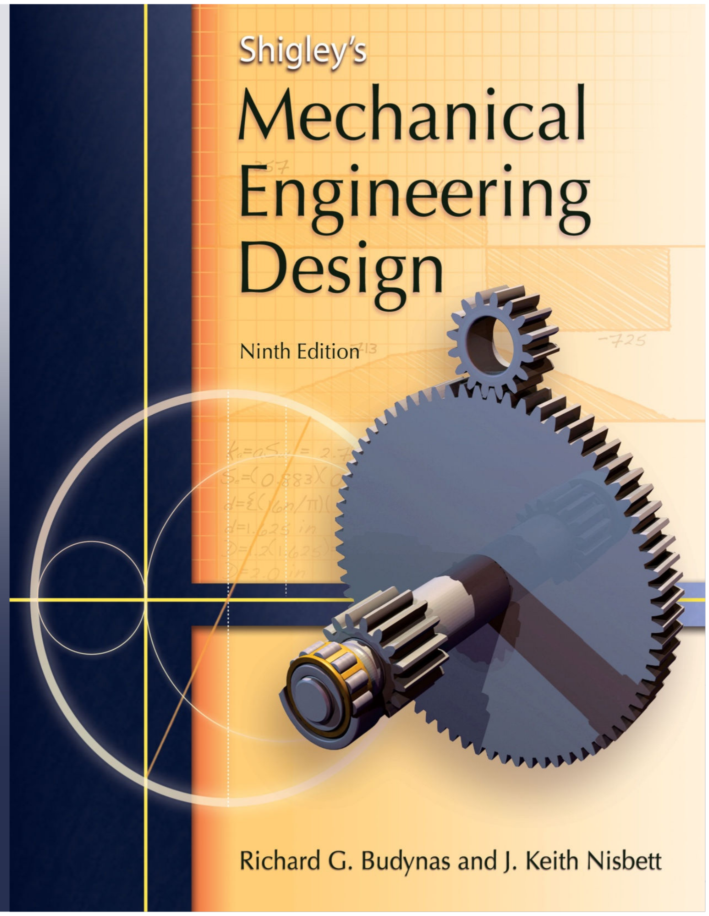
Lecture Slides



Chapter 3

Load and Stress Analysis

Chapter Outline

Shear Force and Bending Moments in Beams

- Cut beam at any location x_1
- Internal shear force *V* and bending moment *M* must ensure equilibrium

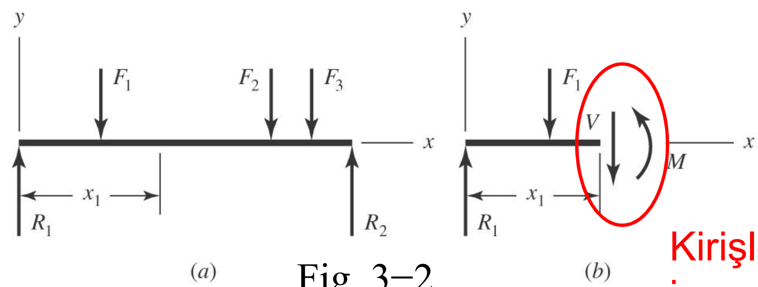
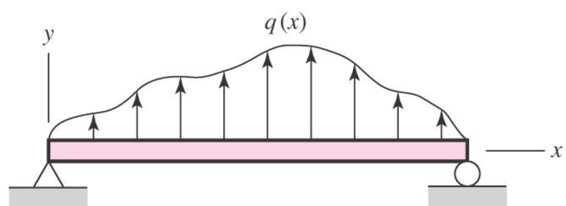


Fig. 3−2

Kirişler için işaret uylaşımı

- Distributed load $q(x)$ is taken positive *upwards*



Relationships between Load, Shear, and Bending

$$V = \frac{dM}{dx} \qquad (3\text{–}3)$$

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q \qquad (3\text{–}4)$$

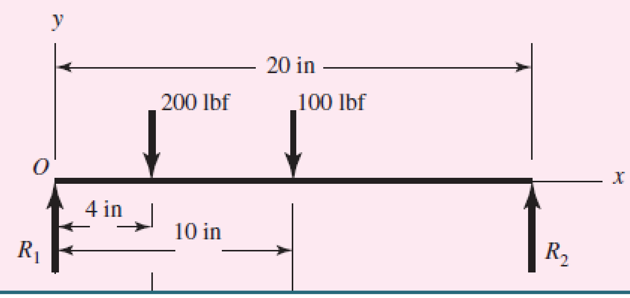
$$\int_{V_A}^{V_B} dV = V_B - V_A = \int_{x_A}^{x_B} q\, dx \qquad (3\text{–}5)$$

$$\int_{M_A}^{M_B} dM = M_B - M_A = \int_{x_A}^{x_B} V\, dx \qquad (3\text{–}6)$$

- The change in shear force from *A* to *B* is equal to the area of the loading diagram between x_A and x_B.
- The change in moment from *A* to *B* is equal to the area of the shear-force diagram between x_A and x_B.

Shear-Moment Diagrams

(a)



Plot the shear and moment diagrams.

BASE UNITS			DERIVED UNITS			
LENGTH	MASS	FORCE	STRESS	ENERGY	VELOCITY	ACCELER.
m	kg	N	Pa	J	m/s	m/s²
in	lbm	lbf	psi	lbf-in	in/s	in/s²

Multiply Input X	By Factor A	To Get Output Y
inch, in	0.0254	meter, m
inch, in	25.4	millimeter, mm
foot, ft	0.305	meter, m
pound, lbf	4.45	newton, N
kilopound/inch^2, kpsi (ksi)	6.89	megapascal, MPa (N/mm^2)

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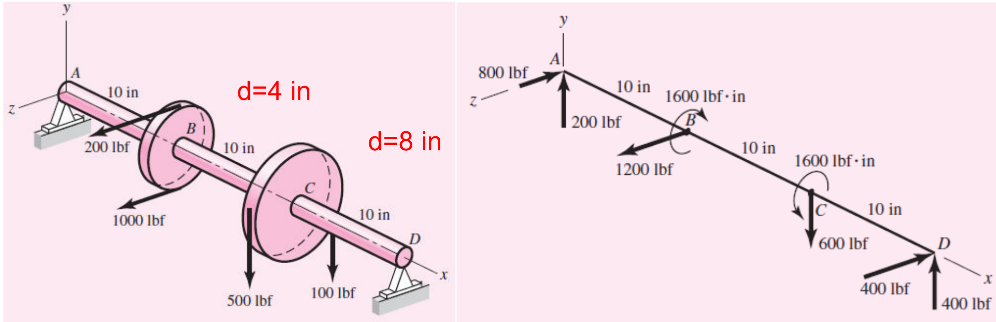
Moment Diagrams – Two Planes



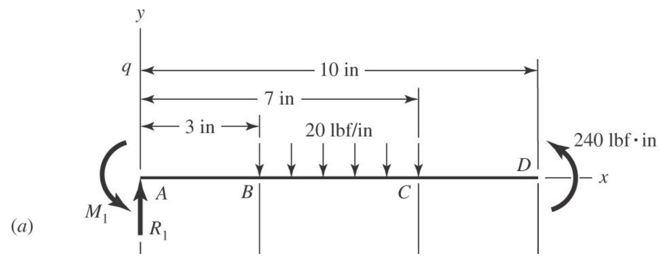
Compute the maximum bending moment developed in the shaft.

Continued..

Example 3-3

Figure 3–6*a* shows the loading diagram for a beam cantilevered at *A* with a uniform load of 20 lbf/in acting on the portion 3 in $\leq x \leq$ 7 in, and a concentrated counter-clockwise moment of 240 lbf · in at $x = 10$ in. Derive the shear-force and bending-moment relations, and the support reactions M_1 and R_1.

