MAK 206 – STRENGTH OF MATERIALS

(adopted from Hibbeler's book)

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CHAPTER 1. STRESS

OUTLINE

1.1-1.2. Recall Statics

A body can be subjected to several different types of **external loads**. They can be classified as either

- surface forces (concentrated or distributed)
- body forces

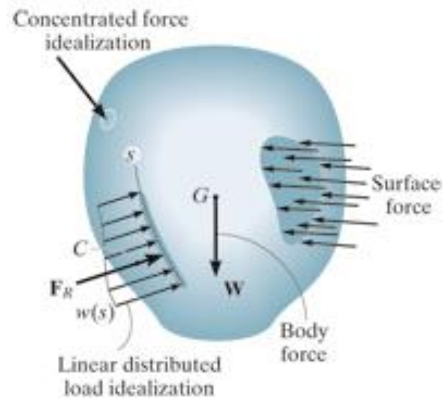


Fig. 1–1

In addition to the external loads, the **support reactions** should also be shown in free body diagrams (FBD).
-- If the support prevents **translation**, a **reaction force** is developed on the body
-- If the support prevents **rotation**, a **reaction moment** is developed on the body

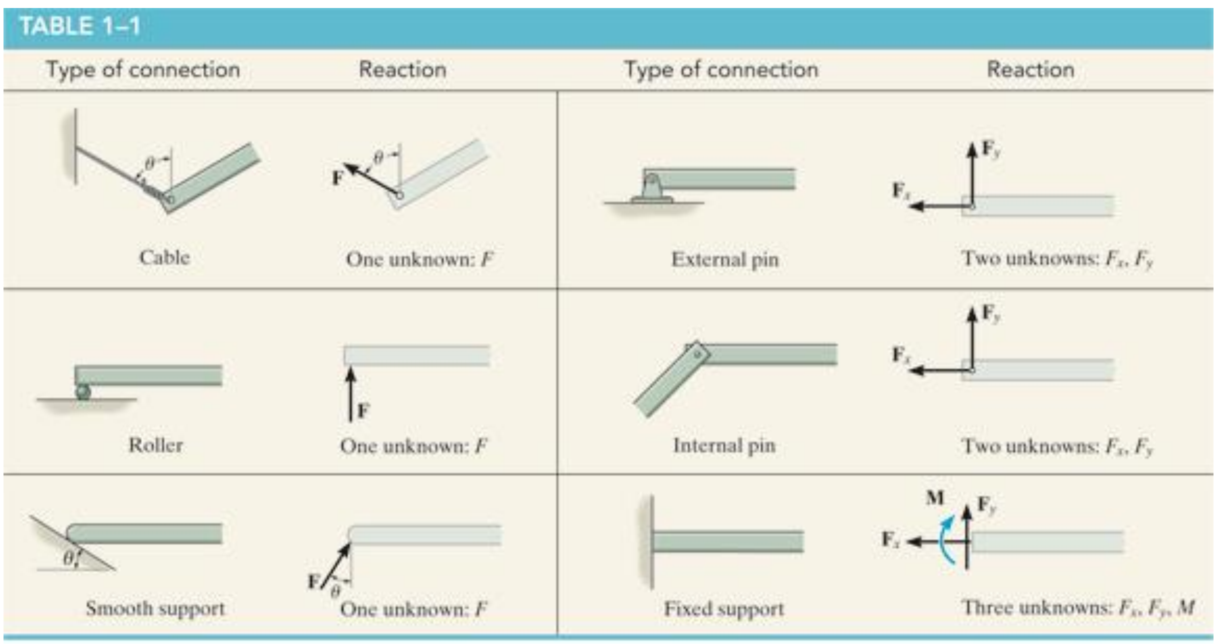


TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
Cable	One unknown: F	External pin	Two unknowns: F_x, F_y
Roller	One unknown: F	Internal pin	Two unknowns: F_x, F_y
Smooth support	One unknown: F	Fixed support	Three unknowns: F_x, F_y, M

<u>Equations of equilibrium (*Denge denklemleri*)</u>

Use equations of equilibrium to compute

- unknown support reactions
- Internal resultant loadings (N,V,M,T)

<u>Internal resultant loadings (*İç bileşke kuvvetler*)</u>

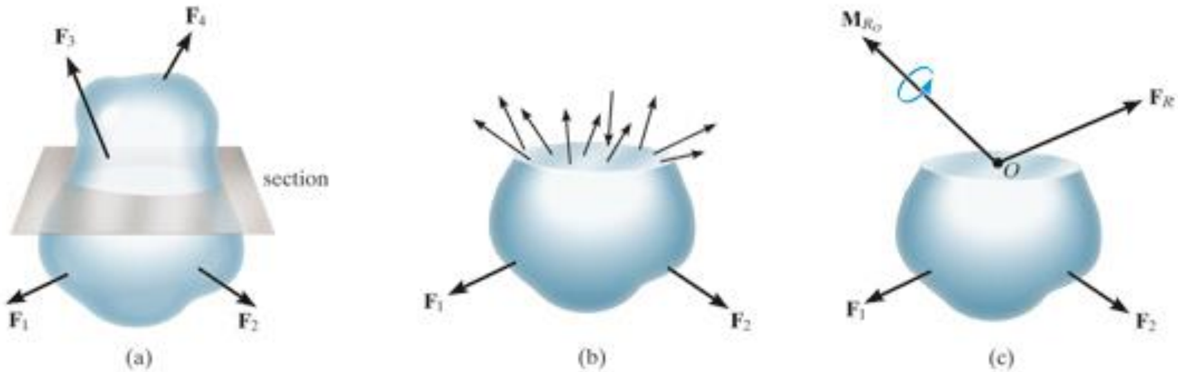


Fig. 1–2

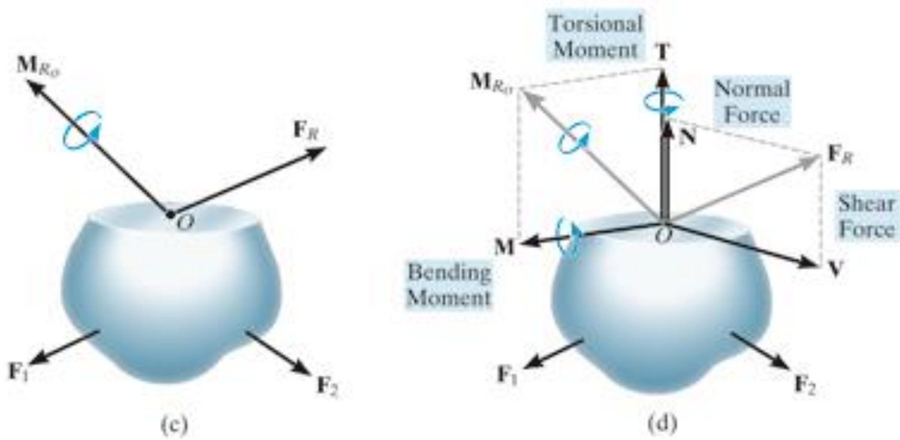


Fig. 1–2 (cont.)

<u>Coplanar Loading (*Düzlemsel yükleme*)</u>

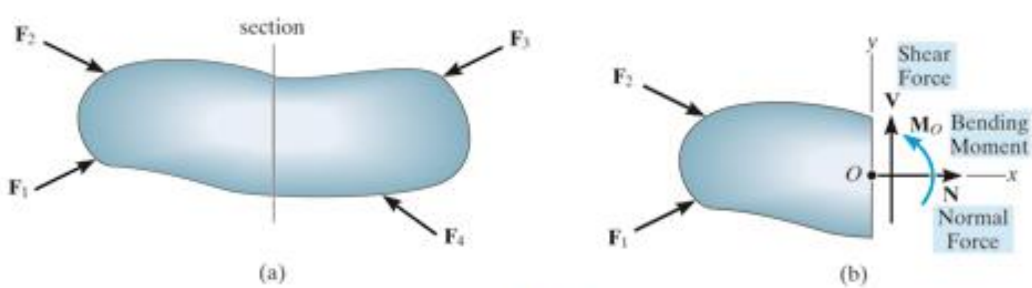
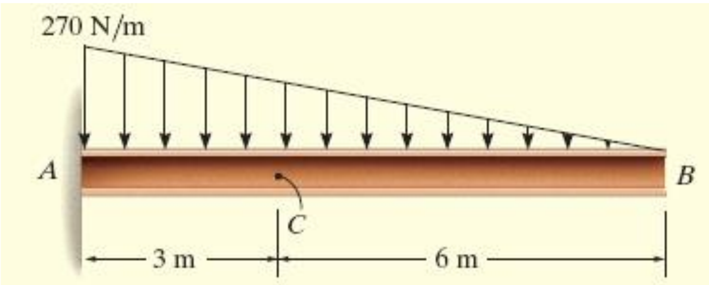


Fig. 1–3

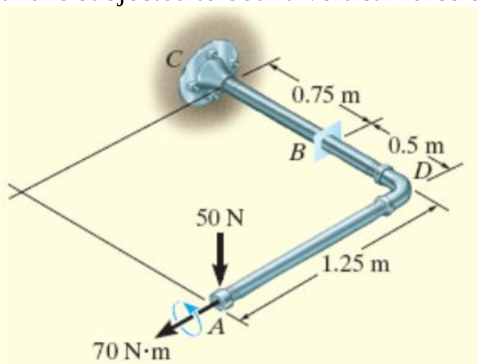
Example 1:

Determine the resultant internal loadings acting on the cross section at C of the beam.

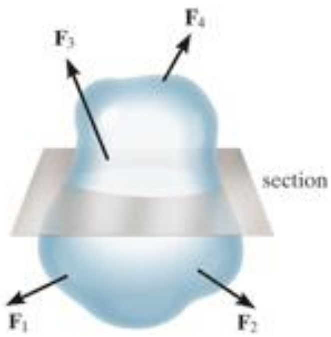


Example 2:

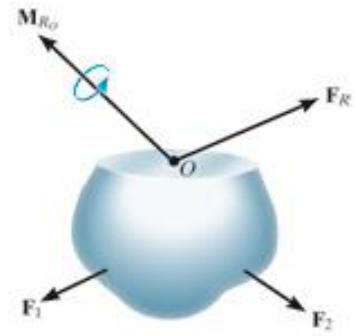
Determine the resultant internal loadings acting on the cross section at *B* of the pipe. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a moment of 70 N·m at its end *A*. It is fixed to the wall at *C*.



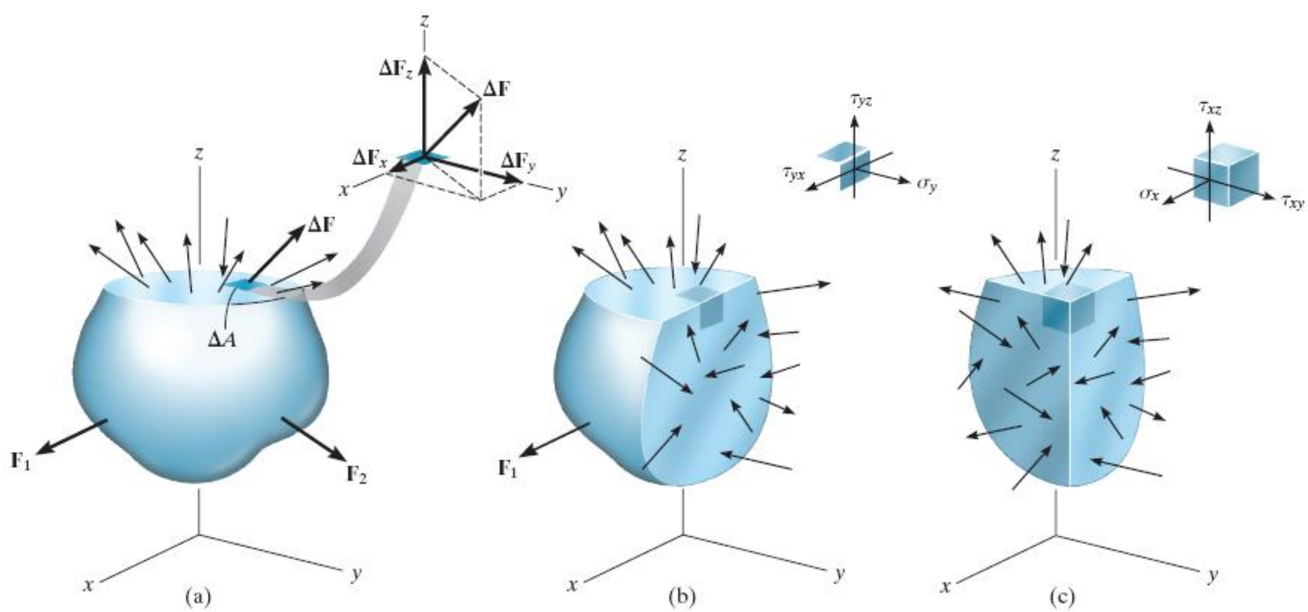
1.3. Stress



MAK 104 STATICS



MAK 206 MECHANICS OF MATERIALS



Normal stress:

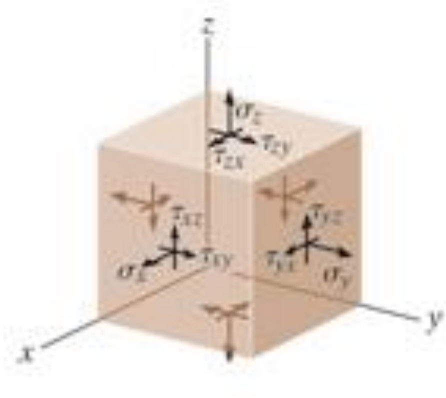
- denoted with σ.
- described with one index only.

