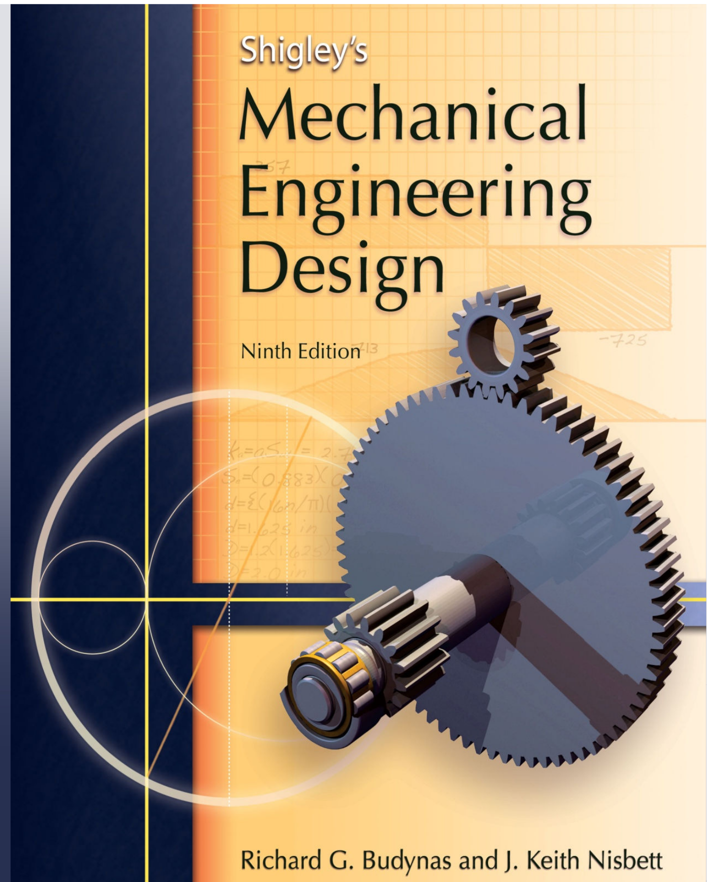
Lecture Slides



Chapter 5

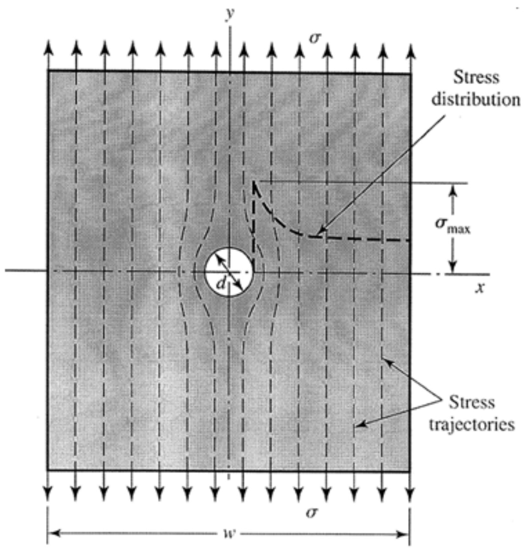
Failures Resulting from Static Loading

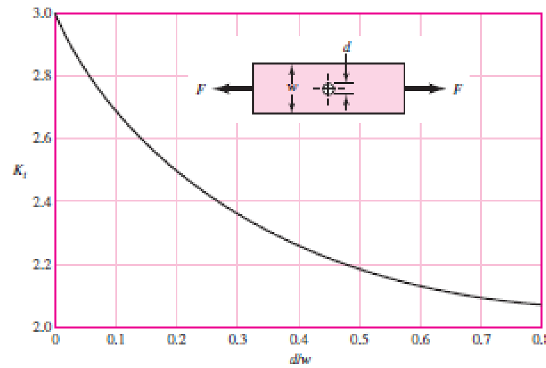
Chapter Outline

Stress Concentration

- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0} \qquad (3\text{–}48)$$





Stress Concentration for Static and Ductile Conditions

- Stress concentration effect is **commonly ignored** for static loads on ductile materials
  - Upon localized yielding, the load is shared with neighboring fibers
  - The structure can carry load untill all fibers yield
- Stress concentration **must be included** for dynamic loading on ductile materials (See Chapter 6)
- Stress concentration **must be included** for brittle materials, since localized yielding may reach brittle failure rather than sharing the load.

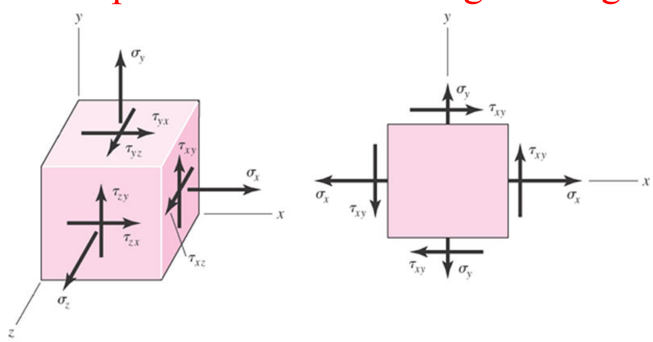
- **In this course, we will include stress concentration effect for all materials and all loading conditions, to be conservative.**