Fig. 3-6



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Stress

- *Normal stress* is normal to a surface, designated by σ
- *Tangential shear stress* is tangent to a surface, designated by τ

- Normal stress acting outward on surface is *tensile stress*
- Normal stress acting inward on surface is *compressive stress*

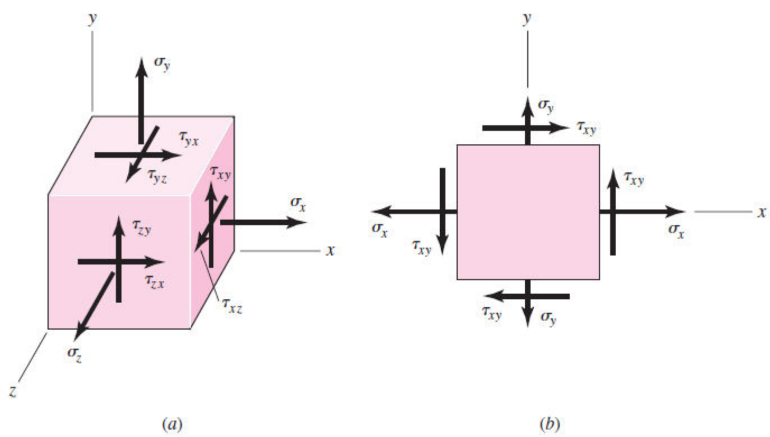
- U.S. Customary units of stress are pounds per square inch (psi)
- 1 lbf/in^2 = 1 psi

- SI units of stress are newtons per square meter (N/m^2)
- 1 N/m^2 = 1 pascal (Pa)

Stress element

Figure 3–8

(*a*) General three-dimensional stress. (*b*) Plane stress with "cross-shears" equal.



- Represents stress *at a point*
- Coordinate directions are arbitrary
- Choosing coordinates which result in zero shear stress will produce principal stresses

Cartesian Stress Components

- Defined by three mutually orthogonal surfaces at a point within a body
- Each surface can have normal and shear stress
- Shear stress is often resolved into perpendicular components
- First subscript indicates direction of surface normal
- Second subscript indicates direction of shear stress

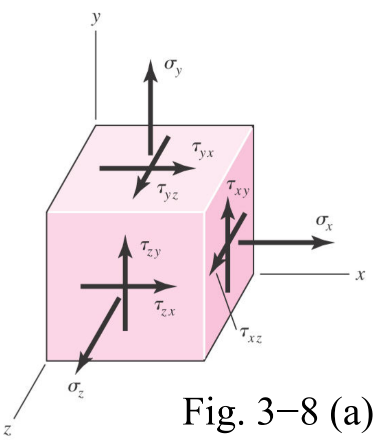


Fig. 3−8 (a)

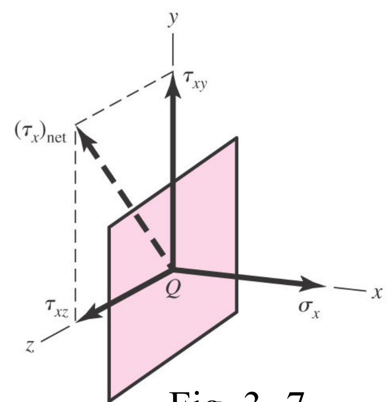


Fig. 3−7

Cartesian Stress Components

- In most cases, "cross shears" are equal

$$\tau_{yx} = \tau_{xy} \qquad \tau_{zy} = \tau_{yz} \qquad \tau_{xz} = \tau_{zx} \tag{3–7}$$

- *Plane stress* occurs when stresses on one surface are zero

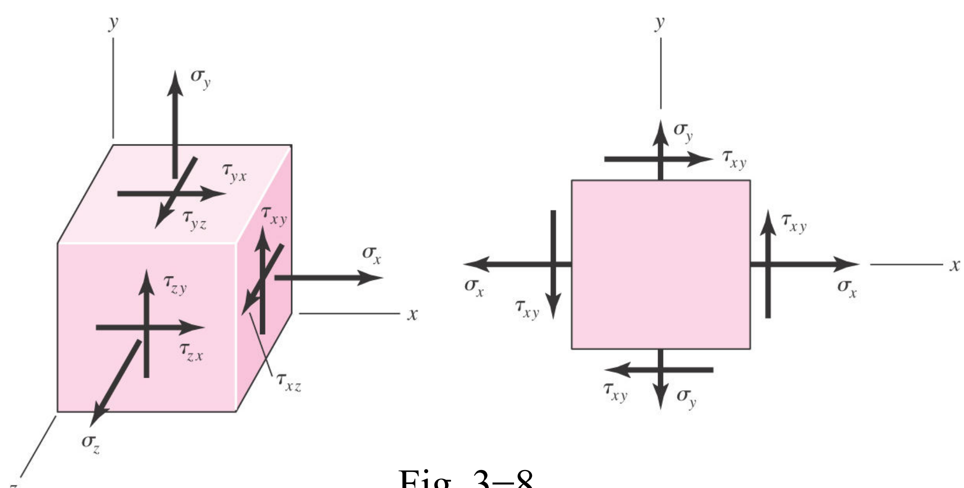


Fig. 3−8

Plane-Stress Transformation Equations

- Cutting plane stress element at an arbitrary angle and balancing stresses gives *plane-stress transformation equations*

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi \qquad (3\text{–}8)$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \qquad (3\text{–}9)$$

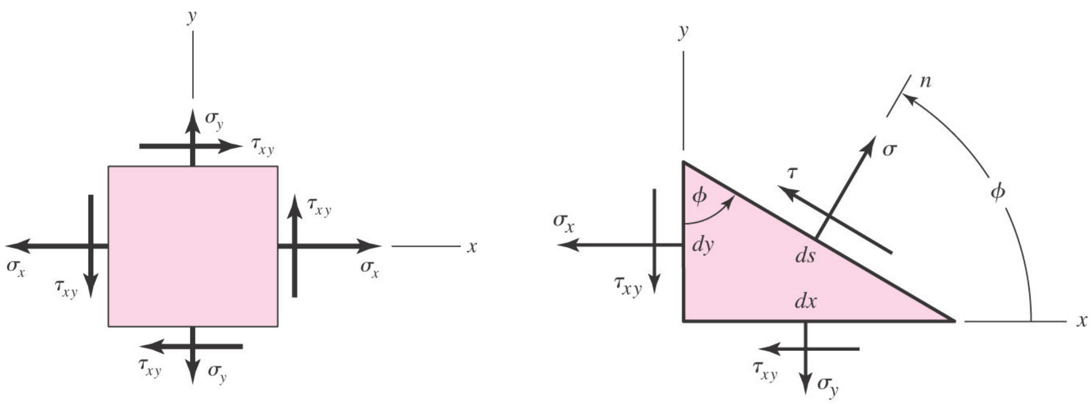


Fig. 3–9

Principal Stresses for Plane Stress

- Differentiating Eq. (3-8) with respect to ϕ and setting equal to zero maximizes σ and gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad (3\text{–}10)$$

- The two values of $2\phi_p$ are the *principal directions.*
- The stresses in the principal directions are the *principal stresses.*
- The principal direction surfaces have zero shear stresses.
- Substituting Eq. (3-10) into Eq. (3-8) gives expression for the non-zero principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (3\text{–}13)$$

- Note that there is a third principal stress, equal to zero for plane stress.

Extreme-value Shear Stresses for Plane Stress

- Performing similar procedure with shear stress in Eq. (3-9), the maximum shear stresses are found to be on surfaces that are ±45º from the principal directions.

- The two extreme-value shear stresses are

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (3\text{–}14)$$

- Direction for maximum shear stress can be found from

$$\tan 2\phi_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

- Since $\tan 2\phi_s = -\frac{1}{\tan 2\phi_p}$, the angle between $2\phi_p$ and $2\phi_s$ is 90º

Maximum Shear Stress

- There are always three principal stresses. One is zero for plane stress.
- There are always three extreme-value shear stresses.