Shear stress:

- denoted with τ.
- described with two indices.
- the first index defines the orientation of the area that the stress acts
- the second index defines the axis along which the stress acts

General state of stress:

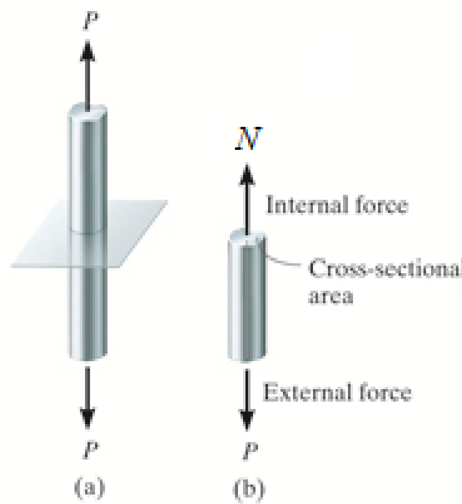
Consider a cubic element taken out of a body under loading



Now, let's use **moment equilibrium** to show that cross shear terms are equal (that is, shear stress tensor is symmetric).

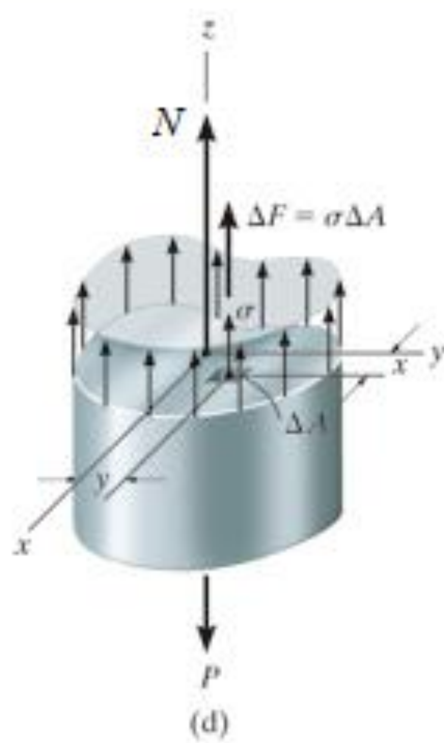
1.4. Average Normal Stress in an Axially Loaded Bar

- truss members
- we can neglect their weight (small compared to loading)



Assumptions

1. The bar remains straight before and after the load is applied, and the cross-section remains plane after deformation → uniform deformation
2. The load should be applied along the centroidal axis
3. The material should be homogenous and isotropic



Equations of equilibrium

$$\sum F_{Rz} = \sum F_z$$

where σ is the average normal stress, N is the internal resultant normal force, and A is the cross-sectional area.

$$\sum M_{Rx} = \sum M_x$$

$$\sum M_{Ry} = \sum M_y$$

Equations (1) and (2) are automatically satisfied, because N pass through the centroid, for which $\int x\,dA = 0$ and $\int y\,dA = 0$.

Maximum average normal stress

$\sigma = \frac{N}{A}$

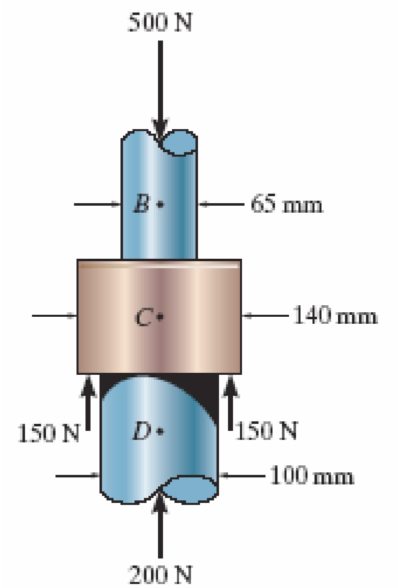
N ← may change along the length of the bar due to various external loads

A ← section area may change along the length of the bar

Thus, it is important to find the maximum of (N/A). It may be helpful to draw a normal force diagram.

Example:

The thrust bearing is subjected to the loads shown. Determine the average normal stress developed on cross sections through points B, C, and D.



1.5. Average Shear Stress (*Ortalama Kayma Gerilmesi*)

τ_{avg} = average shear stress
V = shear force (internal load)
A = shear area

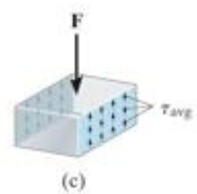


Fig. 1–20

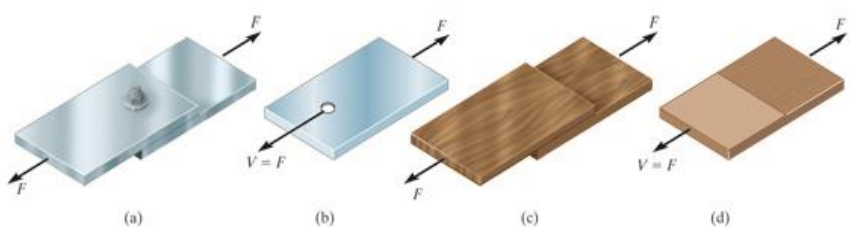


Fig. 1–21

bolted (or riveted) glued

thin plates

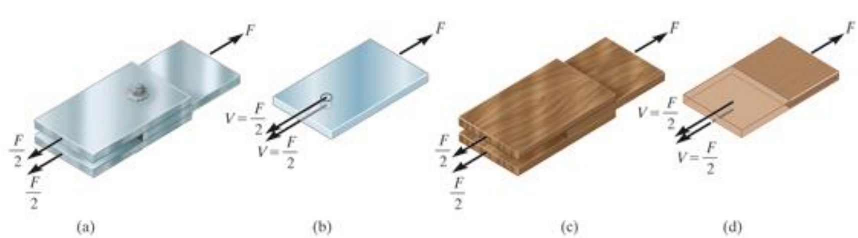


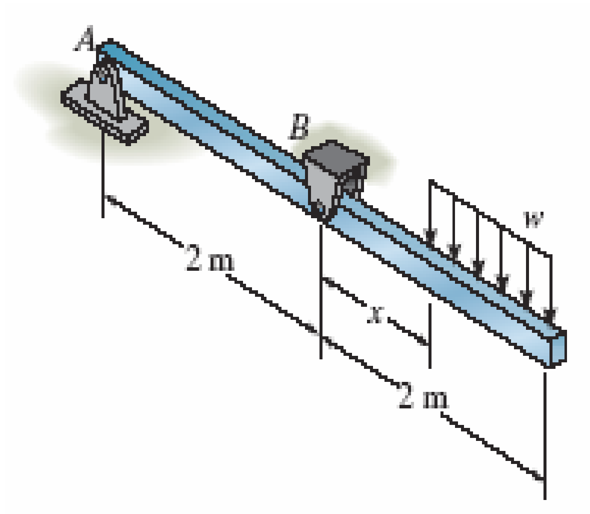
Fig. 1–22

As the plates are **thin**, the bending moment due to F can be neglected.
For thick plates, F will have a bending effect in addition to shearing.

Example:

The bar is held in equilibrium by the pin supports at *A* and *B*. Note that the support at *A* has a single leaf and therefore it involves single shear in the pin, and the support at *B* has a double leaf and therefore it involves double shear.

The allowable shear stress for both pins is $\tau_{allow} = 125$ MPa. If $x = 1$ m and $w = 12$ kN/m, determine the smallest required diameter of pins *A* and *B*. Neglect any axial force in the bar.



1.6. Allowable Stress

- The stress in machine element must remain below certain values.
- These values are selected to be smaller than the true limits of the material.
- Many unknown factors influence the actual stress in a member.
- A factor of safety (F.S.) is needed to obtained allowable load.
- F.S. is a ratio of the failure load divided by the allowable load

- Usually, the load and stress are linearly related, so we have

1.7. Design of simple connections

A) Tension member

Fig. 1–27

B) Connector (bolt, pin)

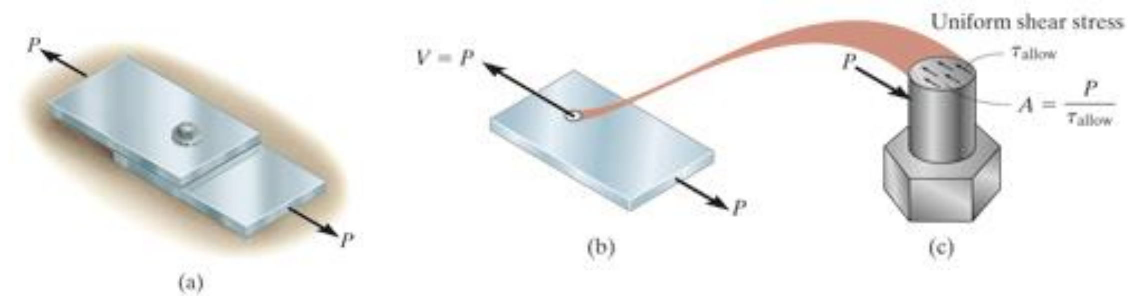


Fig. 1–28

C) Bearing member

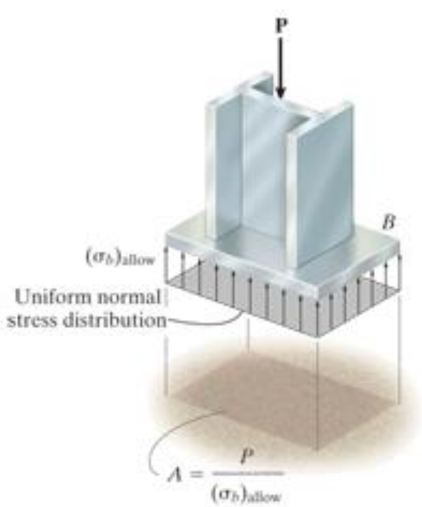


Fig. 1–29

D) Shear due to axial loading

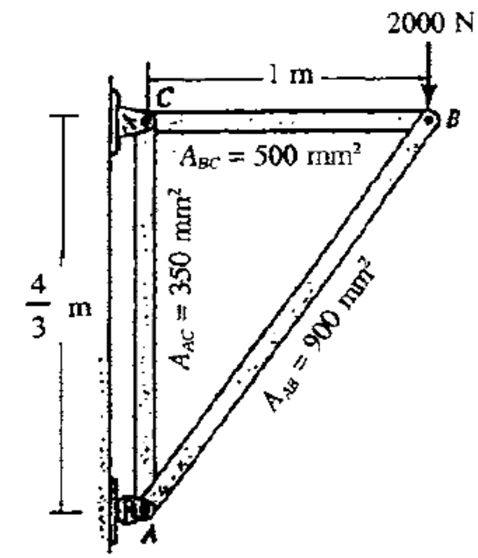
Fig. 1–30

Example 1:

The specimen failed in a tension test at an angle of 52° when the axial load was 100 kN. If the diameter of the specimen is 12 mm, determine the average normal and average shear stress acting on the area of the inclined failure plane. Also, what is the average normal stress acting on the *cross section* when failure occurs?

Example 2:

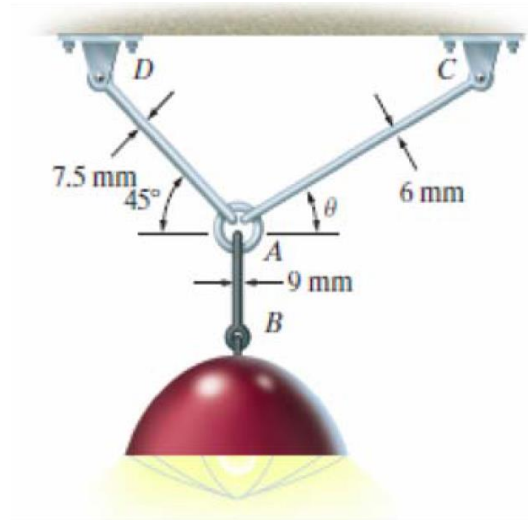
The truss is made from three pin-connected members having the cross-sectional areas shown in the figure. Determine the average normal stress developed in each member when the truss is subjected to the load shown. State whether the stress is tensile or compressive.