Need for Static Failure Theories

- Uniaxial stress element (e.g. tension test)

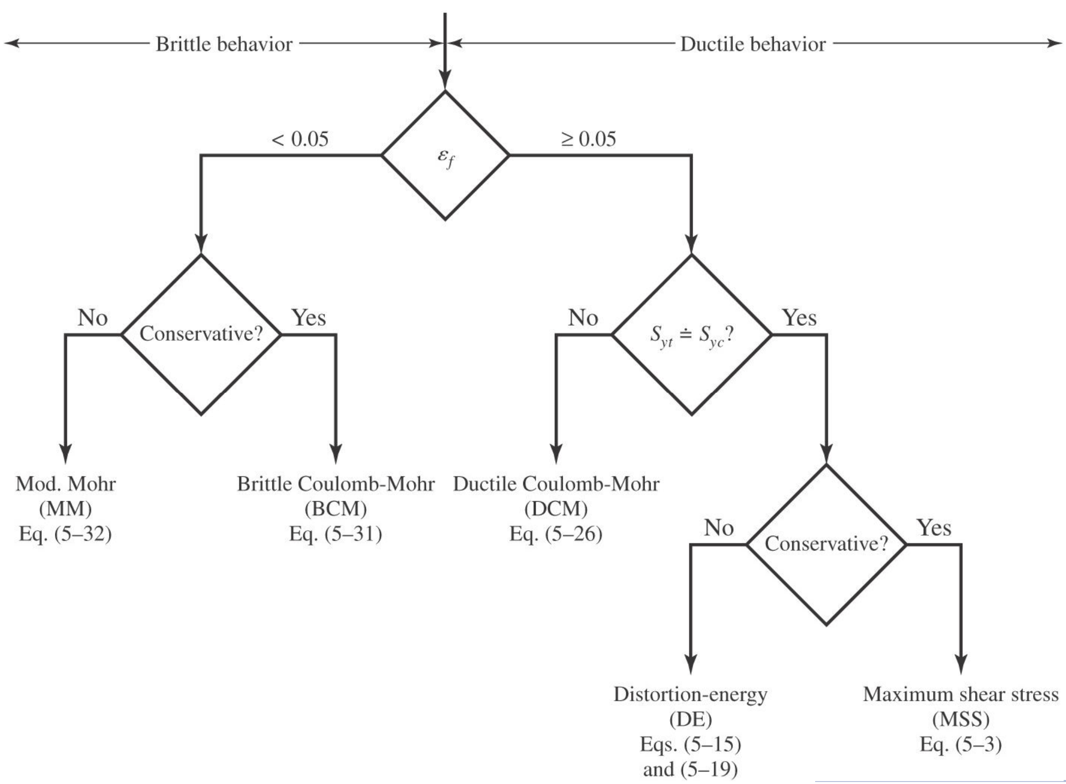
$$n = \frac{Strength}{Stress} = \frac{S}{\sigma}$$

- Multi-axial stress element
  - One strength, multiple stresses
  - How to compare stress state to single strength?



(a) (b)

Selection of Failure Criteria in Flowchart Form

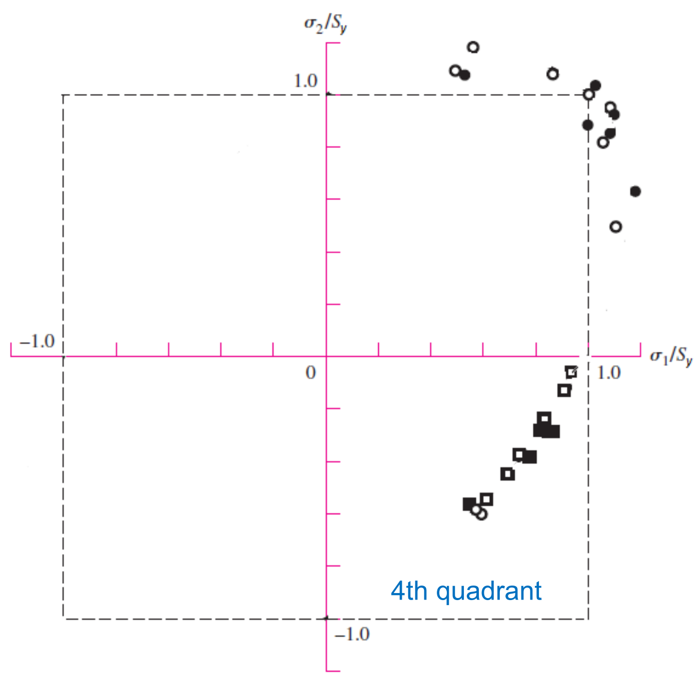


Maximum Normal (Principal) Stress Theory

- **Theory:** Yielding begins when *the maximum principal stress* in a stress element exceeds the yield strength.
- For any stress element, use **Mohr's circle** to find the principal stresses.
- Compare the largest principal stress to the yield strength.
- Is it a good theory? **No!**

Maximum Normal (Principal) Stress Theory

- Experimental data shows the theory is unsafe in the 4th quadrant.
- Similar behavior is also observed for brittle materials.
- Not recommended for use
- Included for historical comparison



Maximum Shear Stress Theory (MSS)

- **Theory:** Yielding begins when the *maximum shear stress* in a stress element exceeds the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield.

