fourth-order polynomial curve fit of the underlying data of Table 6–4 can be used in place of the table, if desired.

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \qquad (6\text{–}27)$$

where $70 \le T_F \le 1000$ °F.

Reliability Factor k_e

- From Fig. 6–17, $S'_e = 0.5\ S_{ut}$ is typical of the data and represents 50% reliability.
- Reliability factor adjusts to other reliabilities.
- *Only* adjusts Fig. 6–17 assumption. *Does not* imply overall reliability.

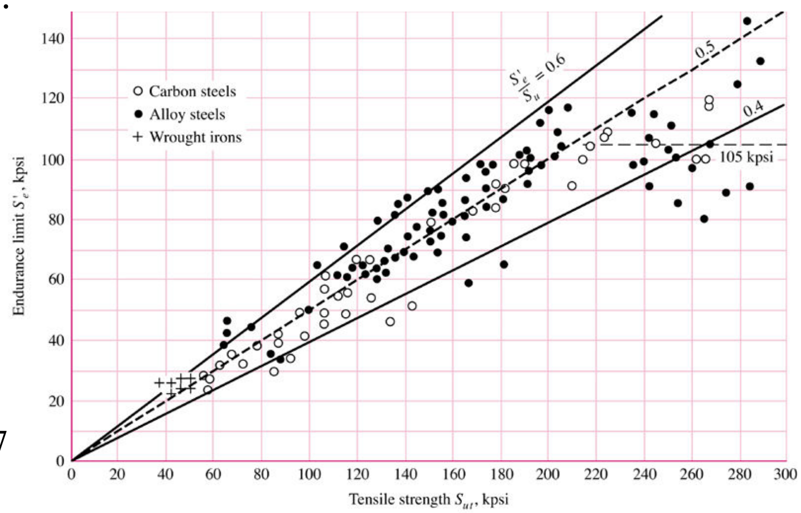


Fig. 6–17

Reliability Factor k_e

- Simply obtain k_e for desired reliability from Table 6–5.

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Table 6–5

Miscellaneous-Effects Factor k_f

- Reminder to consider other possible factors.
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available.
- May require research or testing.

Stress Concentration and Notch Sensitivity

- For dynamic loading, stress concentration effects must be applied.
- Obtain K_t as usual (e.g. Appendix A–15)
- For fatigue, some materials are not fully sensitive to K_t, therefore **a reduced value** can be used.
- Define K_f as the *fatigue stress-concentration factor*.
- Define q as *notch sensitivity,* ranging from 0 (not sensitive) to 1 (fully sensitive).

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \qquad (6\text{–}31)$$

- For $q = 0$, $K_f = 1$
- For $q = 1$, $K_f = K_t$

Notch Sensitivity

- Obtain q for bending or axial loading from Fig. 6–20.
- Then get K_f from Eq. (6–32): $K_f = 1 + q(K_t - 1)$

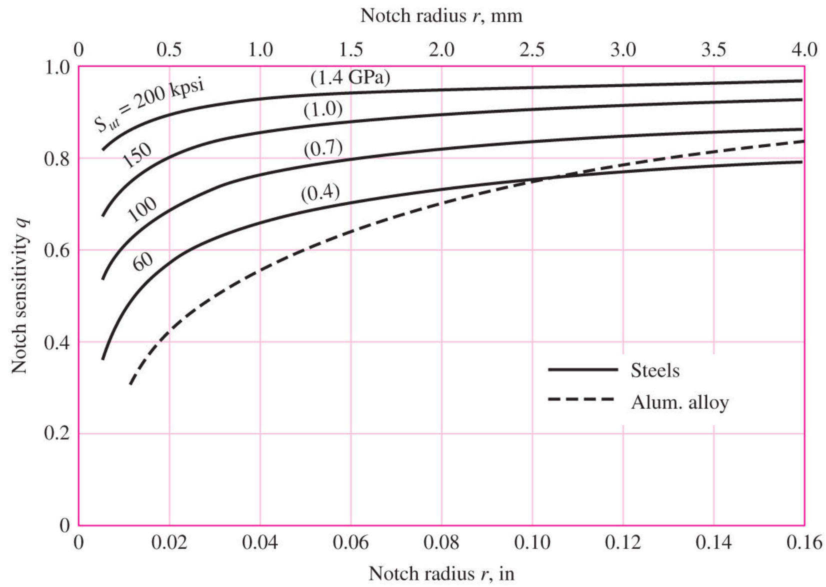


Fig. 6–20

Notch Sensitivity

- Obtain q_s for torsional loading from Fig. 6–21.
- Then get K_{fs} from Eq. (6–32): $K_{fs} = 1 + q_s(K_{ts} - 1)$
- Note that Fig. 6–21 is updated in 9th edition.