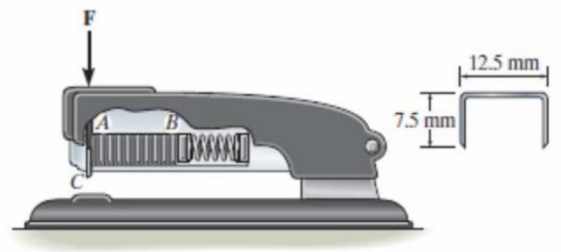
Example 3:

The 250-N lamp is supported by three steel rods connected by a ring at *A*. The diameter of each rod is given in the figure.

Determine the angle of orientation θ of *AC* such that the average normal stress in rod *AC* is twice the average normal stress in rod *AD*. What is the magnitude of stress in each rod?



Example 4:



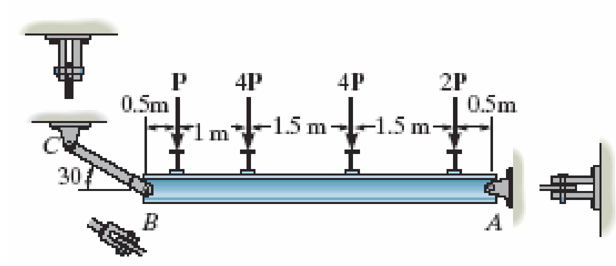
The row of staples *AB* contained in the stapler is glued together so that the maximum shear stress the glue can withstand is τ_{max} = 84 kPa.

Determine the minimum force **F** that must be placed on the plunger in order to shear off a staple from its row and allow it to exit undeformed through the Groove at *C*.

The outer dimensions of the staple are shown in the figure. It has a thickness of 1.25 mm Assume all the other parts are rigid and neglect friction.

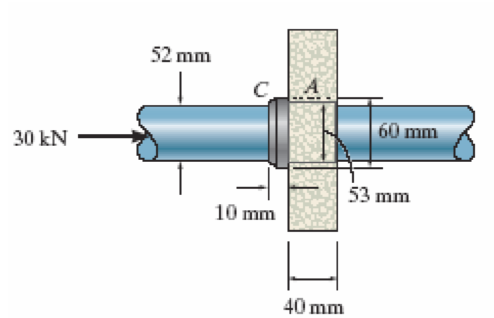
Example 5:

The beam is supported by a pin at *A* and a short link *BC*. Determine the maximum magnitude *P* of the loads the beam will support if the average shear stress in each pin is not to exceed 80 MPa. All pins have the same diameter of 18 mm.



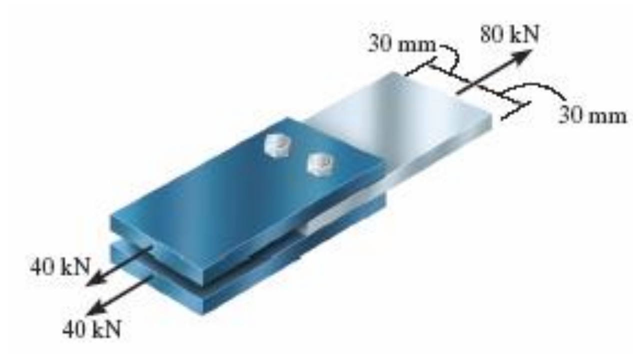
Example 6:

The shaft is subjected to the axial force of 30 kN. If the shaft passes through the 53-mm diameter hole in the fixed support *A*, determine the bearing stress acting on the collar *C*. Also, what is the average shear stress acting along the inside surface of the collar where it is fixed connected to the 52-mm diameter shaft?



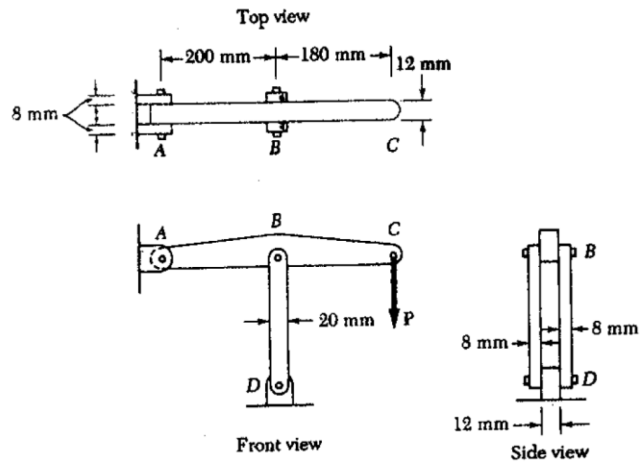
Example 7:

The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is τ_{fail} = 350 MPa. Use a factor of safety for shear of F.S. = 2.5.



Example 8:

The pins at A, B and D are made of steel, for which the shear failure stress is given as 100 MPa.
Use a factor of safety of 2.5 to design the pin diameters.



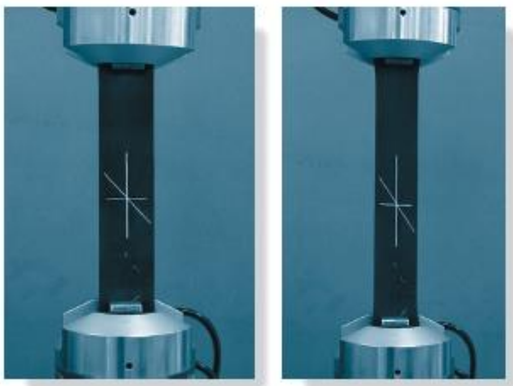
CHAPTER 2. STRAIN

OUTLINE

2.1. Normal Strain and Shear Strain

Deformation

- When a force is applied to a body, it will change the body's shape and size.
- These changes are called **deformation**.

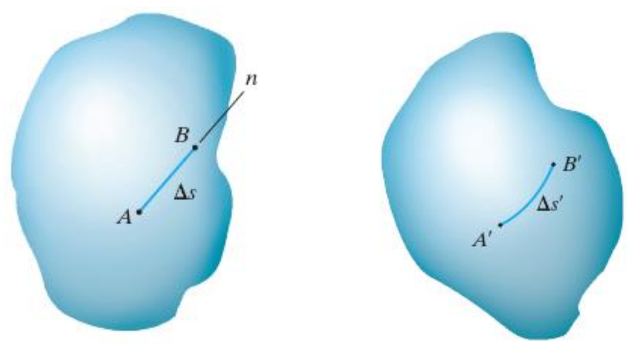


Note the before and after positions of 3 line segments where the material is subjected to tension.

Normal strain

- The elongation / contraction of a line segment per unit of length is referred to as normal strain.
- **Average normal strain** is defined as

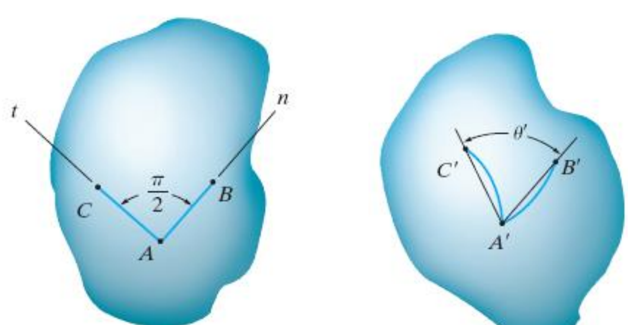
- If the normal strain is known, then the approximate final length is



Undeformed body Deformed body

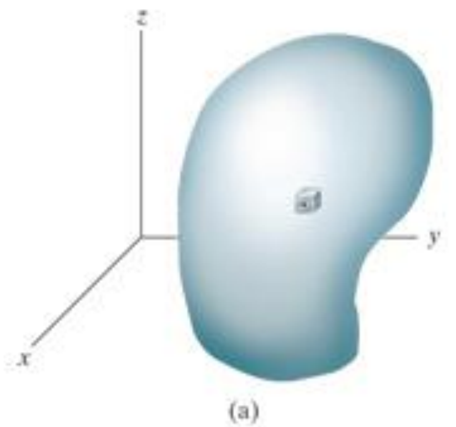
Shear strain

- Change in angle between two line segments that were perpendicular to one another refers to shear strain.

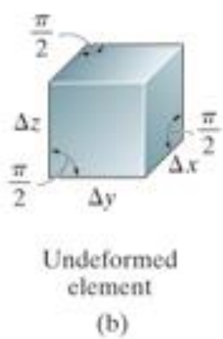


Undeformed body Deformed body

2.2. Cartesian Strain Components



Assume that a deformed body is subdivided into small rectangular elements, and we concentrate on such an element



Normal strain components ➔ **volume change**
(lengths of the sides of the rectangular element changes)

Δx ➔
Δy ➔
Δz ➔

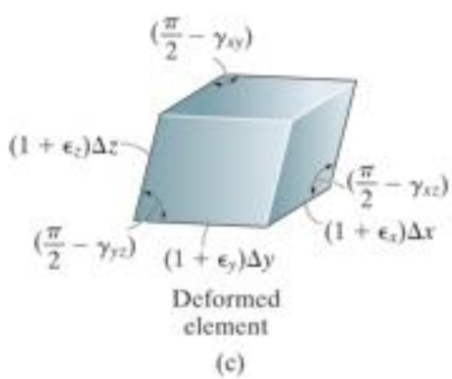


Fig. 2–3

Shear strain components ➔ **shape change**
(angles of the sides of the rectangular element changes)

$\frac{\pi}{2}$ ➔

$\frac{\pi}{2}$ ➔

$\frac{\pi}{2}$ ➔

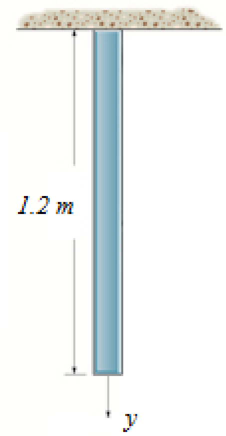
Small strain analysis:

Most engineering materials undergo small deformations, so small strains develop. **ε << 1**
This allows us to use the following approximate values: $\sin \varepsilon \approx \varepsilon$, $\cos \varepsilon \approx 1$, $\tan \varepsilon \approx \varepsilon$.

Example 1:

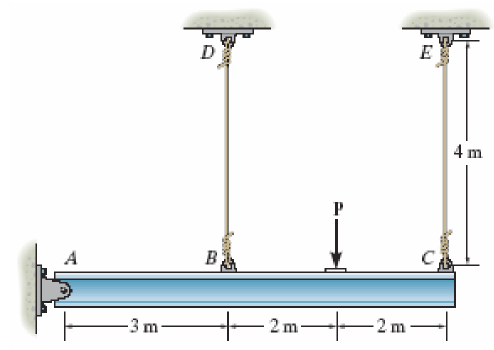
The beam is hanging under its own weight such that $\varepsilon_y = 10^{-3}\sqrt{y}$

a) Determine displacement of end A
b) Compute the average normal strain



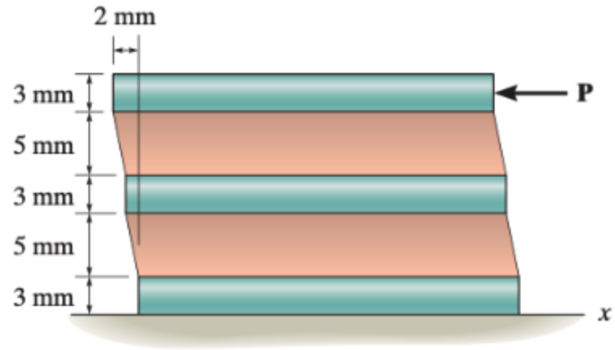
Example 2

The rigid beam is supported by a pin at *A* and wires *BD* and *CE*.
The load **P** on the beam causes the end *C* to be displaced 10 mm downward.
Determine the normal strain developed in wire *BD*.

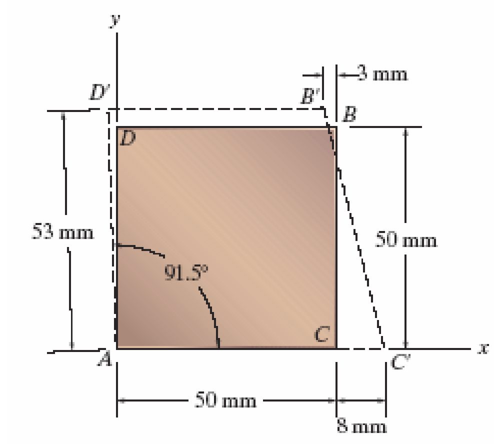


Example 3
Nylon strips are fused to glass plates. When moderately heated, the nylon will become soft while the glass stays approximately rigid.

Determine the average shear strain in each layer of nylon due to the load P when the assembly deforms as shown.



Example 4
The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, *A*, *B*, *C*, and *D*. (Side *D'B'* remains horizontal.)