- For a tension test specimen, the maximum shear stress is $\sigma_1/2$.
- At yielding (i.e., $\sigma_1 = S_Y$), the maximum shear stress is $S_Y/2$.

- Could restate the theory as follows:
  - **Theory:** Yielding begins when the *maximum shear stress* in a stress element exceeds $S_Y/2$.

Maximum Shear Stress Theory (MSS)

- For any stress element, use Mohr's circle to find the maximum shear stress. Compare the maximum shear stress to $S_Y/2$.
- Ordering the principal stresses such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \qquad \text{or} \qquad \boxed{\sigma_1 - \sigma_3 \geq S_y} \tag{5–1}$$

- Incorporating a design factor n

$$\tau_{\max} = \frac{S_y}{2n} \qquad \text{or} \qquad \sigma_1 - \sigma_3 = \frac{S_y}{n} \tag{5–3}$$

- Or solving for factor of safety

$$n = \frac{S_Y/2}{\tau_{\max}}$$

Maximum Shear Stress Theory (MSS)

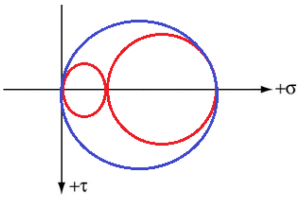
- To compare to experimental data, express τ_{max} in terms of principal stresses and plot.

- To simplify, consider a plane stress state
- Let σ_A and σ_B represent the two non-zero principal stresses, then order them with the zero principal stress such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$

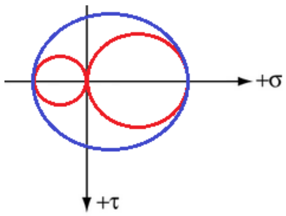
- Assuming $\sigma_A \geq \sigma_B$ there are three cases to consider
  - Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - Case 3: $0 \geq \sigma_A \geq \sigma_B$

Maximum Shear Stress Theory (MSS)

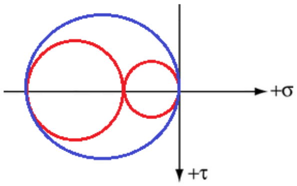
- Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$
  - Failure condition: $\sigma_A \geq S_Y$



- Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$
  - Failure condition: $\sigma_A - \sigma_B \geq S_Y$

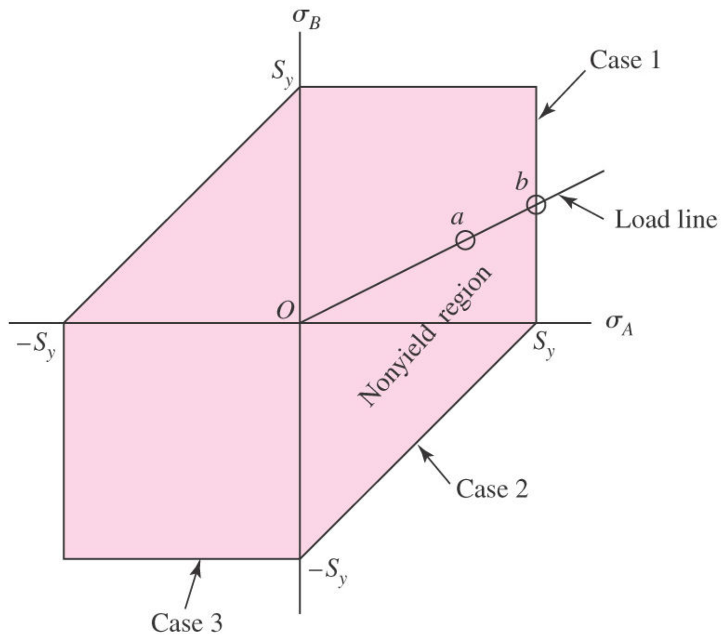


- Case 3: $0 \geq \sigma_A \geq \sigma_B$
  - For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$
  - Failure condition: $\sigma_B \leq -S_Y$ or $|\sigma_B| \geq S_Y$



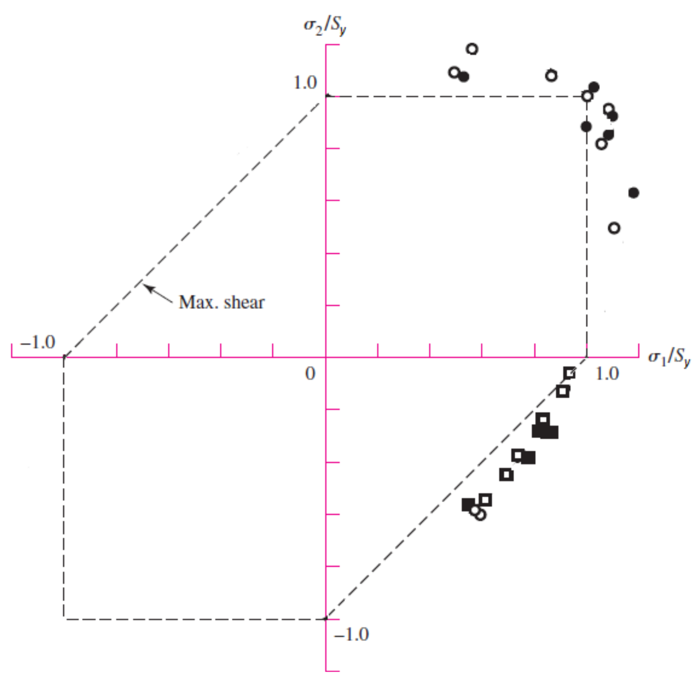
Maximum Shear Stress Theory (MSS)