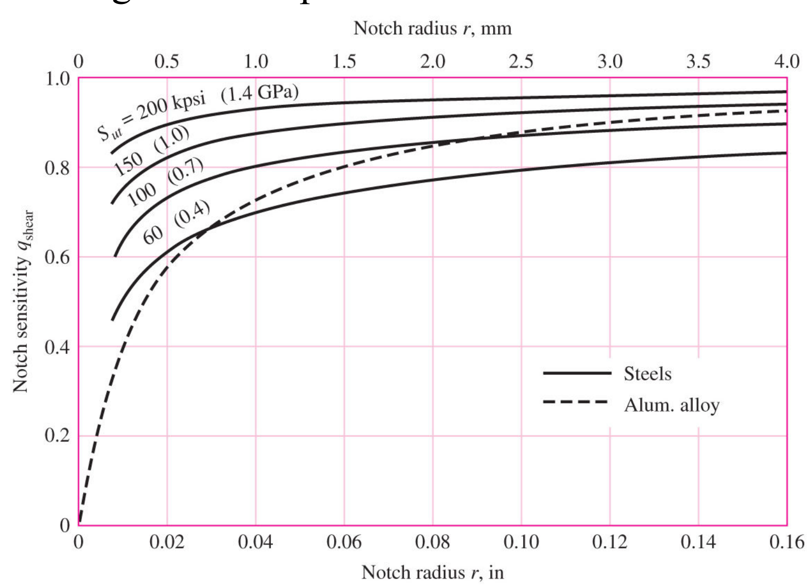


Fig. 6–21

Notch Sensitivity

- Alternatively, can use curve fit equations for Figs. 6–20 and 6–21 to get notch sensitivity, or go directly to K_f.

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \quad (6\text{–}34)$$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \quad (6\text{–}33)$$

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad (6\text{–}35a)$$

Equations apply for steel and when S_{ut} in ksi!!

Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad (6\text{–}35b)$$

Notch Sensitivity for Cast Irons

- Cast irons are already full of discontinuities, which are included in the strengths.
- Additional notches do not add much additional harm.
- Recommended to use $q = 0.2$ for cast irons.

Example 6-6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Solution

Continued..

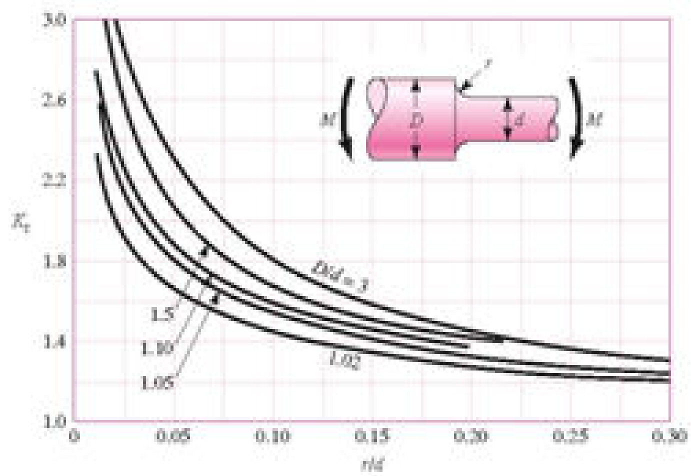
Example 6-6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Solution

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Application of Fatigue Stress Concentration Factor

- Use K_f as a multiplier to increase the nominal stress.
- Some designers (and previous editions of the textbook) sometimes applied $1/K_f$ as a Marin factor to reduce S_e.
- For infinite life, either method is equivalent, since

$$n_f = \frac{S_e}{K_f \sigma} = \frac{(1/K_f)S_e}{\sigma}$$

- For finite life, increasing stress is more conservative.
- Decreasing S_e applies more to high cycle than low cycle.

Example 6-7

For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure. $S_{ut} = 690$ MPa

Solution

Continued..