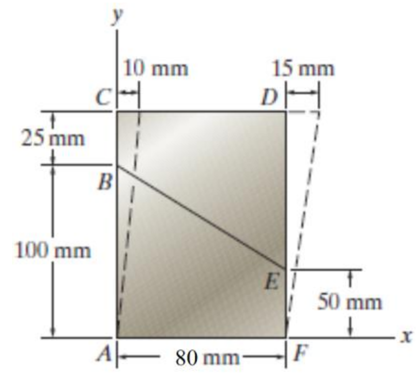


Example 5

The plate distorts into the dashed position shown. Determine

(a) the average normal strains and the shear strain at A,

(b) the average normal strain along line BE.



CHAPTER 3. MECHANICAL PROPERTIES OF MATERIALS

OUTLINE

CHAPTER 3: MECHANICAL PROPOERTIES OF MATERIALS

3.1. THE TENSION / COMPRESSION TEST

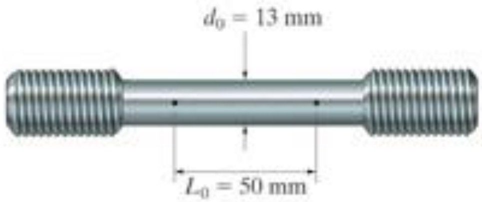


Fig. 3–1

Used primarily to determine the relationship between the average normal stress and the average normal strain

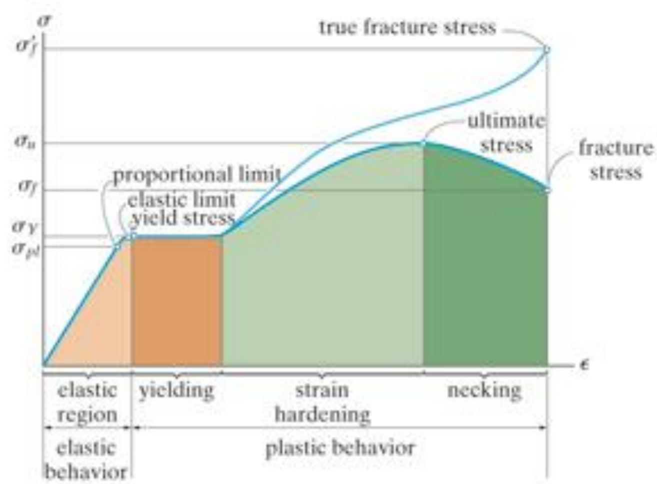
- Metals
- Ceramics
- Polymers
- Composite materials

When a metal specimen is tested, generally its initial diameter is d_0=13 mm. Two punch marks with L_0 = 50 mm distance is used. The specimen is stretched at a very slow, constant rate until it reaches the breaking point. During the test, the applied load P and the elongations $\delta = L - L_0$ are recorded at frequent intervals.

3.2. THE STRESS-STRAIN DIAGRAM

Engineering stress $\sigma = \dfrac{P}{A_0}$ True stress $\sigma = \dfrac{P}{A}$

Engineering strain $\varepsilon = \dfrac{\delta}{L_0}$ True strain $\varepsilon = \ln\left(\dfrac{L}{L_0}\right)$



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Fig. 3–4

Proportional limit …: lineer gerilme gerinim davranışı (doğrusallık sınırı)
Elastic limit ………….: yük kaldırıldığında orjinal şekline geri döner
Yielding …………………: kalıcı deformasyon
Strain hardening ……: gerinim sertleşmesi
Necking …………………: boyun yapma
Failure …………………..: kopma

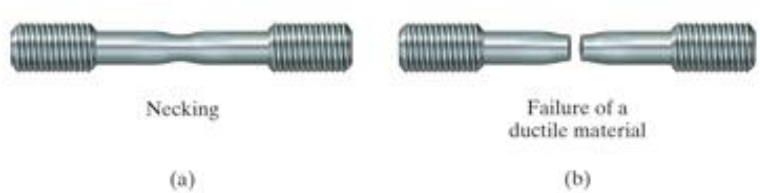


Fig. 3–5

CHAPTER 3 NOTES

3.3. STRESS-STRAIN BEHAVIOR OF DUCTILE AND BRITTLE MATERIALS

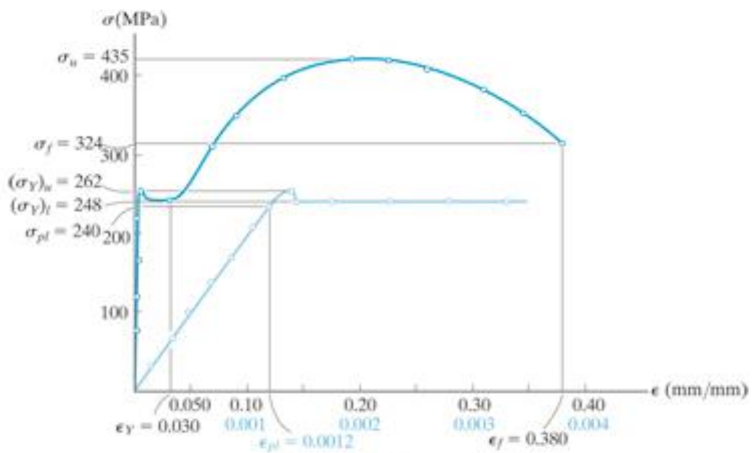
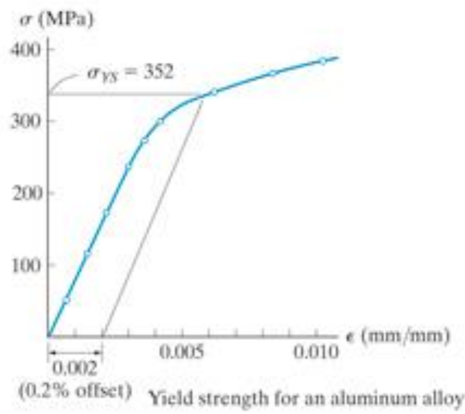
Ductile materials (Sünek malzemeler)

- Large strain before rupture
- Capability of absorbing shock and energy
- Steel, aluminum, brass, zinc

<u>Measures of ductility</u>

- Percent elongation = $\dfrac{L_f - L_0}{L_0}$ (100%) (Percent elongation in Fig. 3.6 is 38%)
- Percent reduction of area = $\dfrac{A_f - A_0}{A_0}$ (100%)

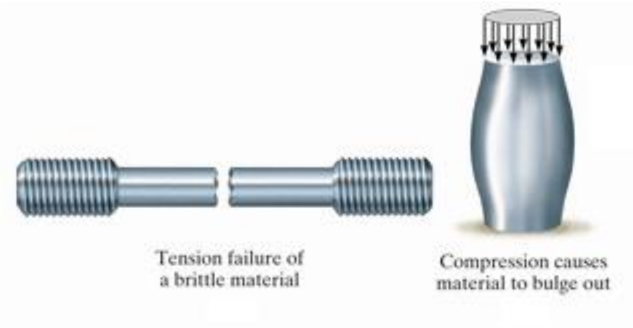
<u>Engineering stress-strain diagram for low carbon steel and aluminum</u>

 Stress-strain diagram for mild steel **Fig. 3–6**	 Yield strength for an aluminum alloy **Fig. 3–7**
Elastic behavior → yielding at constant stress → strain hardening → necking → failure	There is no well-defined yield point! Offset method is used to find the yield strength

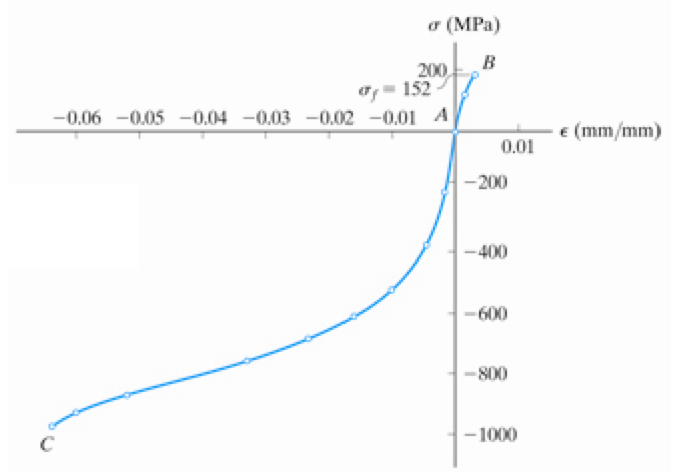
* For aluminum proportional limit, elastic limit, and yield point are all the same (unless otherwise specified).

Brittle materials (Gevrek malzemeler)

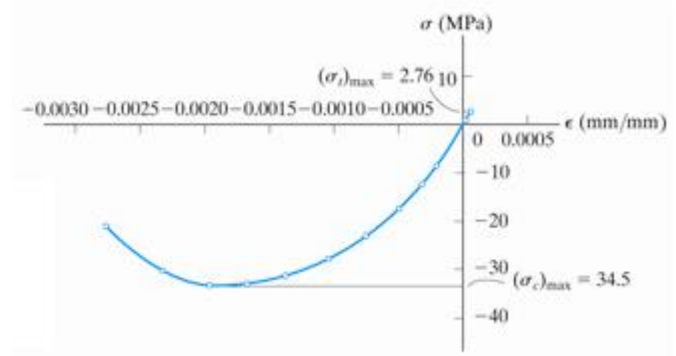
- Little or no yielding before failure
- Much larger resistance to compression than tension
- Gray cast iron, concrete



<u>Engineering stress-strain diagram for gray cast iron and concrete</u>



σ–ϵ diagram for gray cast iron

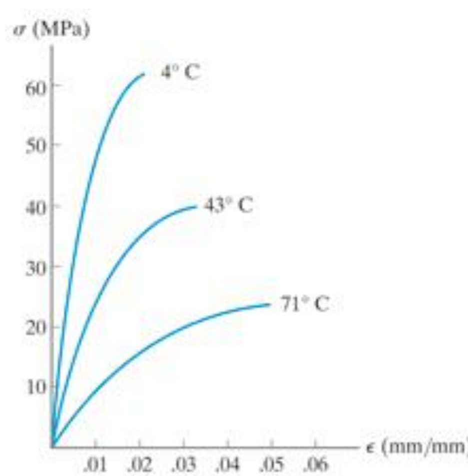


σ–ϵ diagram for typical concrete mix

Both ductile and brittle behavior

Most materials exhibit both ductile and brittle behavior

- Steel has brittle behavior when it contains high **carbon content**, and it is ductile when the carbon content is reduced.
- Materials become more brittle at low **temperature**s, whereas they become more ductile at high temperatures



σ–ϵ diagrams for a methacrylate plastic

Fig. 3–12

3.4. HOOKE'S LAW

Linear relationship between stress and strain $\sigma = E\varepsilon$ (similar to F= k x)

- E − modulus of elasticity (a stiffness property)
  For steel, E=200 GPa
  For aluminum, E=70 GPa
  For rubber, E=0.7 MPa
- To use $\sigma = E\varepsilon$, the linear-elastic behavior must be maintained. If the stress in the material exceeds the proportional limit, $\sigma = E\varepsilon$ is no longer valid!

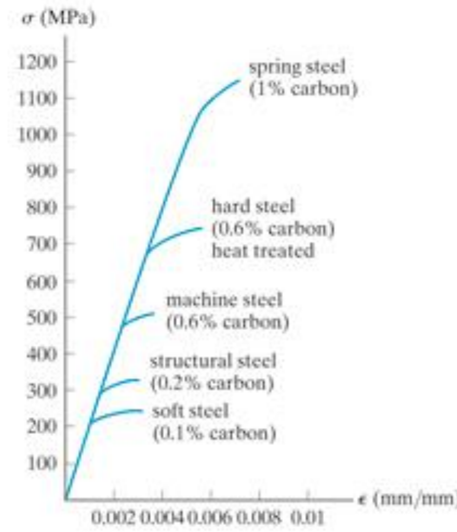


Fig. 3–13

Strain energy

If a ductile material is loaded into the plastic region and then unloaded, elastic strain is recovered whereas the plastic strain remains (Fig.3-14a). As a result, the material is subjected to a permanent set.

If the material is loaded again, the yielding occurs at a higher yield point A' (Fig.3-14a). As a consequence, the material hardens.

If the loading/unloaded is applied in a cyclic manner, some heat or energy will be lost and mechanic hysteresis occurs (the colored area in Fig. 3-14b).