- Plot three cases on principal stress axes
- Case 1: $\sigma_A \geq \sigma_B \geq 0$
  - $\sigma_A \geq S_Y$
- Case 2: $\sigma_A \geq 0 \geq \sigma_B$
  - $\sigma_A - \sigma_B \geq S_Y$
- Case 3: $0 \geq \sigma_A \geq \sigma_B$
  - $\sigma_B \leq -S_Y$
- Other lines are symmetric cases
- **Inside envelope is predicted safe zone**



Maximum Shear Stress Theory (MSS)

- Comparison to experimental data
- Conservative in all quadrants
- Commonly used for design situations

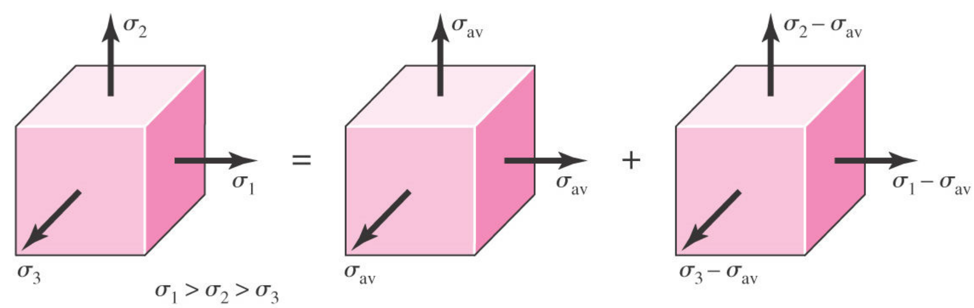


Distortion Energy (DE) Failure Theory

- Also known as:
  - Octahedral Shear Stress
  - Shear Energy
  - Von Mises
  - Von Mises – Hencky
- Originated from observation that ductile materials stressed hydrostatically (equal principal stresses) exhibited yield strengths greatly in excess of expected values.
- Theorizes that if strain energy is divided into hydrostatic volume changing energy and angular distortion energy, the yielding is primarily affected by the distortion energy.

Distortion Energy (DE) Failure Theory

- **Theory:** Yielding occurs when the *distortion strain energy* per unit volume reaches the distortion strain energy per unit volume for yield in simple tension or compression of the same material.



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Fig. 5–8

Derivation of the Distortion Energy

- To be handled in the class.

Continued..

Von Mises Stress

- In terms of *principal stresses*, in 3-D

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]^{1/2} \quad (5\text{–}12)$$

- For plane stress, if the 2-D principal stresses are σ_A and σ_B

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} \quad (5\text{–}13)$$

- In terms of *xyz* components, in 3-D

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2} \quad (5\text{–}14)$$

- For plane stress, in terms of *xyz* components

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} \quad (5\text{–}15)$$

Distortion Energy Theory With Von Mises Stress

- Von Mises Stress can be thought of as a single, equivalent, or effective stress for the entire general state of stress in a stress element.
- Distortion Energy failure theory simply compares von Mises stress to yield strength.

$$\sigma' \geq S_y \quad (5\text{–}11)$$

- Introducing a design factor,

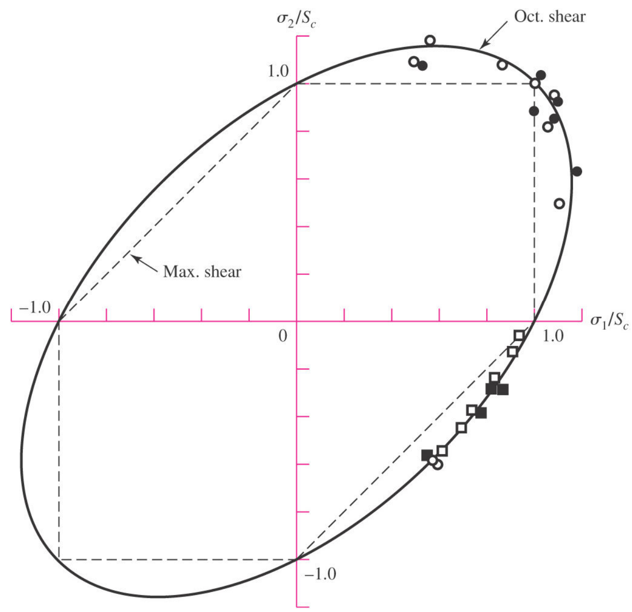
$$\sigma' = \frac{S_y}{n} \quad (5\text{–}19)$$

- Expressing as factor of safety,

$$n = \frac{S_y}{\sigma'}$$

DE Theory Compared to Experimental Data

- Plot von Mises stress on principal stress axes to compare to experimental data (and to other failure theories)
- DE curve is *typical* of data
- Commonly used for analysis situations
- MSS is more conservative than DE