Example 6-9

Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part. S_{ut} = 690 MPa

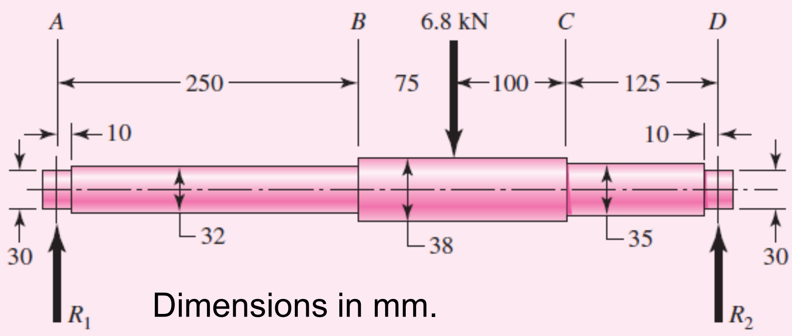


Fig. 6–22 (*a*)

For all fillets r = 3 mm

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Characterizing Fluctuating Stresses

- The *S-N* diagram is applicable for ***completely reversed*** stresses
- Other fluctuating stresses exist
- Sinusoidal loading patterns are common, but not necessary

Fluctuating Stresses

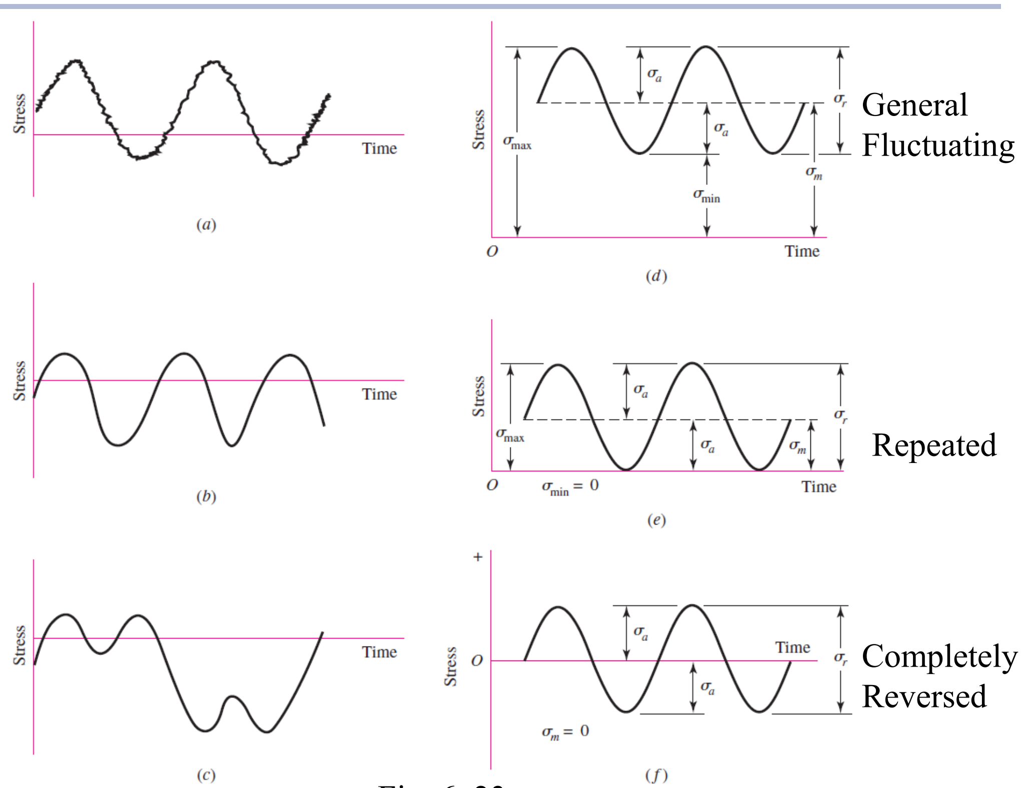


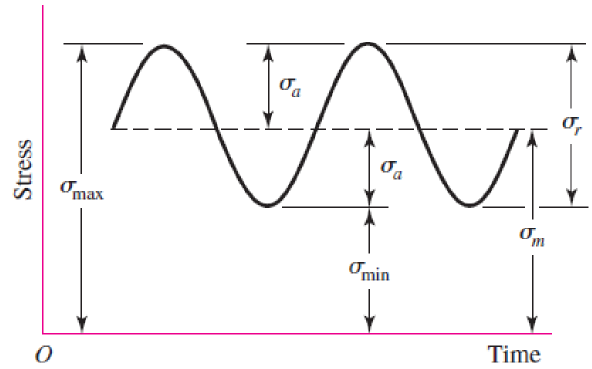
Fig. 6–23

Characterizing Fluctuating Stresses

Mean and alternating stresses:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$