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \qquad (3\text{–}16)$$

- The *maximum shear stress* is always the greatest of these three.
- Eq. (3-14) will not give the *maximum* shear stress in cases where there are two non-zero principal stresses that are both positive or both negative.
- If principal stresses are ordered so that $\sigma_1 > \sigma_2 > \sigma_3$, then $\tau_{max} = \tau_{1/3}$

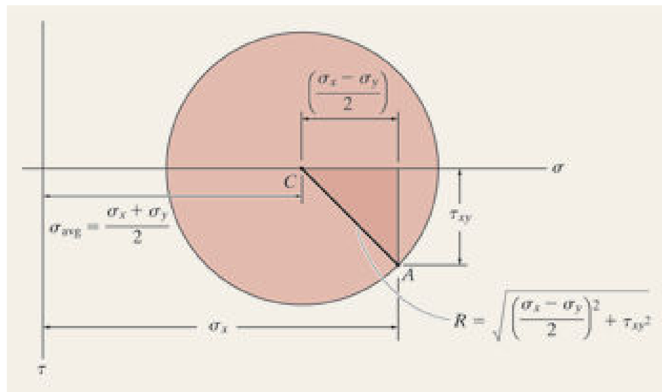
Mohr's Circle Diagram

- A graphical method for visualizing the stress state at a point
- Represents relation between x-y stresses and principal stresses
- Parametric relationship between σ and τ (with 2ϕ as parameter)
- Relationship is a circle with center at

$$C = (\sigma, \tau) = [(\sigma_x + \sigma_y)/2, 0\,]$$

and radius of

$$R = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$



Mohr's Circle Diagram

Construction of circle

- Establish a coordinate system where abscissa represents the normal stress σ (positive to the right), and the ordinate represents the shear stress τ (positive downward).
- Locate the center of the circle C, which lies on σ axis at a distance $\sigma_{avg} = \left(\sigma_x + \sigma_y\right)/2$ from the origin.
- Locate a reference point A, which has coordinates $A\left(\sigma_x, \tau_{xy}\right)$.
- Connect the points C and A, and compute the distance CA (the radius of the circle) by trigonometry.

Principal stresses

- The circle intersect the σ axis at two points (B and D). They are the principal stresses $\sigma_1 \geq \sigma_2$.
- The angle between CA and CB is $2\theta_{p1}$, and the angle between CA and CD is $2\theta_{p2}$.
- A rotation of 2θ in the circle corresponds to a rotation of θ in the element.

Maximum shear stress

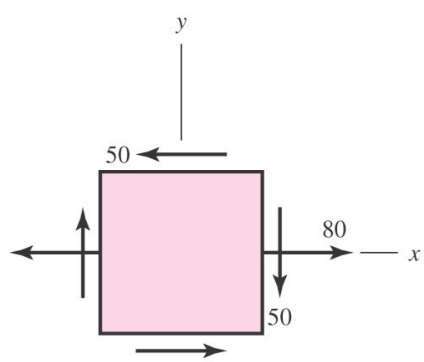
- The radius of circle is equal to the maximum shear stress value.
- The angle between CA and CE is $2\theta_{s1}$, and the angle between CA and CF is $2\theta_{s2}$.
- Again, a rotation of 2θ in the circle corresponds to a rotation of θ in the element.

Example 3-4

A stress element has $\sigma_x = 80$ MPa and $\tau_{xy} = 50$ MPa cw, as shown in Fig. 3–11*a*.

(*a*) Using Mohr's circle, find the principal stresses and directions, and show these on a stress element correctly aligned with respect to the *xy* coordinates. Draw another stress element to show τ_1 and τ_2, find the corresponding normal stresses, and label the drawing completely.

(*b*) Repeat part *a* using the transformation equations only.



(*a*)

Continued..

General Three-Dimensional Stress

- All stress elements are actually 3-D.
- Plane stress elements simply have one surface with zero stresses.
- For cases where there is no stress-free surface, the principal stresses are found from the roots of **the cubic equation**

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma$$
$$- (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0 \qquad (3\text{–}15)$$

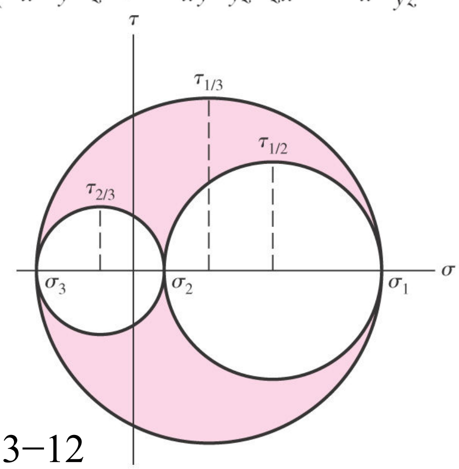


Fig. 3–12

The coefficients of this equation is called "stress invariants." They are independent of the coordinate system used.

The Cubic Equation*

A stress vector parallel to the normal vector $\mathbf{n}$ is given by:

$$\mathbf{T}^{(\mathbf{n})} = \lambda\mathbf{n} = \sigma_\mathbf{n}\mathbf{n}$$

where λ is a constant of proportionality, and in this particular case corresponds to the magnitudes $\sigma_\mathbf{n}$ of the normal stress vectors or principal stresses.

Knowing that $T_i^{(n)} = \sigma_{ij}n_j$ and $n_i = \delta_{ij}n_j$, we have

$$T_i^{(n)} = \lambda n_i$$
$$\sigma_{ij}n_j = \lambda n_i$$
$$\sigma_{ij}n_j - \lambda n_i = 0$$
$$(\sigma_{ij} - \lambda\delta_{ij})\, n_j = 0$$

This is a homogeneous system, i.e. equal to zero, of three linear equations where n_j are the unknowns. To obtain a nontrivial (non-zero) solution for n_j, the determinant matrix of the coefficients must be equal to zero, i.e. the system is singular.

Thus,

$$|\sigma_{ij} - \lambda\delta_{ij}| = \begin{vmatrix} \sigma_{11} - \lambda & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \lambda & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \lambda \end{vmatrix} = 0$$

Expanding the determinant leads to the *characteristic equation*

$$|\sigma_{ij} - \lambda\delta_{ij}| = -\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$$

where

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$$
$$= \sigma_{kk}$$
$$I_2 = \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}$$
$$= \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2$$
$$= \frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji})$$
$$I_3 = \det(\sigma_{ij})$$
$$= \sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{12}^2\sigma_{33} - \sigma_{23}^2\sigma_{11} - \sigma_{31}^2\sigma_{22}$$

The material presented here is taken from Wikipedia.

General Three-Dimensional Stress

- Always three extreme shear values

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \qquad (3\text{–}16)$$

- *Maximum Shear Stress* is the largest
- Principal stresses are usually ordered such that $\sigma_1 > \sigma_2 > \sigma_3$, in which case $\tau_{max} = \tau_{1/3}$

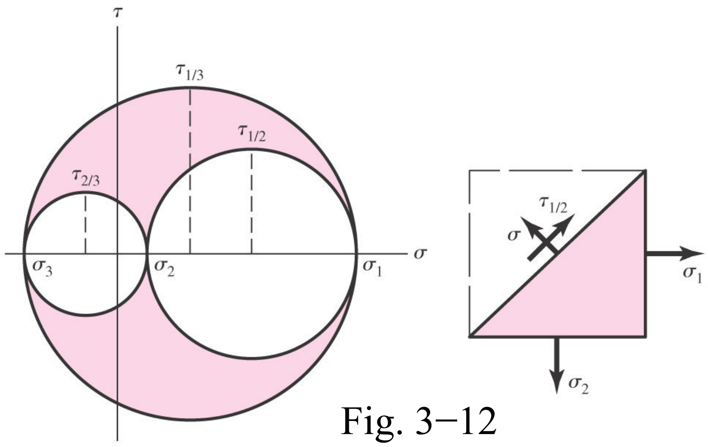


Fig. 3−12

Elastic Strain

- *Hooke's law*

$$\sigma = E\epsilon \qquad (3\text{–}17)$$

- E is Young's modulus, or modulus of elasticity
- Tension in on direction produces negative strain (contraction) in a perpendicular direction.
- For axial stress in x direction,

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu \frac{\sigma_x}{E} \qquad (3\text{–}18)$$

- The constant of proportionality ν is *Poisson's ratio*
- See Table A-5 for values for common materials.