MSS vs DE Theory – Special Case

- If the stress state consists only of
  - a single normal stress component (σ)
  - and a single shear stress component (τ)

- MSS:

$$\tau_{\max} = \sqrt{(\sigma/2)^2 + \tau^2} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$n = \frac{S_y/2}{\tau_{\max}} = \frac{S_y}{\sqrt{\sigma^2 + 4\tau^2}}$$

- DE:

$$\sigma' = \sqrt{\sigma^2 + 3\tau^2}$$

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\sqrt{\sigma^2 + 3\tau^2}}$$

Example 5-1

A hot-rolled steel has a yield strength of $S_{yt} = S_{yc} = 100$ kpsi and a true strain at fracture of $\varepsilon_f = 0.55$. Estimate the factor of safety for the following principal stress states:

(*a*) $\sigma_x = 70$ kpsi, $\sigma_y = 70$ kpsi, $\tau_{xy} = 0$ kpsi

(*b*) $\sigma_x = 60$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = -15$ kpsi

(*c*) $\sigma_x = 0$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 45$ kpsi

(*d*) $\sigma_x = -40$ kpsi, $\sigma_y = -60$ kpsi, $\tau_{xy} = 15$ kpsi

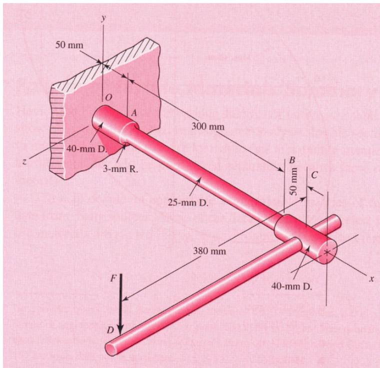
(*e*) $\sigma_1 = 30$ kpsi, $\sigma_2 = 30$ kpsi, $\sigma_3 = 30$ kpsi

Solution

Continued..

Example 5-3

- A certain force F is applied at D. OABC bar is made of AISI 1035 steel, having yield strength of 560 MPa. Find the value of F that initiates yielding, using MSS and DE failure theories.



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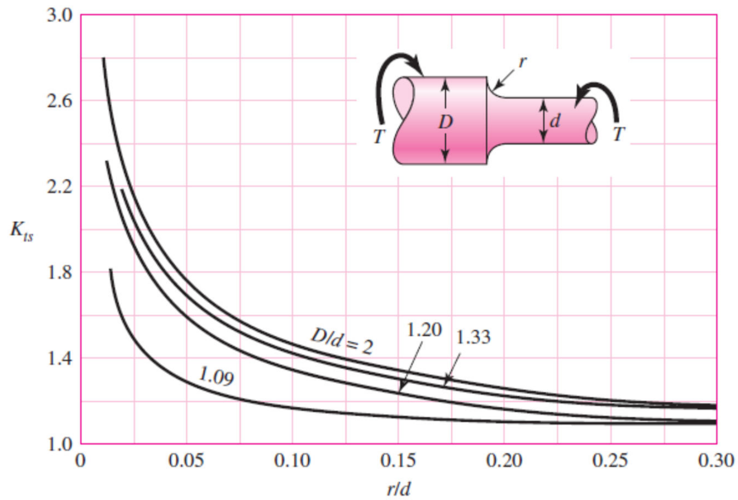
Example 5-3

Figure A–15–8

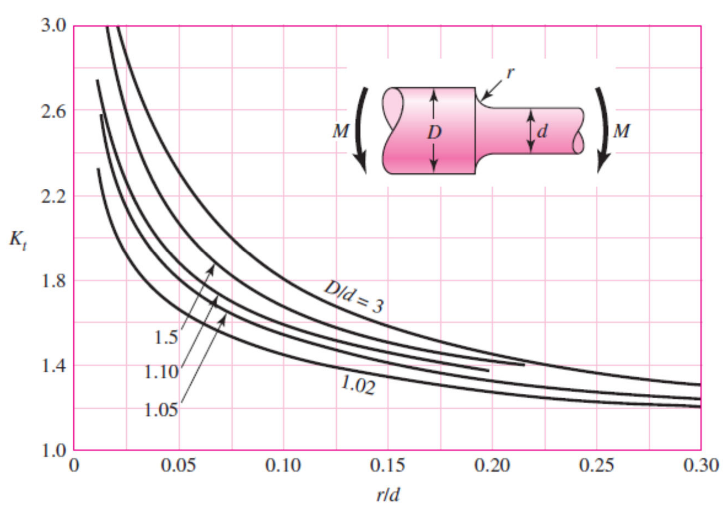
Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



Example 5-3

Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Mohr Theory

- Some materials have compressive strengths different from tensile strengths
- *Mohr theory* is based on three simple tests: tension, compression, and shear
- Plotting Mohr's circle for each, bounding curve defines failure envelope