Stress ratio:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \qquad (6\text{–}37)$$

Amplitude ratio:

$$A = \frac{\sigma_a}{\sigma_m} \qquad (6\text{–}38)$$

Application of K_f for Fluctuating Stresses

- K_f should be applied to both alternating and midrange stress components.
- The *Dowling method* recommends applying K_f to the alternating stress and K_{fm} to the mid-range stress, where K_{fm} is

$$K_{fm} = K_f \qquad K_f|\sigma_{\max,o}| < S_y$$

$$K_{fm} = \frac{S_y - K_f\sigma_{ao}}{|\sigma_{mo}|} \qquad K_f|\sigma_{\max,o}| > S_y \qquad (6\text{–}39)$$

$$K_{fm} = 0 \qquad K_f|\sigma_{\max,o} - \sigma_{\min,o}| > 2S_y$$

Plot of Alternating vs Midrange Stress

- Experimental data on normalized plot of σ_a vs σ_m
- Demonstrates little effect of negative midrange stress

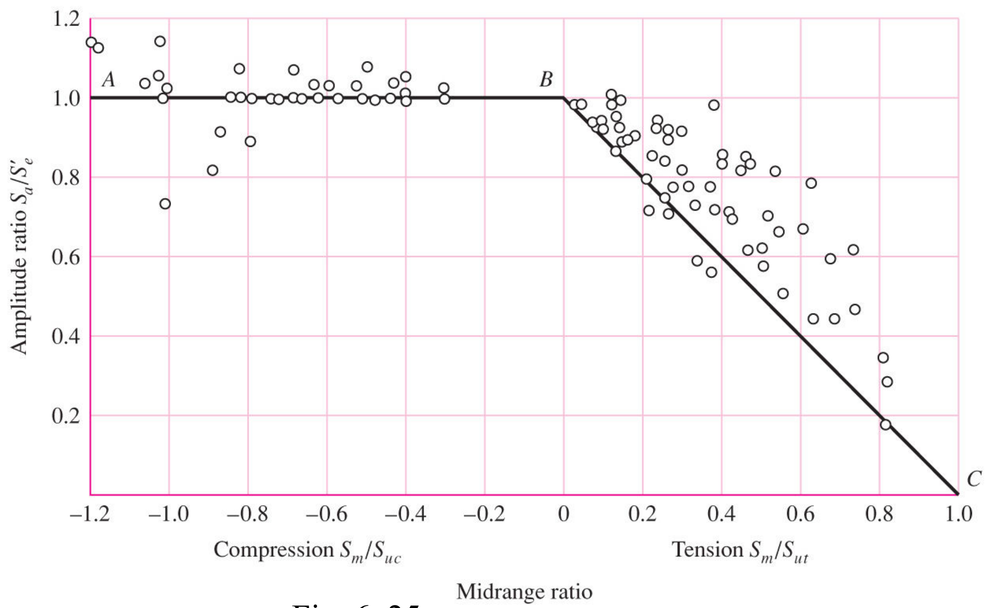


Fig. 6–25

Commonly Used Failure Criteria

- Five commonly used failure criteria
  (limiting boundary for infinite life)