ν = 0.333 for aluminum
ν = 0.292 for carbon steel

Elastic Strain

- For a stress element undergoing σ_x, σ_y, and σ_z, simultaneously,

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$
$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad (3\text{–}19)$$
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Elastic Strain

- Hooke's law for shear:

$$\tau = G\gamma \qquad (3\text{–}20)$$

- *Shear strain* γ is the change in a right angle of a stress element when subjected to pure shear stress.
- G is the *shear modulus of elasticity* or *modulus of rigidity.*
- For a linear, isotropic, homogeneous material,

$$E = 2G(1 + \nu) \qquad (3\text{–}21)$$

Uniformly Distributed Stresses

- Uniformly distributed stress distribution is often assumed for pure tension, pure compression, or pure shear.
- For tension and compression,

$$\sigma = \frac{F}{A} \qquad (3\text{–}22)$$

- For direct shear (no bending present),

$$\tau = \frac{V}{A} \qquad (3\text{–}23)$$

Normal Stresses for Beams in Bending

- Straight beam in positive bending
- *x* axis is *neutral axis*
- *xz* plane is *neutral plane*
- *Neutral axis* is coincident with the *centroidal axis* of the cross section
  - requires symmetric cross section

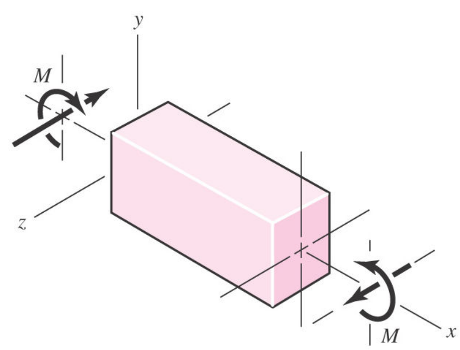


Fig. 3−13

Normal Stresses for Beams in Bending

- Bending stress varies linearly with distance from neutral axis, *y*

$$\sigma_x = -\frac{My}{I} \qquad (3\text{–}24)$$

- *I* is the *second-area moment* about the *z* axis

$$I = \int y^2 dA \qquad (3\text{–}25)$$

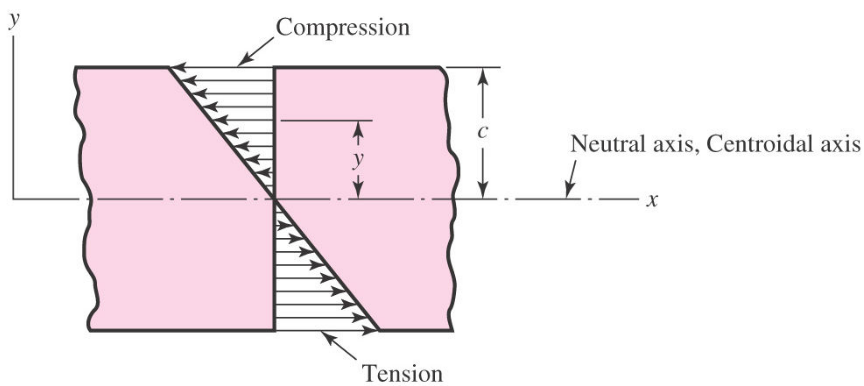


Fig. 3−14

Assumptions for Normal Bending Stress

- Pure bending (though effects of axial, torsional, and shear loads are often assumed to have minimal effect on bending stress)
- Material is isotropic and homogeneous
- Material obeys Hooke's law
- Beam is initially straight with constant cross section
- Beam has axis of symmetry in the plane of bending
- Proportions are such that failure is by bending rather than crushing, wrinkling, or sidewise buckling
- Plane cross sections remain plane during bending

Example 3-5

A beam having a T section with the dimensions shown in Fig. 3–15 is subjected to a bending moment of 1600 N · m, about the negative z axis, that causes tension at the top surface. Locate the neutral axis and find the maximum tensile and compressive bending stresses.

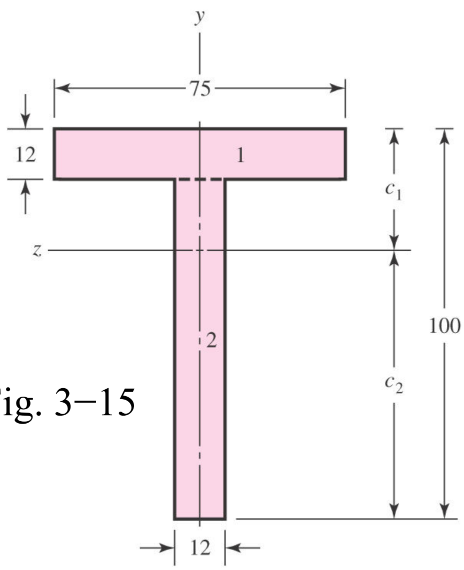


Fig. 3−15

Dimensions in mm

Continued..

Two-Plane Bending

- Consider bending in both *xy* and *xz* planes
- Cross sections with one or two planes of symmetry only

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad (3\text{–}27)$$

- For solid circular cross section, the maximum bending stress is

$$\sigma_m = \frac{Mc}{I} = \frac{(M_y^2 + M_z^2)^{1/2}(d/2)}{\pi d^4/64} = \frac{32}{\pi d^3}(M_y^2 + M_z^2)^{1/2} \qquad (3\text{–}28)$$

Example 3-6

As shown in Fig. 3–16*a*, beam *OC* is loaded in the *xy* plane by a uniform load of 50 lbf/in, and in the *xz* plane by a concentrated force of 100 lbf at end *C*. The beam is 8 in long.

(*a*) For the cross section shown determine the maximum tensile and compressive bending stresses and where they act.

(*b*) If the cross section was a solid circular rod of diameter, $d = 1.25$ in, determine the magnitude of the maximum bending stress.

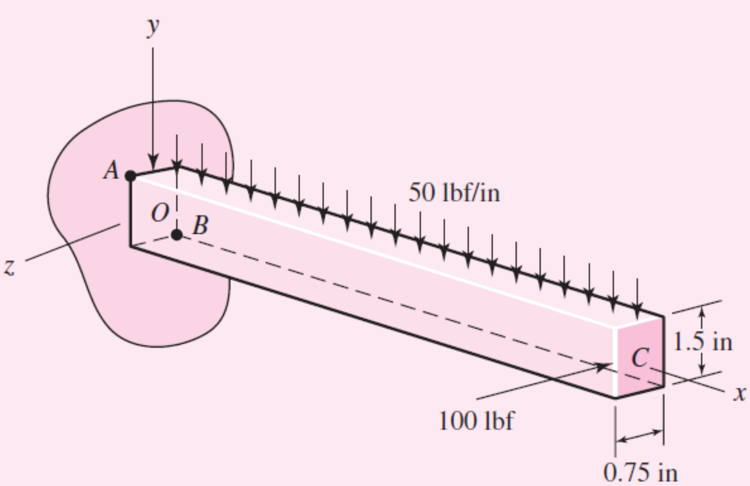


Fig. 3−16 (*a*)

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Shear Stresses for Beams in Bending

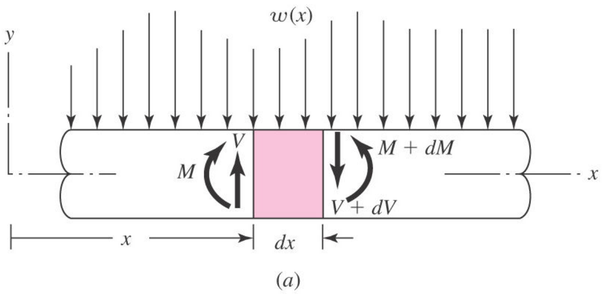
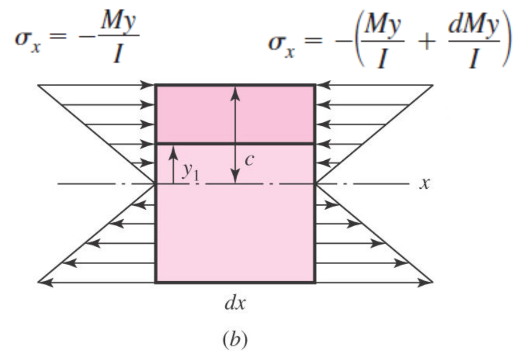
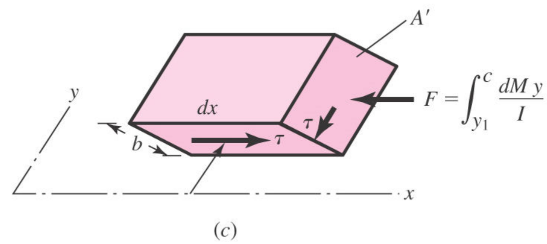