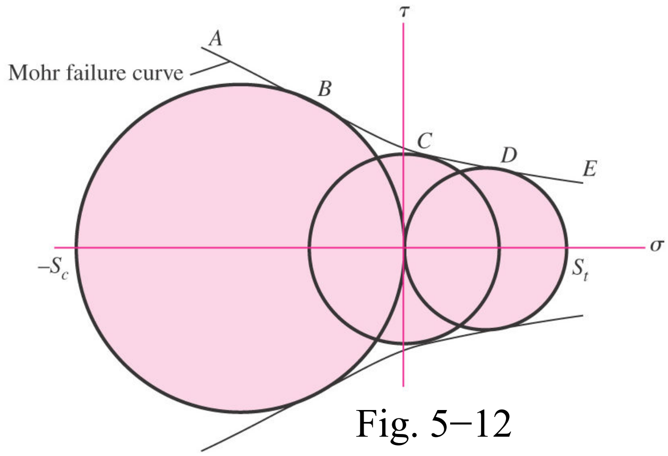


Fig. 5−12

Coulomb-Mohr Theory

- Curved failure curve is difficult to determine analytically
- *Coulomb-Mohr theory* simplifies to linear failure envelope using only tension and compression tests (dashed circles)

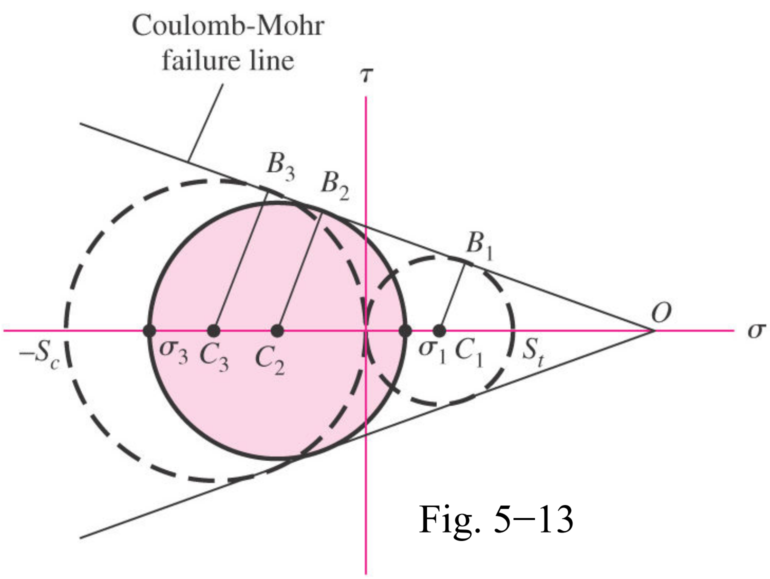


Fig. 5−13

Coulomb-Mohr Theory

- The failure criteria is

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

- Using factor of safety

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

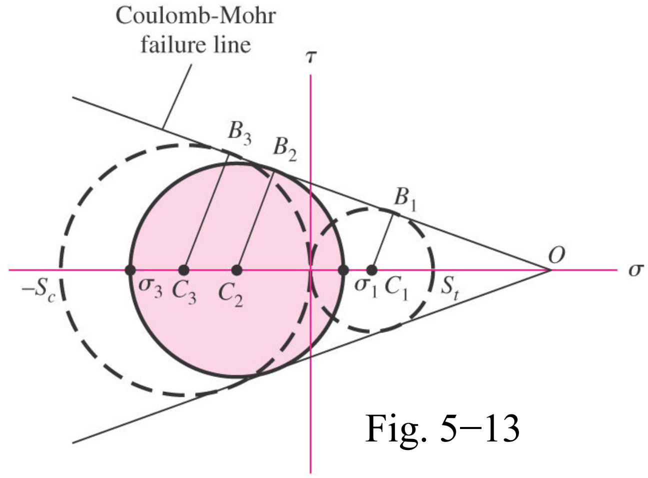


Fig. 5−13

- For ductile material, use tensile and compressive yield strengths
- For brittle material, use tensile and compressive ultimate strengths

Coulomb-Mohr Theory

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1 \qquad (5\text{−}22)$$

- Consider three cases
- **Case 1: $\sigma_A \geq \sigma_B \geq 0$** **For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$**
  - Eq. (5−22) reduces to

$$\sigma_A \geq S_t \qquad (5\text{−}23)$$

- **Case 2: $\sigma_A \geq 0 \geq \sigma_B$** **For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$**
  - Eq. (5-22) reduces to

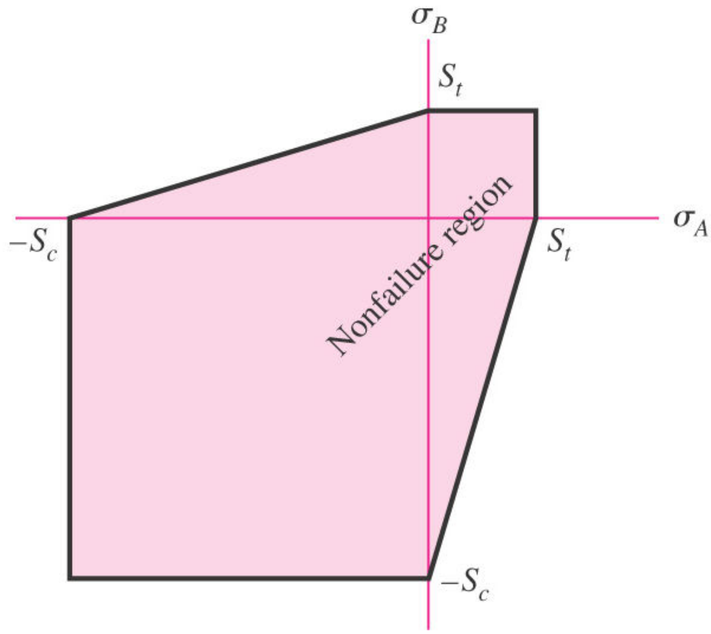
$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1 \qquad (5\text{−}24)$$