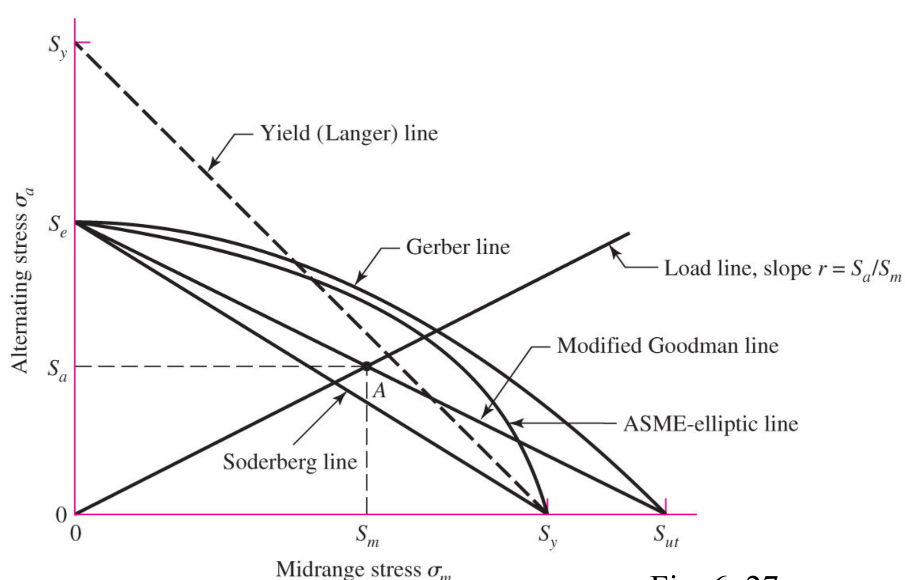


Fig. 6–27

Commonly Used Failure Criteria

- Gerber passes through the data
- ASME-elliptic passes through data and incorporates rough yielding check
- Modified Goodman is linear, so simple to use for design. It is more conservative than Gerber.
- Soderberg provides a very conservative single check of both fatigue and yielding.
- Langer line represents standard yield check.
  - It is equivalent to comparing maximum stress to yield strength.

Equations for Commonly Used Failure Criteria

- Intersecting a constant slope load line with each failure criteria produces design equations
- n is the design factor or factor of safety for infinite fatigue life

Soderberg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \qquad (6\text{–}45)$$

mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad (6\text{–}46)$$

Gerber $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad (6\text{–}47)$$

ASME-elliptic $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad (6\text{–}48)$$

Fatigue safety factors (n_f)

- Fatigue safety factors can be obtained from Eqs. (6.45) to (6.48).

- Soderberg $$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_y}}$$

- Mod-Goodman $$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$$

- Gerber $$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right]$$

- ASME-elliptic $$n_f = \sqrt{\frac{1}{\left(\dfrac{\sigma_a}{S_e}\right)^2 + \left(\dfrac{\sigma_m}{S_y}\right)^2}}$$

Example 6-10

A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find ~~S_a and S_m and~~ the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line ~~and (*b*) the ASME-elliptic fatigue line.~~

Solution

S_y=84 ksi, S_{ut}=100 ksi

Continued..

Continued..

Example 6-12

A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e =$ 40 kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) Modified Goodman criterion.

~~(*b*) Gerber criterion.~~

Solution

Continued..

Fatigue Criteria for Brittle Materials

- Smith Dolan fatigue locus is commonly used.
- **For the first quadrant**, that is $r = \sigma_a / \sigma_m > 0$

$$S_a = \frac{r S_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4r S_{ut} S_e}{(r S_{ut} + S_e)^2}}\right]$$

- **In the second quadrant**, that is $r = \sigma_a / \sigma_m < 0$

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1\right) S_m$$

- The safety factor is computed from

$$n = S_a / \sigma_a$$

- Table A–24 gives properties of gray cast iron, including endurance limit
- The endurance limit already includes k_a and k_b
- The average k_c for axial and torsional is 0.9.

Example 6-13

A grade 30 gray cast iron is subjected to a load F applied to a 1 by $\frac{3}{8}$-in cross-section link with a $\frac{1}{4}$-in-diameter hole drilled in the center as depicted in Fig. 6–31*a*. The surfaces are machined. In the neighborhood of the hole, what is the factor of safety guarding against failure under the following conditions:
(*a*) The load $F = 1000$ lbf tensile, steady.
(*b*) The load is 1000 lbf repeatedly applied.
(*c*) The load fluctuates between -1000 lbf and 300 lbf without column action.
Use the Smith-Dolan fatigue locus.