Fig. 3−17





Equilubrium in x-direction:

$$\tau b\, dx = \int_{y_1}^{c} \frac{(dM)y}{I} dA$$

$$\tau = \frac{V}{Ib} \int_{y_1}^{c} y dA \qquad (3\text{–}29)$$

Transverse Shear Stress

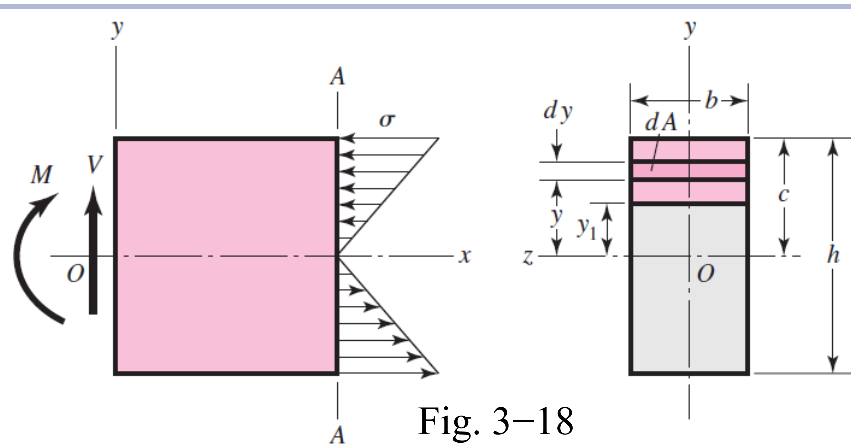


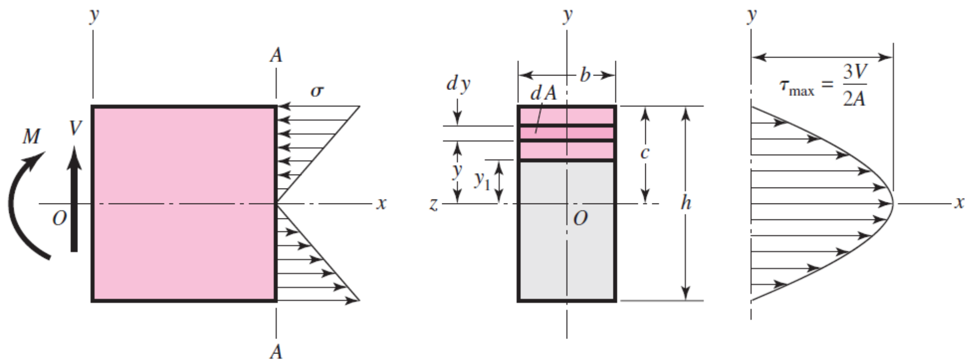
Fig. 3−18

$$Q = \int_{y_1}^{c} y dA = \bar{y}' A' \qquad (3\text{–}30)$$

$$\boxed{\tau = \frac{VQ}{Ib}} \qquad (3\text{–}31)$$

- Transverse shear stress is always accompanied with bending stress.

Transverse Shear Stress in a Rectangular Beam



(a) (b) (c)

$$Q = \int_{y_1}^{c} y\,dA = b\int_{y_1}^{c} y\,dy = \left.\frac{by^2}{2}\right|_{y_1}^{c} = \frac{b}{2}\left(c^2 - y_1^2\right)$$

$$\tau = \frac{VQ}{Ib} = \frac{V}{2I}\left(c^2 - y_1^2\right) \qquad I = \frac{Ac^2}{3}$$

$$\tau = \frac{3V}{2A}\left(1 - \frac{y_1^2}{c^2}\right) \qquad (3\text{–}33)$$

parabolic

Maximum Values of Transverse Shear Stress

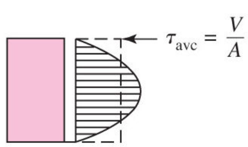
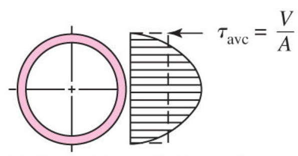
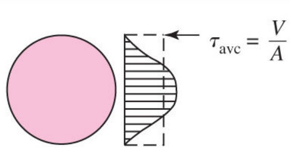
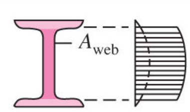
Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{max} = \frac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{max} = \frac{2V}{A}$
 Circular	$\tau_{max} = \frac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{max} \doteq \frac{V}{A_{web}}$

Table 3–2

Significance of Transverse Shear Compared to Bending

- **Example: Cantilever beam, rectangular cross section**
- The beam is subjected to bending and shear
- Maximum shear stress, including bending stress (*My/I*) and transverse shear stress (*VQ/Ib*),

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{3F}{2bh}\sqrt{4(L/h)^2(y/c)^2 + \left[1 - (y/c)^2\right]^2}$$

- The effect of the transverse shear is negligible if L > 10h.

Example 3-7

A beam 12 in long is to support a load of 488 lbf acting 3 in from the left support, as shown in Fig. 3–20*a*. The beam is an I beam with the cross-sectional dimensions shown. To simplify the calculations, assume a cross section with square corners, as shown in Fig. 3–20*c*. Points of interest are labeled (*a*, *b*, *c*, and *d*) at distances *y* from the neutral axis of 0 in, 1.240^- in, 1.240^+ in, and 1.5 in (Fig. 3–20*c*). At the critical axial location along the beam, find the following information.

(*a*) Determine the profile of the distribution of the transverse shear stress, obtaining values at each of the points of interest.

(*b*) Determine the bending stresses at the points of interest.

(*c*) Determine the maximum shear stresses at the points of interest, and compare them.

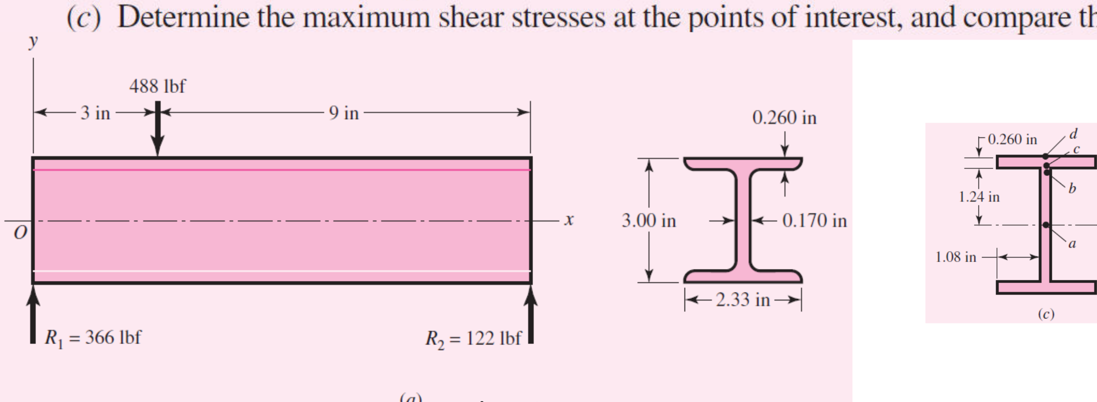


Fig. 3−20

Continued..

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Power, Speed, and Torque

- Power equals torque times speed

$$H = T\omega \qquad (3\text{–}43)$$

where H = power, W
T = torque, N · m
ω = angular velocity, rad/s

- A convenient conversion with speed in rpm

$$T = 9.55\frac{H}{n} \qquad (3\text{–}44)$$

where H = power, W
n = angular velocity, revolutions per minute

Power, Speed, and Torque

- In U.S. Customary units, with unit conversion built in

$$H = \frac{FV}{33\,000} = \frac{2\pi Tn}{33\,000(12)} = \frac{Tn}{63\,025} \qquad (3\text{–}42)$$

where H = power, hp
T = torque, lbf · in
n = shaft speed, rev/min
F = force, lbf
V = velocity, ft/min