- **Case 3: $0 \geq \sigma_A \geq \sigma_B$** **For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$**
  - Eq. (5−22) reduces to

$$\sigma_B \leq -S_c \qquad (5\text{−}25)$$

Coulomb-Mohr Theory

- Plot three cases on principal stress axes
- Similar to MSS theory, except with different strengths for compression and tension



Example 5-2

A 25-mm-diameter shaft is statically torqued to 230 N · m. It is made of cast 195-T6 aluminum, with a yield strength in tension of 160 MPa and a yield strength in compression of 170 MPa. It is machined to final diameter. Estimate the factor of safety of the shaft.

Solution

Failure Theories for Brittle Materials

- Experimental data indicates some differences in failure for brittle materials.
- Failure criteria is generally ultimate fracture rather than yielding
- Compressive strengths are usually larger than tensile strengths

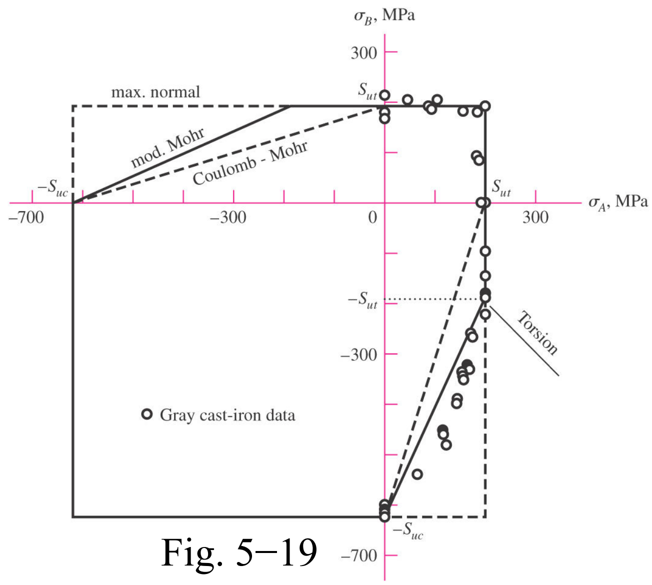


Fig. 5−19

Brittle Coulomb-Mohr

- Same as previously derived, using ultimate strengths for failure
- Failure equations dependent on quadrant

Quadrant condition	Failure criteria	
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–31*a*)
$\sigma_A \geq 0 \geq \sigma_B$	$\dfrac{\sigma_A}{S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$	(5–31*b*)
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\dfrac{S_{uc}}{n}$	(5–31*c*)

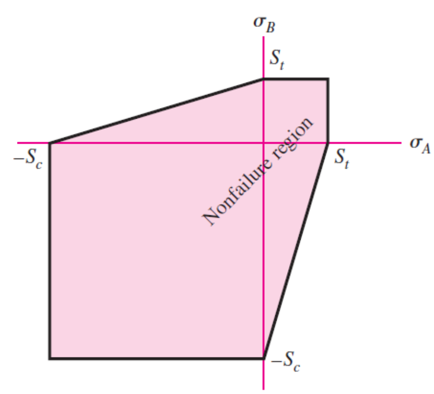


Fig. 5−14

Brittle Failure Experimental Data

- Coulomb-Mohr is conservative in 4th quadrant
- *Modified Mohr* criteria adjusts to better fit the data in the 4th quadrant

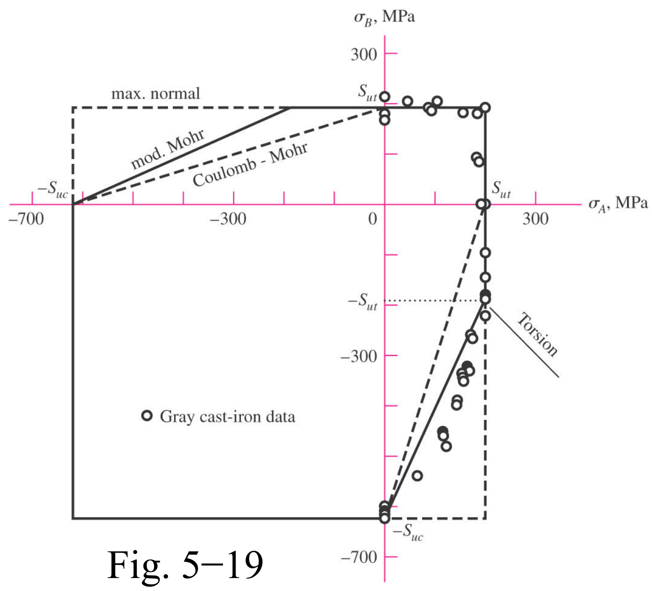
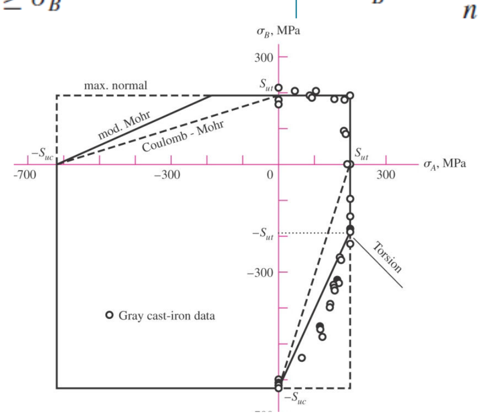