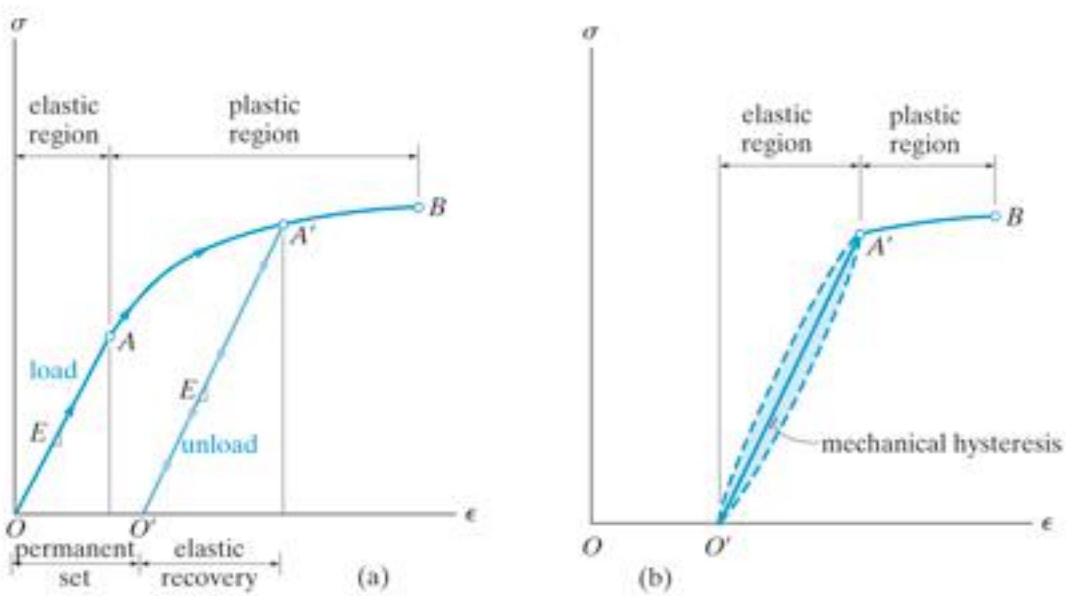


Fig. 3–14

3.5. STRAIN ENERGY

As a material is deformed by external loading, it store energy internally throughout its volume.

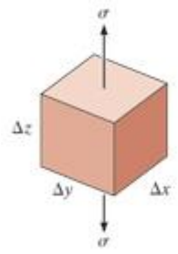


Fig. 3–15

Energy = Work = Force × Displacement

ΔU = ΔF/2 (ε Δz) = ½ σ Δx Δy (ε Δz) = ½ σ ε (Δx Δy Δz)

$$\Delta U = \frac{1}{2}\sigma\,\varepsilon\,\Delta V$$

Strain Energy Density
$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\,\varepsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

Modulus of Resilience

The area under σ-ε diagram within the proportional limit

Modulus of Toughness

The entire area under σ-ε diagram

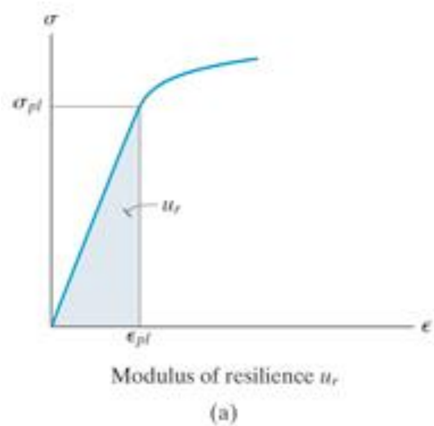


Fig. 3–16

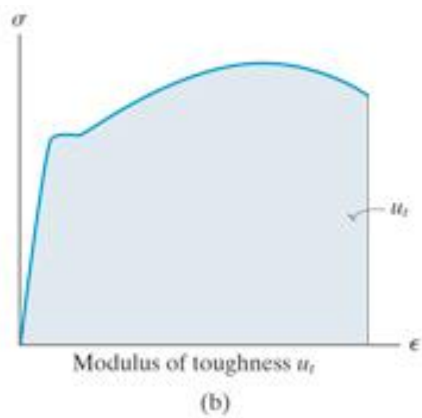
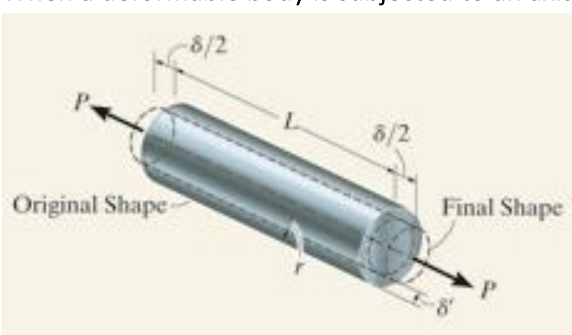


Fig. 3–16 (cont.)

3.6. POISSON'S RATIO

When a deformable body is subjected to an axial tensile force, it elongates but contracts laterally.



Longitudinal and lateral strains

$$\varepsilon_{long} = \frac{\delta}{L} \qquad \varepsilon_{lateral} = \frac{\delta'}{R}$$

Poisson's ratio

$$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{long}}$$

In principle, $0 \le \nu \le 0.5$

For most engineering materials, it ranges between 1/4 and 1/3

3.7. THE SHEAR STRESS-STRAIN DIAGRAM

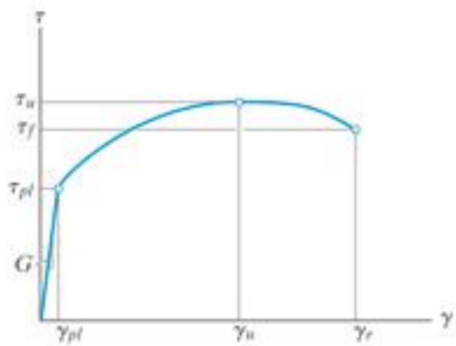


Fig. 3–24

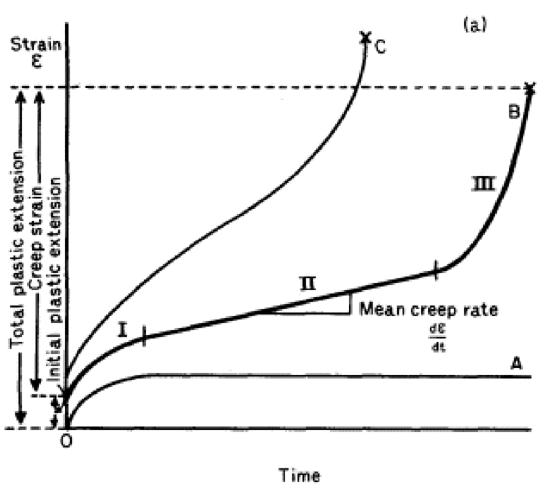
$$\tau = G\gamma$$

$$G = \frac{\tau_{pl}}{\gamma_{pl}} \qquad G = \frac{E}{2(1+\nu)}$$

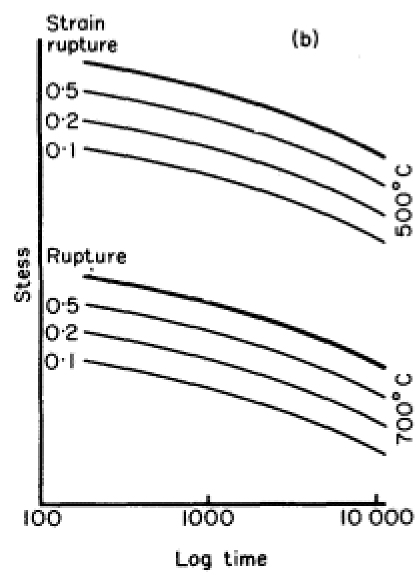
3.8. FAILURE DUE TO CREEP (SÜRÜNME) AND FATIGUE (YORULMA)

Creep

- When a material has to support a load for a very long period of time, the material may continue to deform until a sudden fracture occurs. This time-dependent permanent deformation is known as creep.
- **Temperature** is also an important factor
  - Metals and ceramics creep at high temperature
  - Composite materials creep even at room temperature



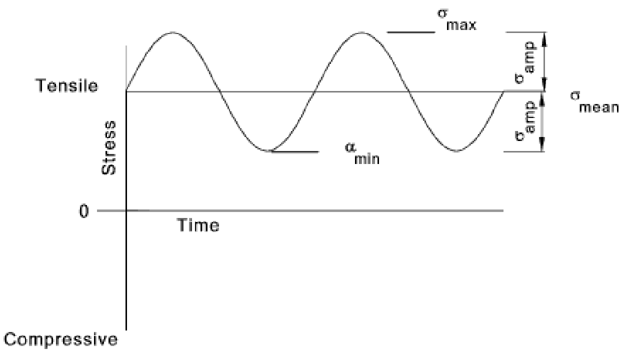
Effect of stress



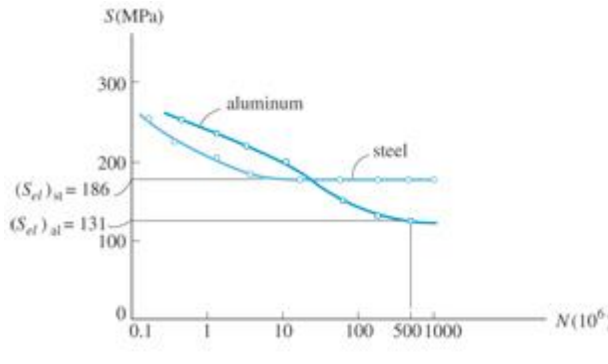
Effect of temperature

Fatigue

- Repeated loading causes materials to fail below yield stress
- Ductile materials exhibit brittle behavior at failure
- Connecting rods, crankshafts, gas turbines



Repeated stress



S–N diagram for steel and aluminum alloys
(N axis has a logarithmic scale)

Fig. 3–28

S-N curve

CHAPTER 4. AXIAL LOAD

OUTLINE

4.1. Saint-Venant's Principle

4.2. Elastic Deformation of an Axially Loaded Member

4.4. Statically Indeterminate Axially Loaded Member

4.3. Principle of Superposition

4.5. The Force Method of Analysis

4.6. Thermal Stress

(Section 4.4 intentionally precedes 4.3)

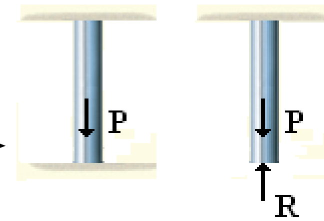
CHAPTER 4: AXIAL LOADING

Chapter 1 → * normal stress at axially loaded member: $\sigma = N/A$

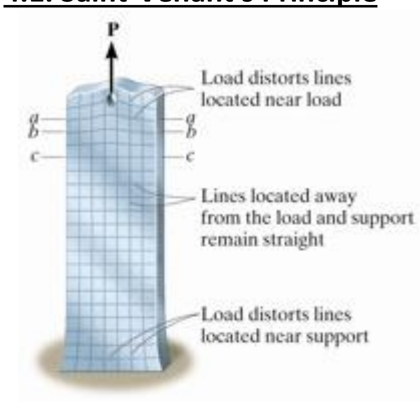
Chapter 4 → * deformation in this member

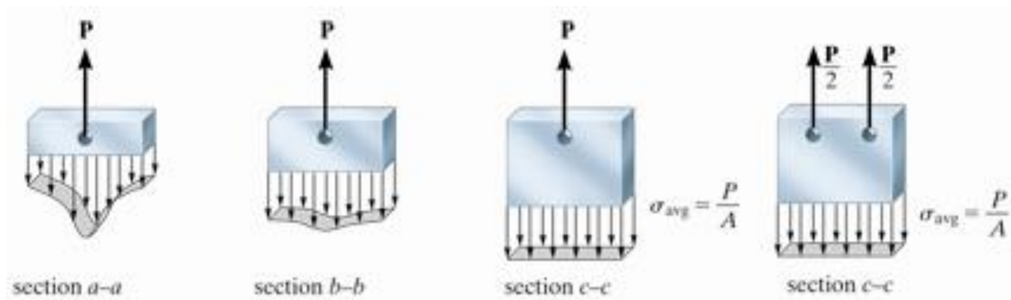
* support reactions that cannot be found via equil. eqs.

* thermal stresses



4.1. Saint-Venant's Principle



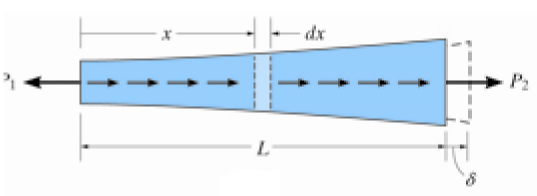


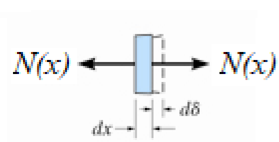
At a considerable distance **away from the localized effects**, stress distribution is the same for all statically equivalent loadings ($\vec{F}_R$, $\vec{M}_R$ should be same).

How far away?

- As a general rule, the largest dimension of the loaded section
- For thin-walled members and loadings with large deformations, the largest dimension is not far enough!