Torsion

- *Torque vector* – a moment vector collinear with axis of a mechanical element
- A bar subjected to a torque vector is said to be in *torsion*
- *Angle of twist*, in radians, for a solid round bar

$$\theta = \frac{Tl}{GJ} \qquad (3\text{–}35)$$

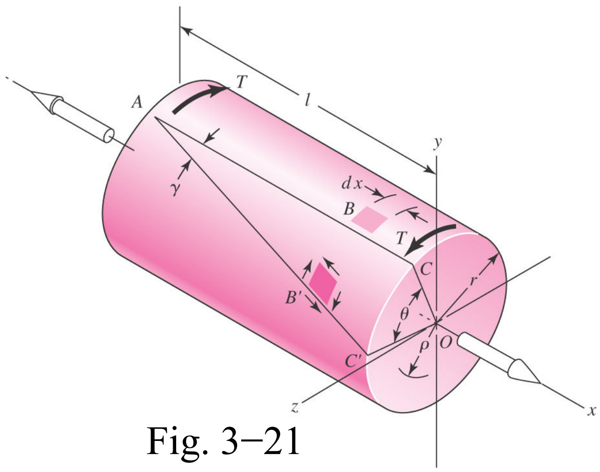


Fig. 3–21

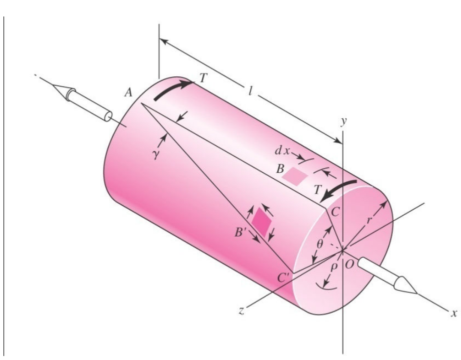
Torsional Shear Stress

- For round bar in torsion, torsional shear stress is proportional to the radius ρ

$$\tau = \frac{T\rho}{J} \qquad (3\text{–}36)$$

- Maximum torsional shear stress is at the outer surface

$$\tau_{\max} = \frac{Tr}{J} \qquad (3\text{–}37)$$



Assumptions for Torsion Equations

- Equations (3-35) to (3-37) are only applicable for the following conditions
  - Pure torque
  - Remote from any discontinuities or point of application of torque
  - Material obeys Hooke's law
  - Adjacent cross sections originally plane and parallel remain plane and parallel
  - Radial lines remain straight
    - Depends on axisymmetry, so does not hold true for noncircular cross sections
- Consequently, only applicable for round cross sections

Torsional Shear in Rectangular Section

- Shear stress does not vary linearly with radial distance for rectangular cross section
- Shear stress is zero at the corners
- Maximum shear stress is at the middle of the longest side
- For rectangular b x c bar, where b is longest side

$$\tau_{\max} = \frac{T}{\alpha b c^2} \doteq \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) \qquad (3\text{–}40)$$

$$\theta = \frac{Tl}{\beta b c^3 G} \qquad (3\text{–}41)$$

b/c	1.00	1.50	1.75	2.00	2.50	3.00	4.00	6.00	8.00	10	∞
α	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.299	0.307	0.313	0.333
β	0.141	0.196	0.214	0.228	0.249	0.263	0.281	0.299	0.307	0.313	0.333

Example 3-8

Figure 3–22 shows a crank loaded by a force $F = 300$ lbf that causes twisting and bending of a $\frac{3}{4}$-in-diameter shaft fixed to a support at the origin of the reference system. In actuality, the support may be an inertia that we wish to rotate, but for the purposes of a stress analysis we can consider this a statics problem.

(*a*) Draw separate free-body diagrams of the shaft AB and the arm BC, and compute the values of all forces, moments, and torques that act. Label the directions of the coordinate axes on these diagrams.

(*b*) Compute the maxima of the torsional stress and the bending stress in the arm BC and indicate where these act.

(*c*) Locate a stress element on the top surface of the shaft at A, and calculate all the stress components that act upon this element.

(*d*) Determine the maximum normal and shear stresses at A.

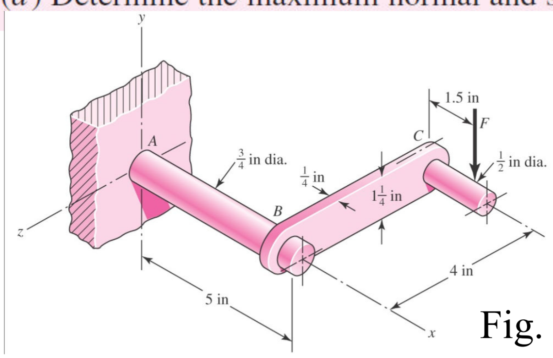


Fig. 3−22

Continued..

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Example 3-9

The 1.5-in-diameter solid steel shaft shown in Fig. 3–24*a* is simply supported at the ends. Two pulleys are keyed to the shaft where pulley *B* is of diameter 4.0 in and pulley *C* is of diameter 8.0 in. Considering bending and torsional stresses only, determine the locations and magnitudes of the greatest tensile, compressive, and shear stresses in the shaft.

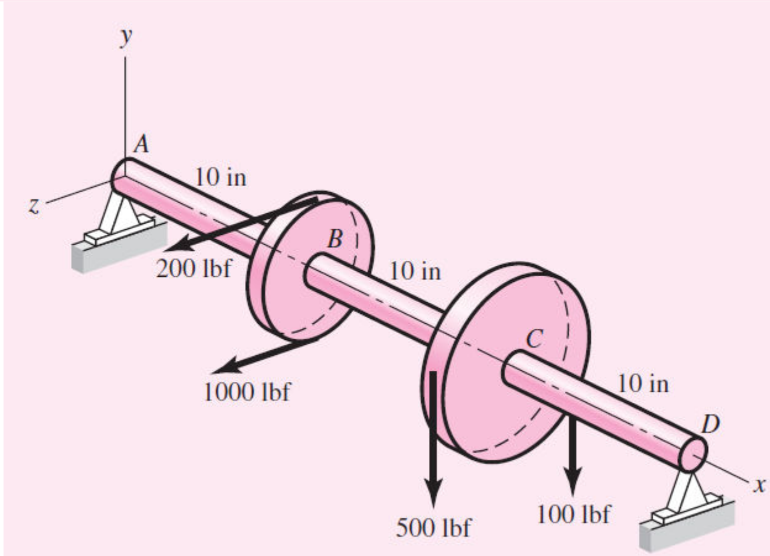

Fig. 3−24 (*a*)

Continued..

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Closed Thin-Walled Tubes

- Wall thickness t << tube radius r
- Product of shear stress times wall thickness is constant
- Shear stress is inversely proportional to wall thickness
- Total torque T is

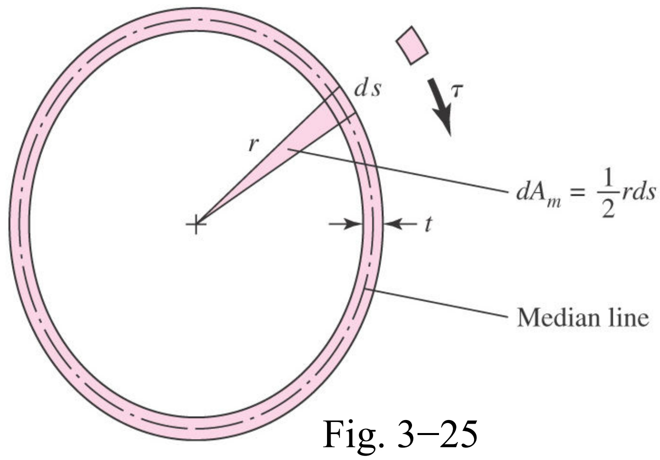


Fig. 3–25

$$T = \int \tau t r \, ds = (\tau t) \int r \, ds = \tau t (2A_m) = 2A_m t \tau$$

- A_m is the area enclosed by the section median line

Closed Thin-Walled Tubes

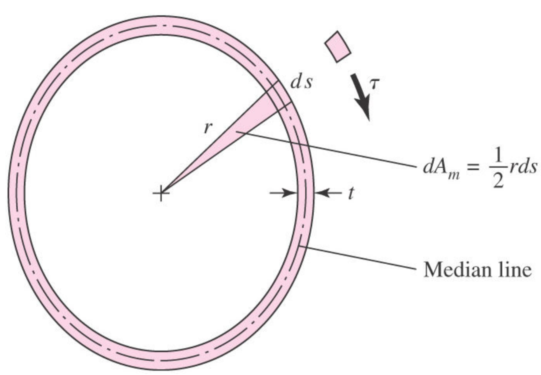
- Solving for shear stress

$$\tau = \frac{T}{2A_m t} \qquad (3\text{–}45)$$

- Angular twist (radians) per unit length

$$\theta_1 = \frac{T L_m}{4G A_m^2 t} \qquad (3\text{–}46)$$

- L_m is the length of the section median line



Example 3-10

A welded steel tube is 40 in long, has a $\frac{1}{8}$-in wall thickness, and a 2.5-in by 3.6-in rectangular cross section as shown in Fig. 3–26. Assume an allowable shear stress of 11 500 psi and a shear modulus of $11.5(10^6)$ psi.