Fig. 5−19

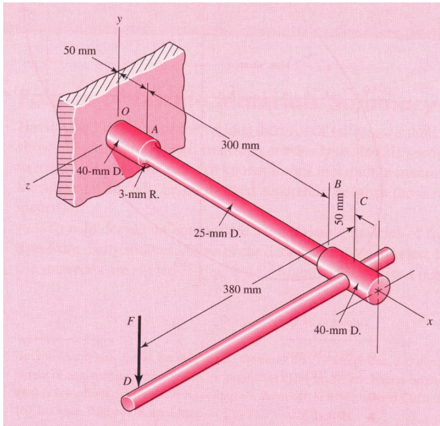
Modified-Mohr

Quadrant condition	Failure criteria	
$\sigma_A \ge \sigma_B \ge 0$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–32*a*)
$\sigma_A \ge 0 \ge \sigma_B$ and $\left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert \le 1$	$\sigma_A = \dfrac{S_{ut}}{n}$	(5–32*a*)
$\sigma_A \ge 0 \ge \sigma_B$ and $\left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert > 1$	$\dfrac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc} S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$	(5–32*b*)
$0 \ge \sigma_A \ge \sigma_B$	$\sigma_B = -\dfrac{S_{uc}}{n}$	(5–32*c*)



Example 5-5

- A certain force F is applied at D. OABC bar is made of a brittle material with 210 MPa ultimate tensile stress and 750 MPa ultimate compressive stress. Find the value of F that leads to fracture, using Coulomb-Mohr and Modified Mohr failure theories.



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Selection of Failure Criteria

- First determine ductile vs. brittle
- **For ductile**
  - MSS is conservative
  - DE is typical, often used for analysis where agreement with experimental data is desired
  - If tensile and compressive strengths differ, use Ductile Coulomb-Mohr
- **For brittle**
  - Brittle Coulomb-Mohr is very conservative in 4th quadrant
  - Modified Mohr is still slightly conservative in 4th quadrant, but closer to typical

Selection of Failure Criteria in Flowchart Form

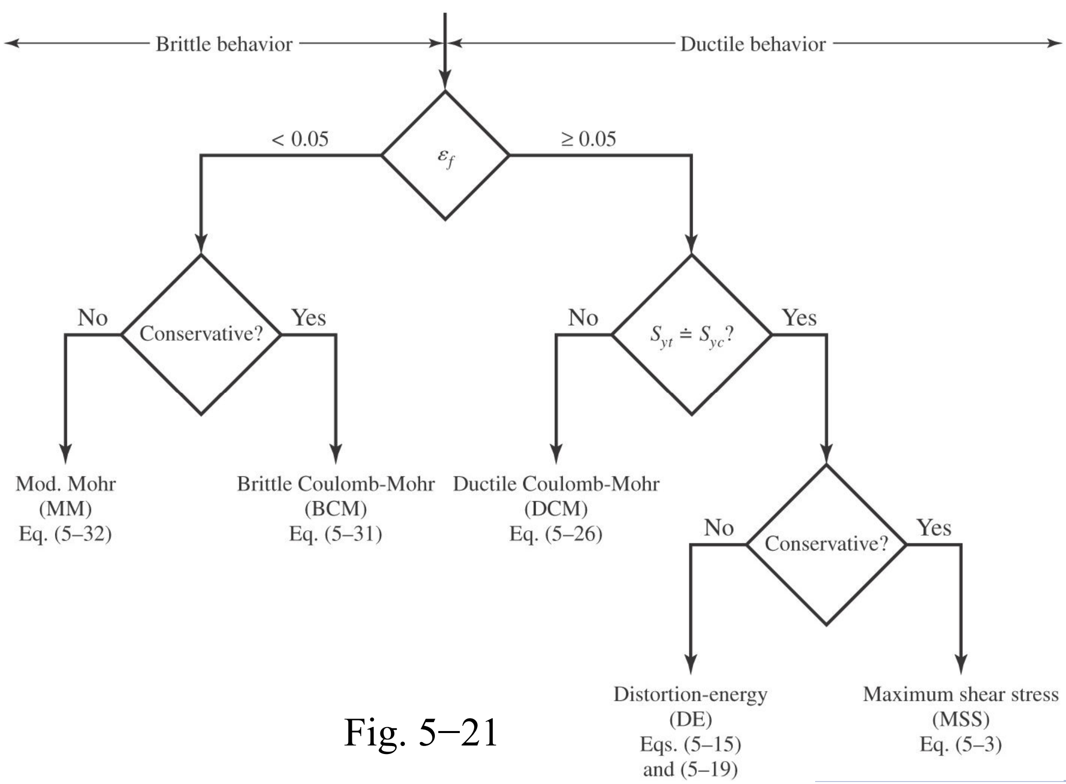

Fig. 5−21