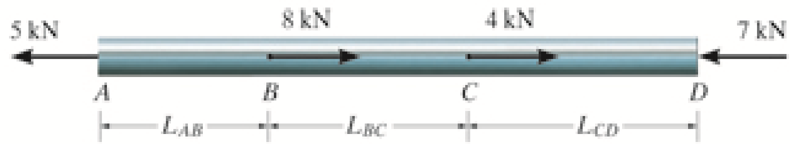
4.2. Elastic deformation of an axially loaded member





Example 1

Compute the axial deformation of the steel rod (E = 200 GPa) shown below.

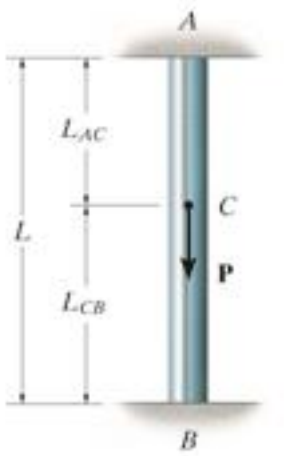


4.4. Statically Indeterminate Axially Loaded Member

(Section 4.4 intentionally precedes 4.3)

A member is **statically indeterminate** when equations of equilibrium are not sufficient to determine the reactions on a member.

Additional equations are needed. ➔ We consider the deformed geometry and write additional equations that ensures a compatible deformation in a deformed body.
These equations are called **compatibility conditions**.

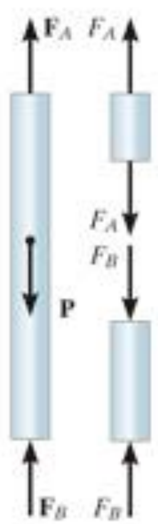


(a)

Equilibrium equation:

Notice that this equation is not sufficient to determine the two reactions on the bar.

Let′s write the compatibility condition that the length of the bar should remain unchanged since we have fixed support at both ends.



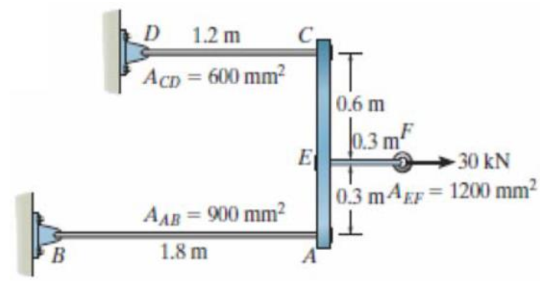
(b) (c)

Now, we can combine equilibrium equation with compatibility condition

Example 1.

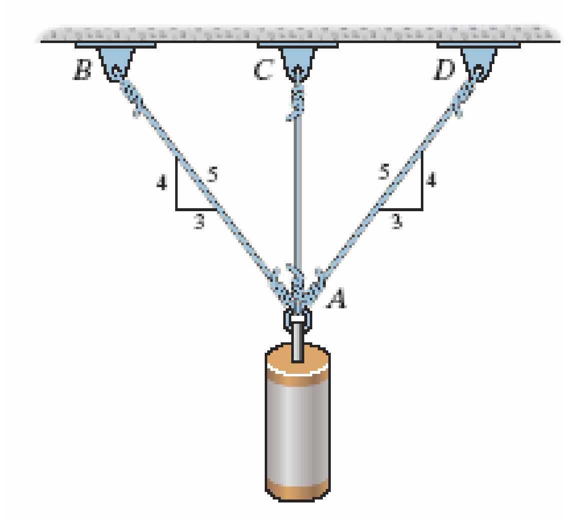
The assembly consists of three titanium rods (E=120GPa) rods and a rigid bar *AC*.

Determine the horizontal displacement of point *F*.



Example 2.

The three A-36 steel wires each have a diameter of 2 mm and unloaded lengths of *LAC* = 1.60 m and *LAB* = *LAD* = 2.00 m. Determine the force in each wire after the 150-kg mass is suspended from the ring at *A*.



4.3. Principle of Superposition

- Principle of superposition is used to simplify stress and displacement problems by subdividing the loading into components and adding the results.
- The following two conditions need to be satisfied to be able to use it

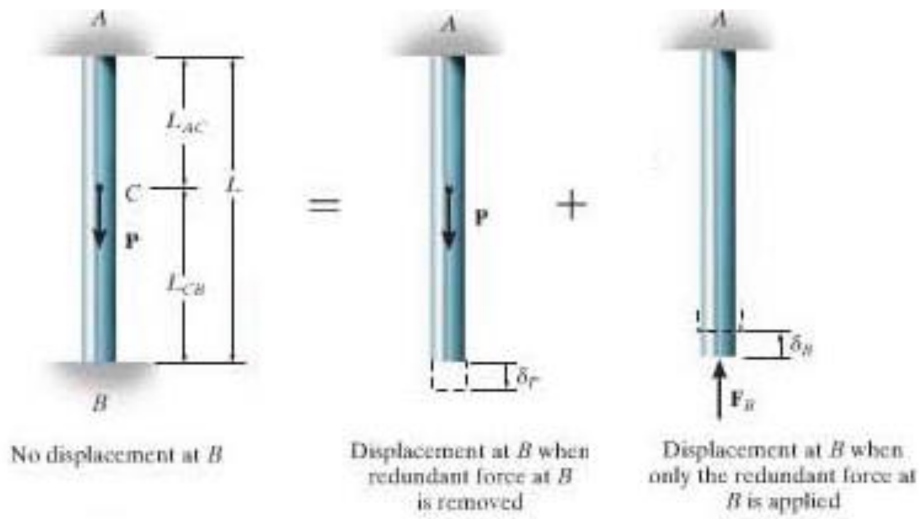
1. ***The loading must be linearly related to the stress or displacement that is to be determined.*** For example, the equations $\sigma = P/A$ and $\delta = PL/AE$ involve a linear relationship between P and σ or δ.

2. ***The loading must not significantly change the original geometry or configuration of the member.*** If significant changes do occur, the direction and location of the applied forces and their moment arms will change. For example, consider the slender rod shown in Fig. 4–10*a*, which is subjected to the load **P**. In Fig. 4–10*b*, **P** is replaced by two of its components, $\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$. If **P** causes the rod to deflect a large amount, as shown, the moment of the load about its support, Pd, will not equal the sum of the moments of its component loads, $Pd \neq P_1d_1 + P_2d_2$, because $d_1 \neq d_2 \neq d$.

4.5. The Force Method of Analysis

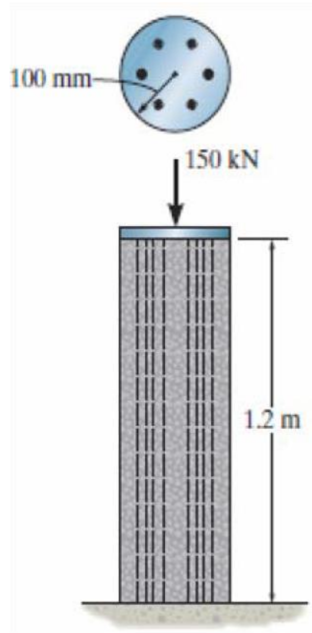
- This method is used to solve statistically indeterminate problems.
- Compatibility equation is written by using principle of superposition.

Consider our running example



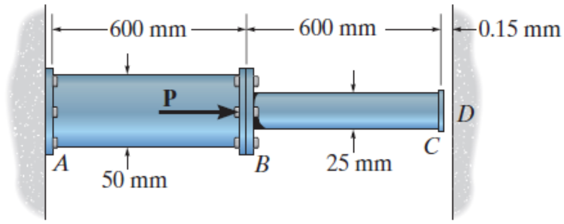
Example 1.

The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 150 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.



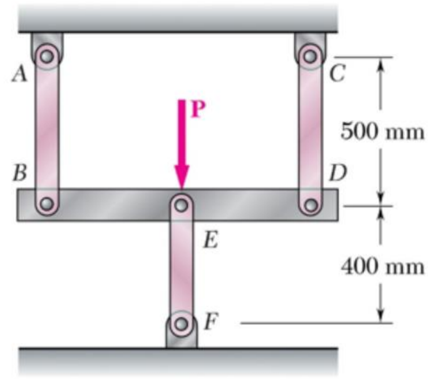
Example 2.

If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force is applied. The assembly is made of A36 steel (E=200GPa).



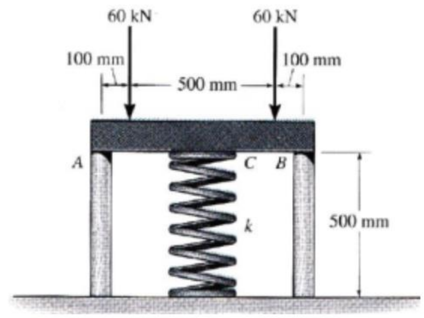
Example 3.

Three steel rods (E 200 GPa) support a 36-kN load P.
Each of the rods AB and CD has a 200-mm^2 cross-sectional area
and rod EF has a 625 mm^2 cross-sectional area.
Determine (a)the change in length of rod EF, (b) the stress in each rod.



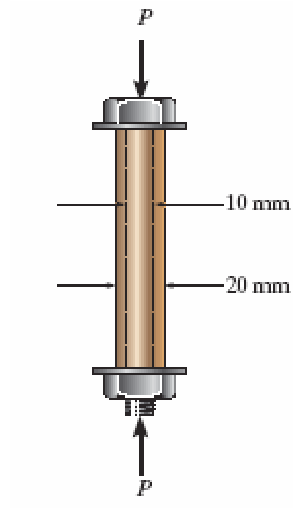
Example 4.

The rigid bar is supported by the two short wooden posts (E_w = 11 GPa) and a spring (k=1.8 MN/m, original length of 520mm).
Each of the posts has length of 500mm and sectional area of 800mm^2.
Determine the vertical displacement of A and B.



Example 5.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st}$ = 640 MPa, and for the bronze $(\sigma_Y)_{br}$ = 520 MPa, determine the magnitude of the largest elastic load *P* that can be applied to the assembly. E_{st} = 200 GPa, E_{br} = 100 GPa.



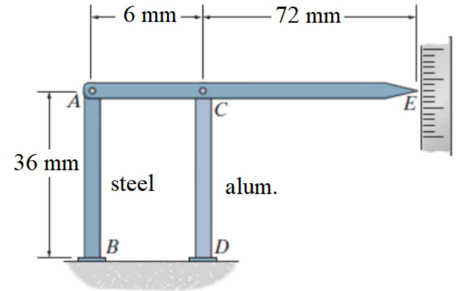
4.6. Thermal Stress

- A change in temperature cause a material to change its dimensions
- The experiments have shown that the expansion (or contraction) is **linearly related** to the temperature change

- If the temperature varies throughout the length of the member

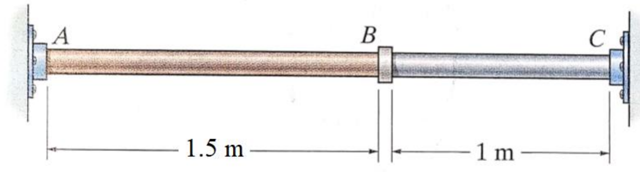
Example 1.

The device is used to measure a change in temperature. Bar AB is made of steel ($\alpha=12\times10^{-6}$ 1/°C), and bar CD is made of aluminum ($\alpha=23\times10^{-6}$ 1/°C). When the temperature is at 25 °C, the rigid bar ACE is in horizontal position.
Determine the vertical displacement of end E when the temperature rises to 75 °C.



Example 2.

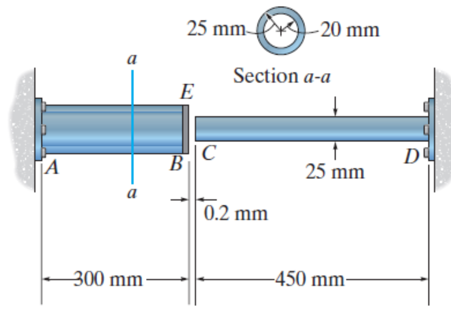
The C83400-red-brass rod AB and 2014-T6-aluminum rod BC are joined at the collar B and fixed connected at their ends. If there is no load in the members when $T_1 = 10°C$, determine the average normal stress in each member when $T_2 = 50°C$. Also, how far will the collar be displaced? The cross-sectional area of each member is 1000 mm^2.



For brass, take $\alpha=18\times10^{-6}$ 1/°C, E=100 GPa
For aluminum, take $\alpha=23\times10^{-6}$ 1/°C, E=70 GPa

Example 3.

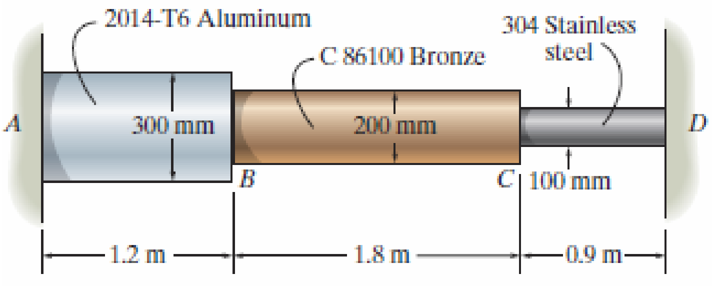
The AM1004-T61 magnesium alloy tube *AB* is capped with a rigid plate *E*.The gap between *E* and end *C* of the 6061-T6 aluminum alloy solid circular rod *CD* is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



For Mg, take $\alpha=26\times10^{-6}$ 1/°C, E=44.7 GPa
For Al, take $\alpha=24\times10^{-6}$ 1/°C, E=68.9 GPa

Example 5.