(*a*) Estimate the allowable torque *T*.

(*b*) Estimate the angle of twist due to the torque.

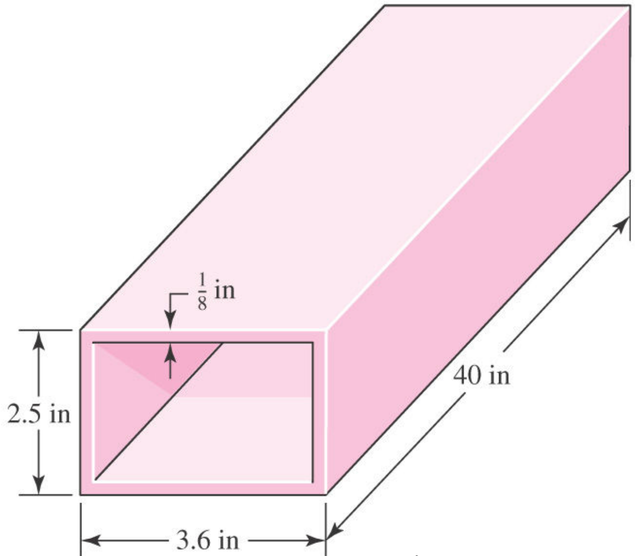


Fig. 3−26

Continued..

Example 3-11

Compare the shear stress on a circular cylindrical tube with an outside diameter of 1 in and an inside diameter of 0.9 in, predicted by Eq. (3–37), to that estimated by Eq. (3–45).

Solution

Eq. (3–37),

$$\tau_{\max} = \frac{Tr}{J}$$

Eq. (3–45),

$$\tau = \frac{T}{2A_m t}$$

Open Thin-Walled Sections

- When the median wall line is not closed, the section is said to be an *open section*
- Some common open thin-walled sections

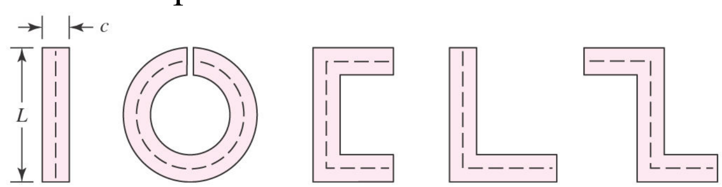


Fig. 3–27

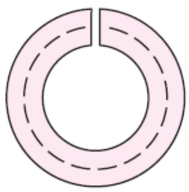
- Torsional shear stress

$$\tau = G\theta_1 c = \frac{3T}{Lc^2} \qquad (3\text{–}47)$$

where T = Torque, L = length of median line, c = wall thickness, G = shear modulus, and θ_1 = angle of twist per unit length

Open Thin-Walled Sections

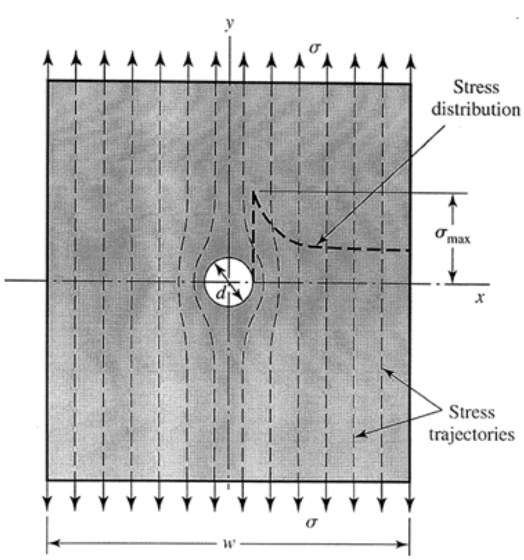
- Shear stress is inversely proportional to c^2
- Angle of twist is inversely proportional to c^3
- For small wall thickness, stress and twist can become quite large
- **Example:**
  - Compare thin round tube with and without slit
  - Ratio of wall thickness to outside diameter of 0.1
  - Stress with slit is 12.3 times greater
  - Twist with slit is 61.5 times greater



Stress Concentration

- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0} \qquad (3\text{–}48)$$



Theoretical Stress Concentration Factor

- Graphs available for standard configurations
- See Appendix A-15 and A-16 for common examples
- Many more in *Peterson's Stress-Concentration Factors*
- Note the trend for higher K_t at sharper discontinuity radius, and at greater disruption

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

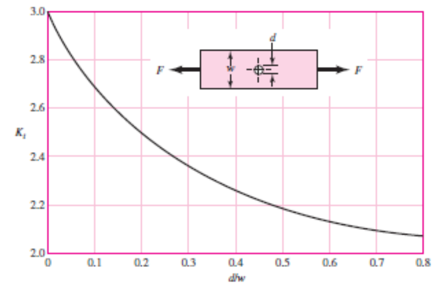
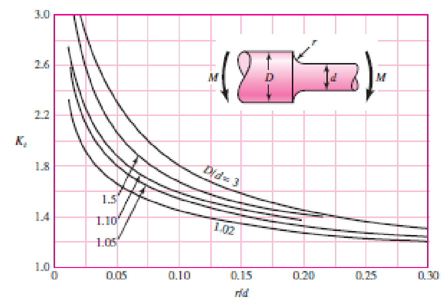


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Example 3-13

The 2-mm-thick bar shown in Fig. 3–30 is loaded axially with a constant force of 10 kN. The bar material has been heat treated and quenched to raise its strength, but as a consequence it has lost most of its ductility. It is desired to drill a hole through the center of the 40-mm face of the plate to allow a cable to pass through it. A 4-mm hole is sufficient for the cable to fit, but an 8-mm drill is readily available. Will a crack be more likely to initiate at the larger hole, the smaller hole, or at the fillet?

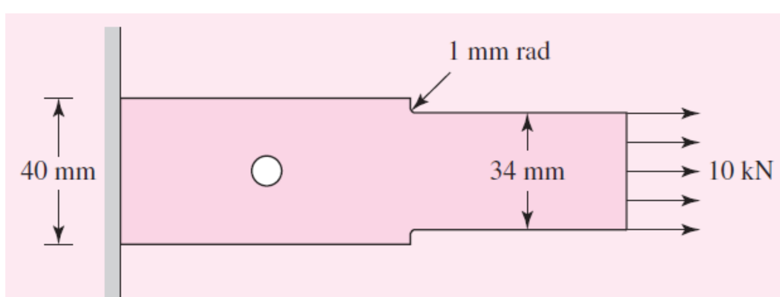
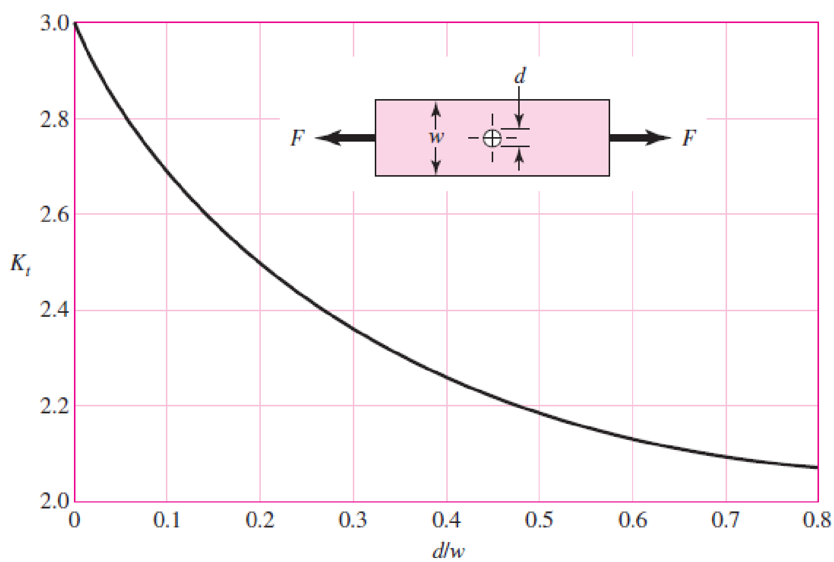


Fig. 3−30

Continued..