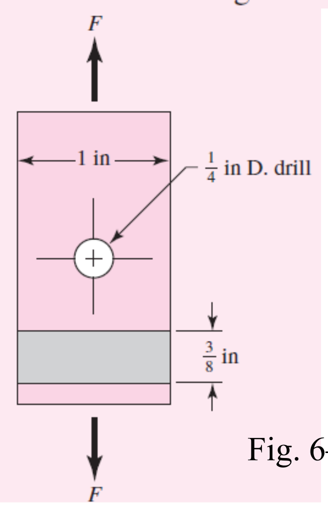


Fig. 6–31*a*

$S_{ut} = 31$ ksi, $S_{uc} = 109$ ksi
$k_a k_b S_e' = 14$ ksi
$q = 0.2$

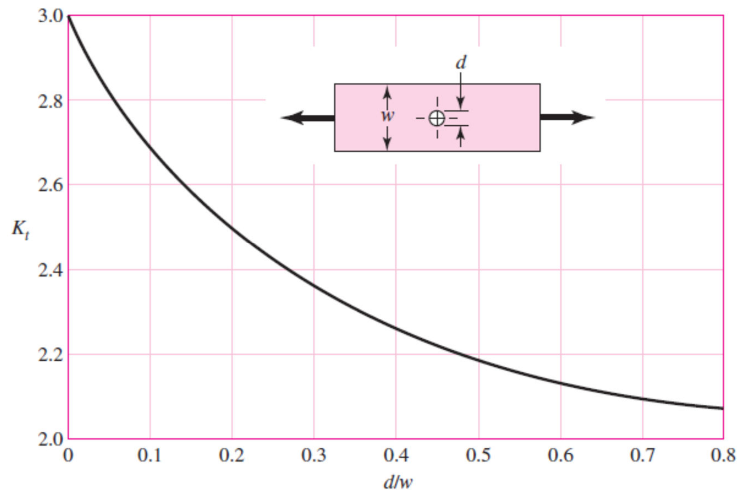
Continued..

Continued..

Example 6-13

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



Torsional Fatigue Strength

- Testing has found that the steady-stress component has no effect on the endurance limit for torsional loading if the material is ductile, polished, notch-free, and cylindrical.
- However, for less than perfect surfaces, the modified Goodman line is more reasonable.
- For pure torsion cases, use $k_c = 0.59$ to convert normal endurance strength to shear endurance strength.
- For shear ultimate strength, recommended to use

$$S_{su} = 0.67 S_{ut} \qquad (6\text{–}54)$$

Combinations of Loading Modes

- When more than one type of loading (bending, axial, torsion) exists, use the Distortion Energy theory to combine them.
- Obtain von Mises stresses for both midrange and alternating components.
- Apply appropriate K_f to each type of stress.
- For load factor, use $k_c = 1$. The torsional load factor ($k_c = 0.59$) is inherently included in the von Mises equations.
- If needed, axial load factor can be divided into the axial stress.

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2 \right\}^{1/2} \quad (6\text{–}55)$$

$$\sigma'_m = \left\{ \left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2 \right\}^{1/2} \quad (6\text{–}56)$$

Static Check for Combination Loading

- Distortion Energy theory still applies for check of static yielding
- Obtain von Mises stress for maximum stresses (sum of midrange and alternating)
- Stress concentration factors are not necessary to check for yielding at first cycle

$$\sigma'_{\text{max}} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2 \right]^{1/2}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}}$$

- Alternate simple check is to obtain conservative estimate of σ'_{max} by summing σ'_a and σ'_m

$$\sigma'_{\text{max}} \doteq \sigma'_a + \sigma'_m$$

Example 6-14

A rotating shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria for the following loading conditions:

(*a*) The shaft is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.

(*b*) The shaft is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m.

$S_{ut} = 440$ MPa, $S_y = 370$ MPa,

Continued..

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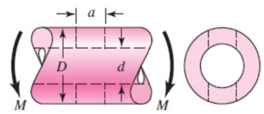
Continued..

Example 6-14

Table A–16

Approximate Stress-Concentration Factor K_t for Bending of a Round Bar or Tube with a Transverse Round Hole

Source: R. E. Peterson, *Stress Concentration Factors*, Wiley, New York, 1974, pp. 146, 235.



The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

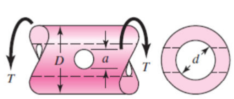
Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Example 6-14

Table A–16 *(Continued)*

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress Concentration Factors*, Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

Cumulative Fatigue Damage

- A common situation is to load at σ_1 for n_1 cycles, then at σ_2 for n_2 cycles, etc.
- The cycles at each stress level contributes to the fatigue damage
- Accumulation of damage is represented by the *Miner's rule*

$$\sum \frac{n_i}{N_i} = c \qquad (6\text{–}57)$$

where n_i is the number of cycles at stress level σ_i and N_i is the number of cycles to failure at stress level σ_i

- c is experimentally found to be in the range $0.7 < c < 2.2$, **with an average value near unity.**
- Defining D as the accumulated damage,

$$D = \sum \frac{n_i}{N_i} \qquad (6\text{–}58)$$