The assembly has the diameters and material make-up indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 20°C$, determine the average normal stress in each material when the temperature reaches $T_2 = 40°C$.

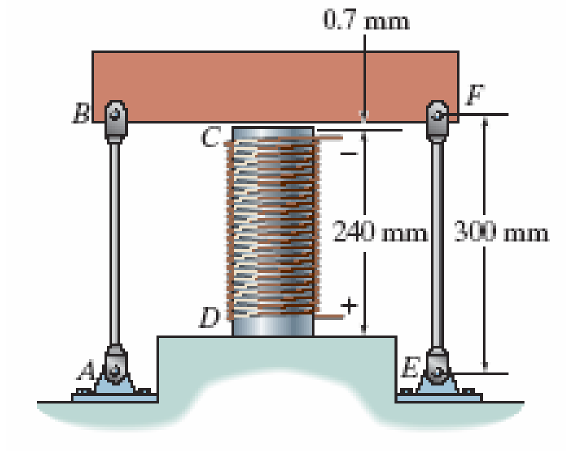


For Al, take $\alpha = 23\times10^{-6}$ 1/ºC, E=73 GPa
For Br, take $\alpha = 17\times10^{-6}$ 1/ºC, E=103 GPa
For SS, take $\alpha = 17\times10^{-6}$ 1/ºC, E=193 GPa

Example 6.

The center rod *CD* of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance heating. At the lower temperature T_1 the gap between *C* and the rigid bar is 0.7 mm. Determine the force in rods *AB* and *EF* caused by the increase in temperature. Rods *AB* and *EF* are made of steel, and each has a cross-sectional area of 125 mm^2. *CD* is made of aluminum and has a cross-sectional area of and 375 mm^2.

$E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°C$, $\alpha_{al} = 23(10^{-6})/°C$.



CHAPTER 5. TORSION

OUTLINE

5.1. Torsional Deformation of a Circular Shaft

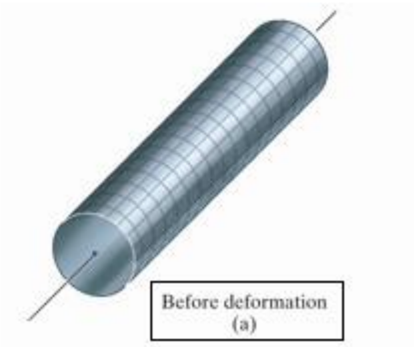
5.2. The Torsion Formula

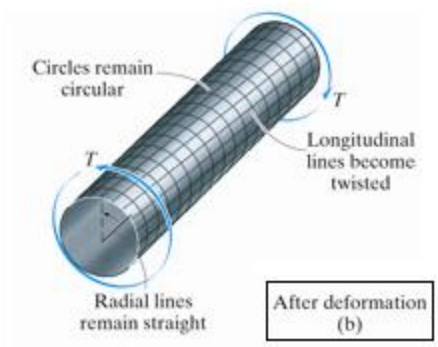
5.3. Power Transmission

5.4. Angle of Twist

5.5. Statically Indeterminate Torque-Loaded Members

5.1. Torsional Deformation of a Circular Shaft



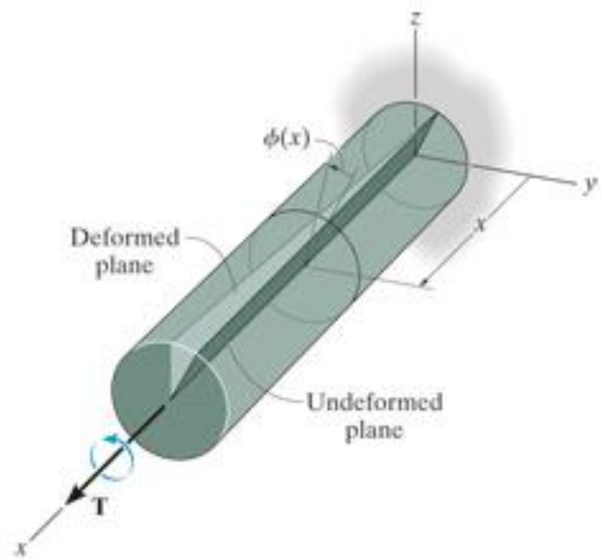


During deformation

- Circles remain circles
- Cross sections at the ends of the shaft remain flat (no warping, no bulging)
- Uniform deformation

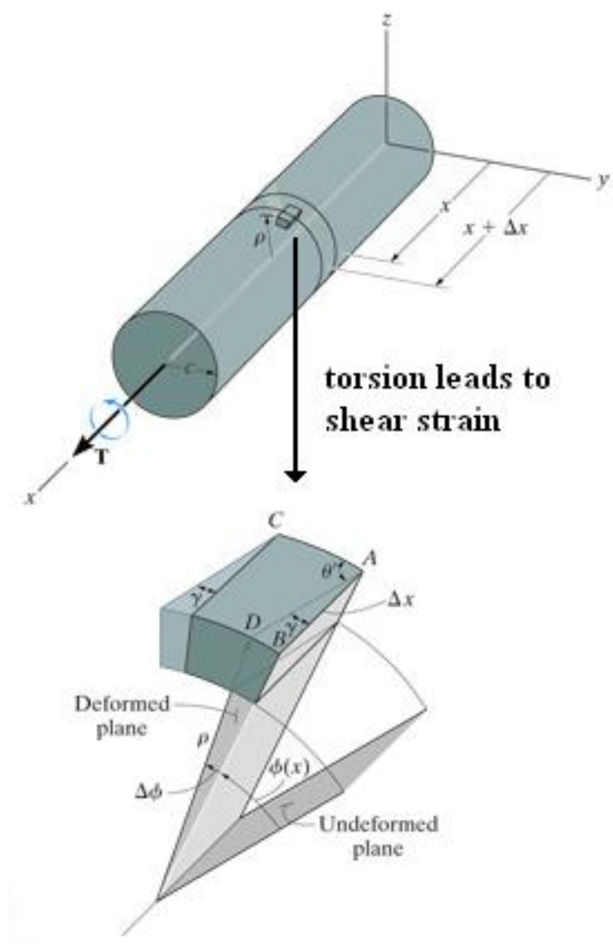
If angle of twist is small,

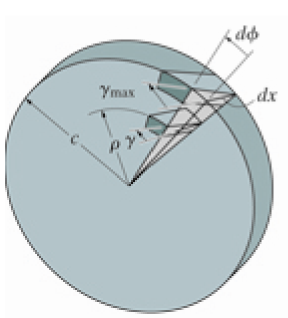
- length and diameter of the shaft do not change.



The angle of twist ϕ(x) increases as x increases.

Fig. 5–2

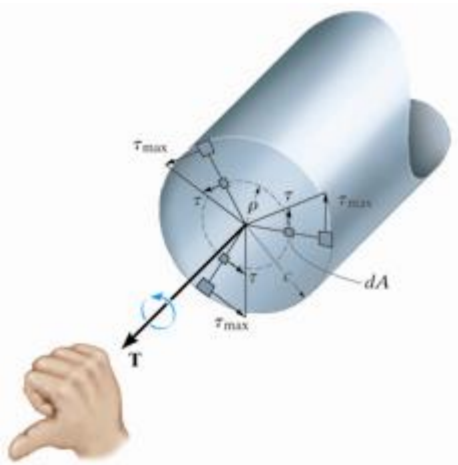




Shear strain linearly changes with

the radial distance (ρ) from the shaft $\gamma = \left(\dfrac{\rho}{c}\right)\gamma_{\max}$

5.2. The Torsion Formula



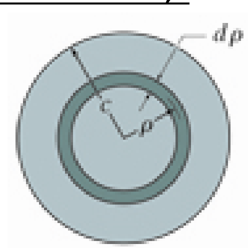
Recall from Section 5.1 that $\gamma = \left(\dfrac{\rho}{c}\right)\gamma_{\max}$

For linear-elastic materials $\tau = G\gamma$

So we have, $\tau = \left(\dfrac{\rho}{c}\right)\tau_{\max}$

Consider the torque produced on the shaft

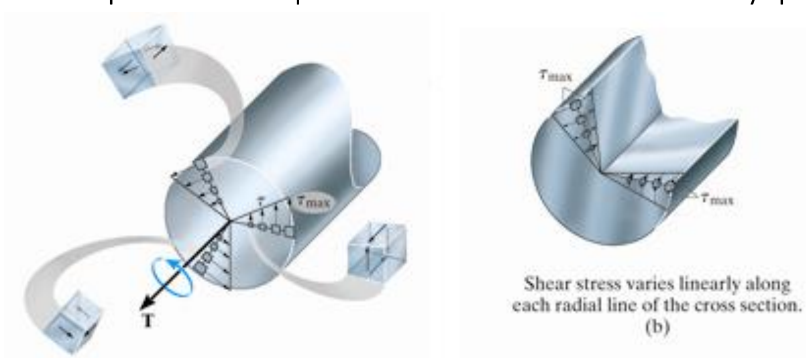
Polar moment of inertia, J



$$J = \int_A \rho^2 dA = \int_0^c \rho^2 \left(2\pi\rho d\rho\right)$$

…

Torque on the shaft produces a linear shear stress distribution in each radial line of the cross section. Similarly, an associated shear stress is developed on an axial plane. This associated shear stress may split the wooden shafts.



(a)

Shear stress varies linearly along each radial line of the cross section.
(b)

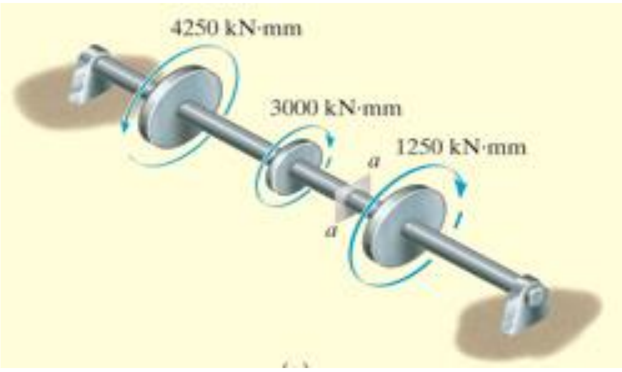
Fig. 5–7

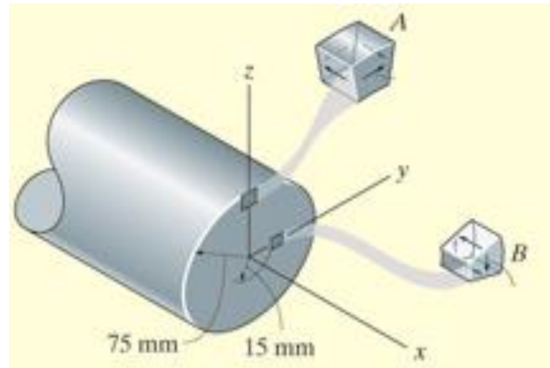
Failure of a wooden shaft due to torsion.

Fig. 5–8

Example 1

The shaft shown is supported by two bearings and is subjected to three torques. Determine the shear stress developed at A and B, located at section *a-a* of the shaft.

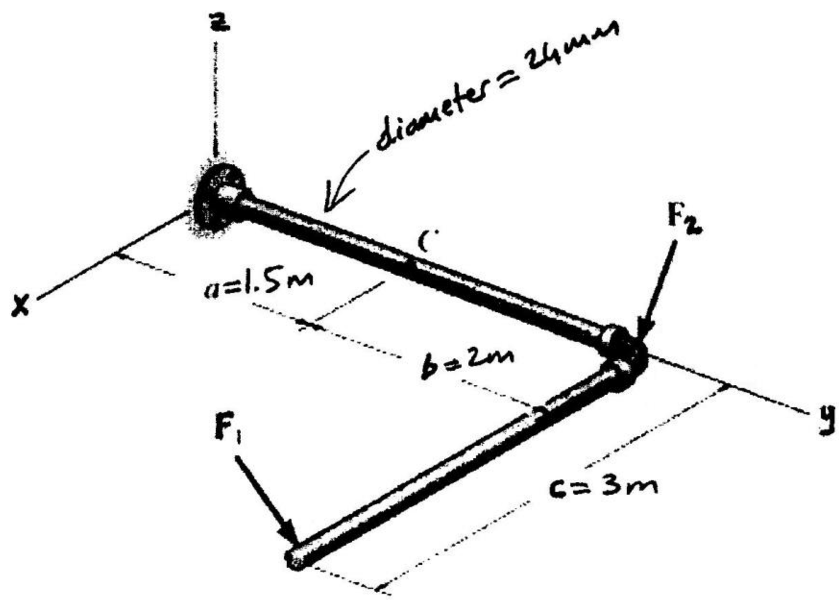




Example 2.