Example 3-13

Maximum stresses at the 4-mm and 8-mm holes

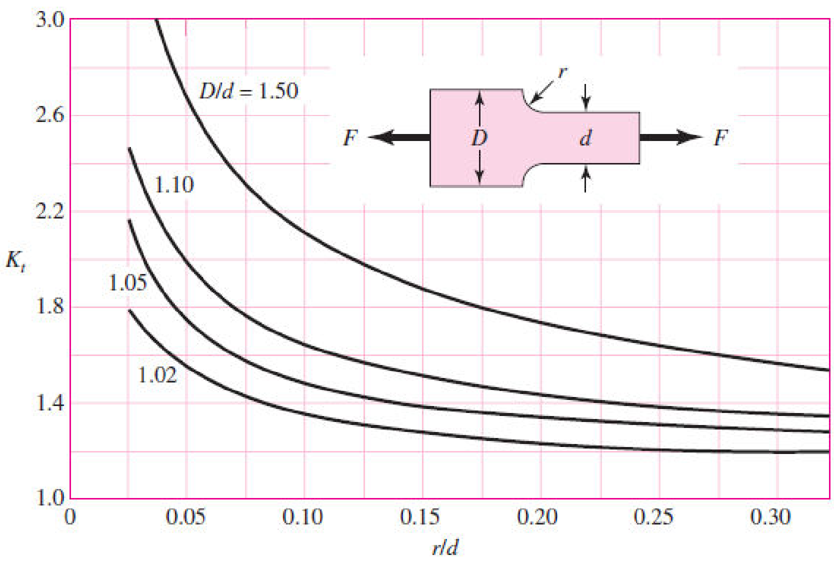


$$\sigma_{max} = K_t \sigma_0$$

Example 3-13

Maximum stress at the fillet

$$\sigma_{\max} = K_t \sigma_0$$



Stresses in Pressurized Cylinders

- Cylinder with inside radius r_i, outside radius r_o, internal pressure p_i, and external pressure p_o
- Tangential and radial stresses,

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2} \quad (3\text{–}49)$$

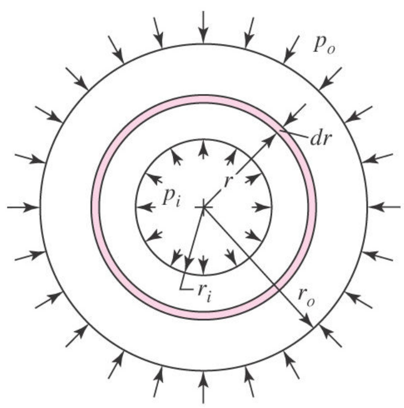


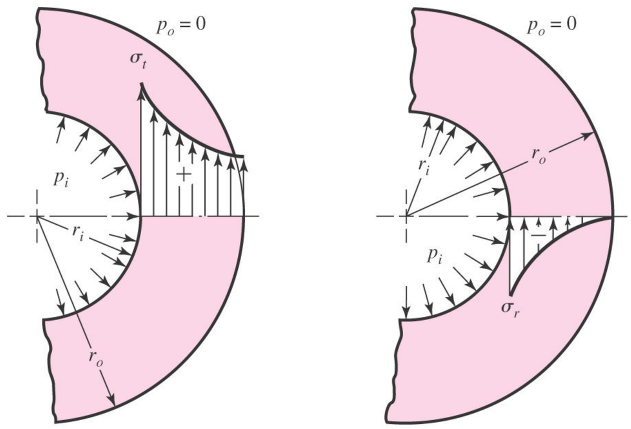
Fig. 3–31

Stresses in Pressurized Cylinders

- Special case of zero outside pressure, $p_o = 0$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \qquad (3\text{–}50)$$



(a) Tangential stress distribution Fig. 3–32 (b) Radial stress distribution

- If ends are closed, then longitudinal stresses also exist

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} \qquad (3\text{–}51)$$

Thin-Walled Vessels

- Cylindrical pressure vessel with wall thickness 1/10 or less of the radius
- Radial stress is quite small compared to tangential stress
- Average tangential stress

$$(\sigma_t)_{\text{av}} = \frac{pd_i}{2t} \qquad (3\text{–}52)$$

- Maximum tangential stress

$$(\sigma_t)_{\text{max}} = \frac{p(d_i + t)}{2t} \qquad (3\text{–}53)$$

- Longitudinal stress (if ends are closed)

$$\sigma_l = \frac{pd_i}{4t} \qquad (3\text{–}54)$$

Example 3-14

An aluminum-alloy pressure vessel is made of tubing having an outside diameter of 8 in and a wall thickness of $\frac{1}{4}$ in.

(*a*) What pressure can the cylinder carry if the permissible tangential stress is 12 kpsi and the theory for thin-walled vessels is assumed to apply?

Solution

Stresses in Rotating Rings

- Rotating rings, such as flywheels, blowers, disks, etc.
- Tangential and radial stresses are similar to thick-walled pressure cylinders, except caused by inertial forces
- Conditions:
  - Outside radius is large compared with thickness (>10:1)
  - Thickness is constant
  - Stresses are constant over the thickness
- Stresses are

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right) \qquad (3\text{–}55)$$

Press and Shrink Fits

- Two cylindrical parts are assembled with *radial interference* δ
- Pressure at interface

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]} \qquad (3\text{–}56)$$

- If both cylinders are of the same material

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right] \qquad (3\text{–}57)$$

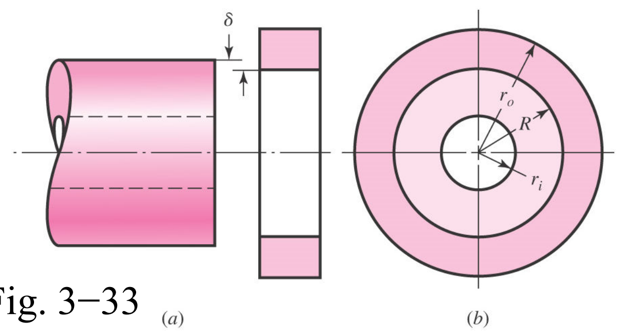


Fig. 3−33

Press and Shrink Fits

- Eq. (3-49) for pressure cylinders applies

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2} \qquad (3\text{–}49)$$

- For the inner member, $p_o = p$ and $p_i = 0$

$$(\sigma_t)_i\Big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} \qquad (3\text{–}58)$$

- For the outer member, $p_o = 0$ and $p_i = p$

$$(\sigma_t)_o\Big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} \qquad (3\text{–}59)$$

Temperature Effects

- Normal strain due to expansion from temperature change

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T) \quad (3\text{–}60)$$

where α is the *coefficient of thermal expansion*

- *Thermal stresses* occur when members are constrained to prevent strain during temperature change
- For a straight bar constrained at ends, temperature increase will create a compressive stress

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \quad (3\text{–}61)$$

- Flat plate constrained at edges

$$\sigma = -\frac{\alpha(\Delta T)E}{1-\nu} \quad (3\text{–}62)$$

Coefficients of Thermal Expansion

Table 3–3

Coefficients of Thermal Expansion (Linear Mean Coefficients for the Temperature Range 0–100°C)

Material	Celsius Scale (°C^{-1})	Fahrenheit Scale (°F^{-1})
Aluminum	$23.9(10)^{-6}$	$13.3(10)^{-6}$
Brass, cast	$18.7(10)^{-6}$	$10.4(10)^{-6}$
Carbon steel	$10.8(10)^{-6}$	$6.0(10)^{-6}$
Cast iron	$10.6(10)^{-6}$	$5.9(10)^{-6}$
Magnesium	$25.2(10)^{-6}$	$14.0(10)^{-6}$
Nickel steel	$13.1(10)^{-6}$	$7.3(10)^{-6}$
Stainless steel	$17.3(10)^{-6}$	$9.6(10)^{-6}$
Tungsten	$4.3(10)^{-6}$	$2.4(10)^{-6}$