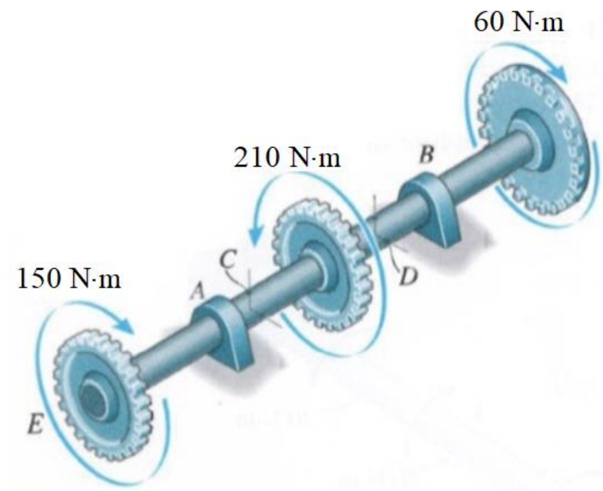
Determine the maximum torsional stress developed at C.

$F_1 = (-80\,i + 200\,j - 300\,k)$ N $\qquad$ $F_2 = (250\,i - 150\,j - 200\,k)$ N



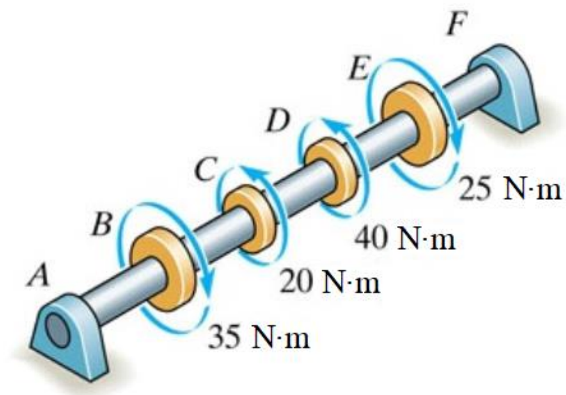
Example 3.

The solid 25-mm-diameter shaft is used to transmit the torques applied to the gears. If it is supported by smooth bearings at *A* and *B*, which do not resist torque, determine the shear stress developed in the shaft at points *C* and *D*. Indicate the shear stress on volume elements located at these points.



Example 4.

The solid shaft has a diameter of 15 mm. If it is subjected to the torques shown, determine the maximum shear stress developed in regions *CD* and *EF* of the shaft. The bearings at *A* and *F* allow free rotation of the shaft.

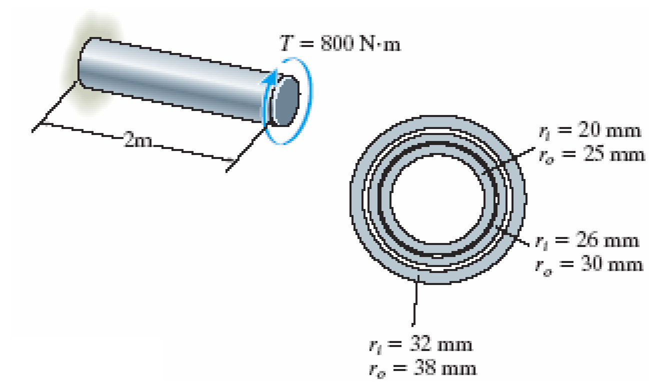


Example 5.

The steel shaft is subjected to the torsional loading shown. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line where it is maximum.

Example 6.

The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of T = 800 N·m is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.



5.3. Power Transmission (*Güç Aktarımı*)

- Shafts and tubes having circular cross sections are often used to transmit power.

$$Power = \frac{Work}{unit\,time} = \frac{d}{dt}(W)$$

Work = Torque × Angle of rotation ($W = T \times \theta$)

So we have, $P = T\frac{d\theta}{dt} = T\omega$ (Watt) ω: angular velocity (rad/s)

For machinery applications, the frequency of rotation of the shaft is often reported.
f = number of cycles per second (Hz)

Angular velocity and frequency are related through $\omega = 2\pi f$

Thus, the transmitted power can be related to the applied torque via $P = 2\pi f T$

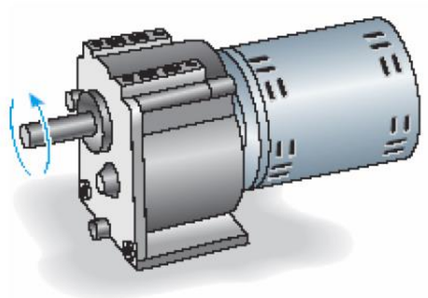
Shaft design

$T = \frac{P}{2\pi f}$ and $\tau_{\max} = \frac{Tc}{J} \rightarrow \tau_{allow}$ are used to design shafts under stress considerations

For tubular shafts, the polar moment of inertia is computed from $J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$, where c_o is the outer radius and c_i is the inner radius.

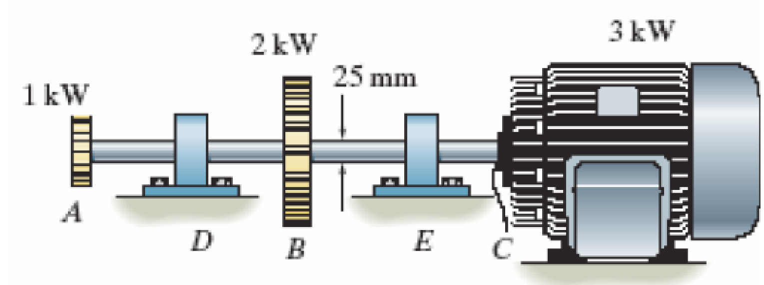
Example 1.

The gear motor can develop 100 W when it turns at 80 rev/min. If the allowable shear stress for the shaft is $\tau_{allow} = 28$ MPa, determine the smallest diameter of the shaft to the nearest multiples of 5mm that can be used.



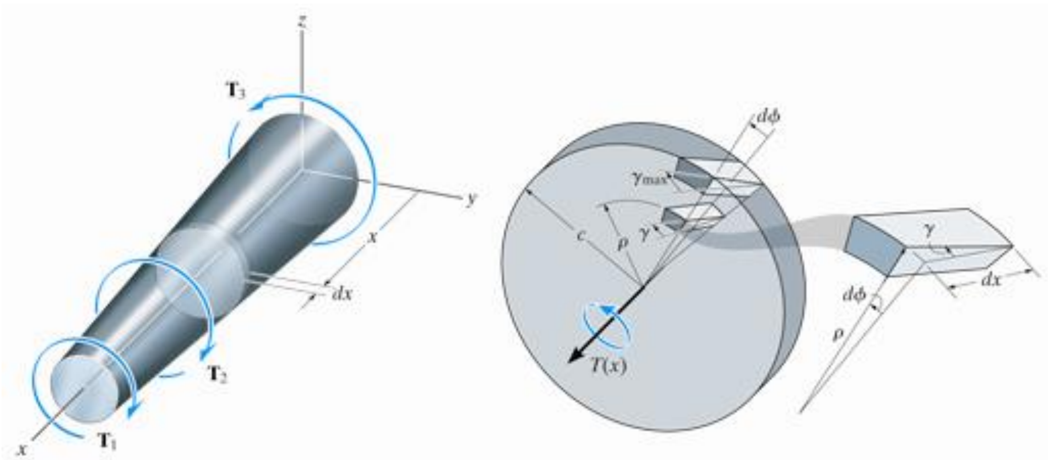
Example 2.

The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



5.4. Angle of Twist (Dönme Açısı), ϕ

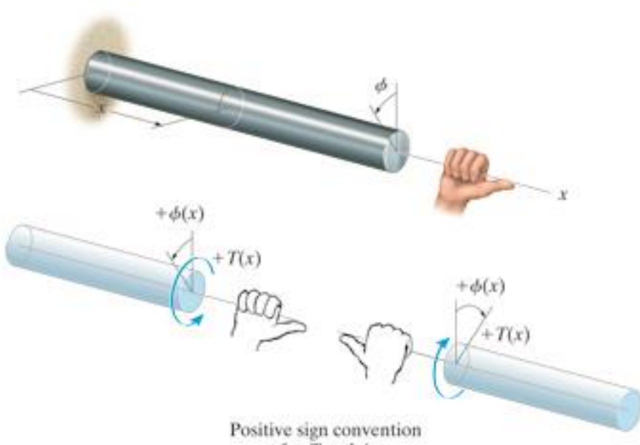
- In shaft design, the angle of twist of shafts are occasionally restricted.
- In addition, angle of twist is important when analyzing the support reactions of statically indeterminate shafts.



Constant torque and cross section

Varying torque or cross section

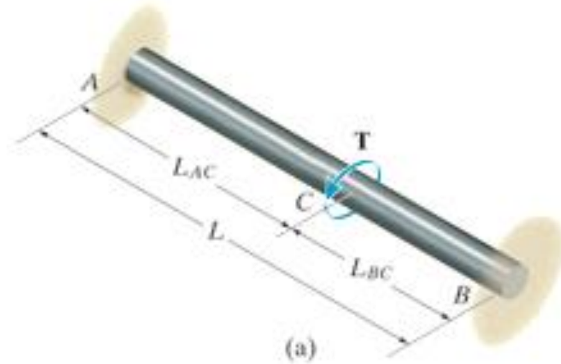
Sign convention



Positive sign convention for T and ϕ.

Fig. 5–18

5.5. Statically Indeterminate Torque-Loaded Members



- If the moment equilibrium equations is inadequate to determine the unknown reactive torques, then additional equation(s) will be required.

- Compatibility equations (in terms of angle of twist) are used to obtain additional equations.

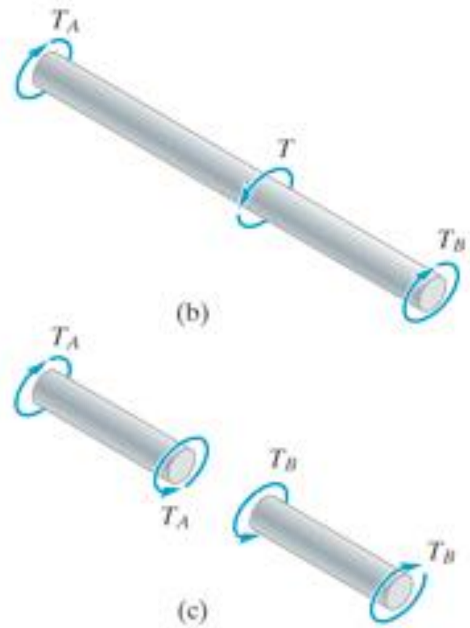
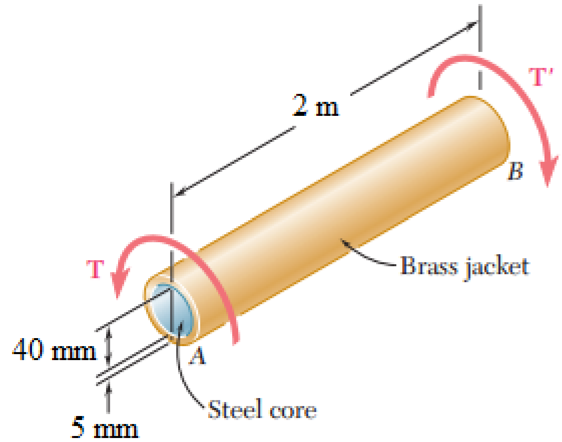


Fig. 5–24

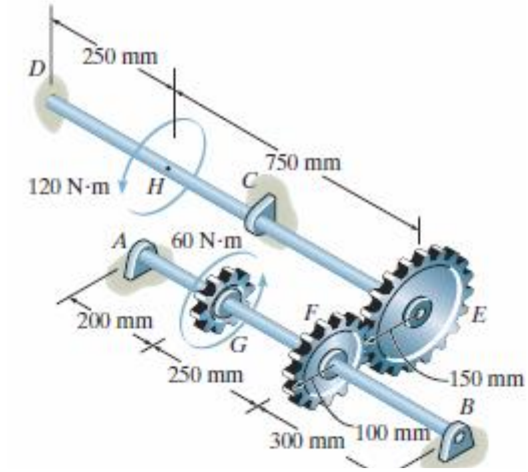
Example 1.

A composite shaft is subjected to T=T'=400 N.m.
Determine the safety factors for the brass jacket and steel core.
$(\tau_{fail})_{brass} = 20$ MPa, $(\tau_{fail})_{steel} = 45$ MPa.
$G_{brass} = 39$ MPa, $G_{steel} = 77.2$ MPa.



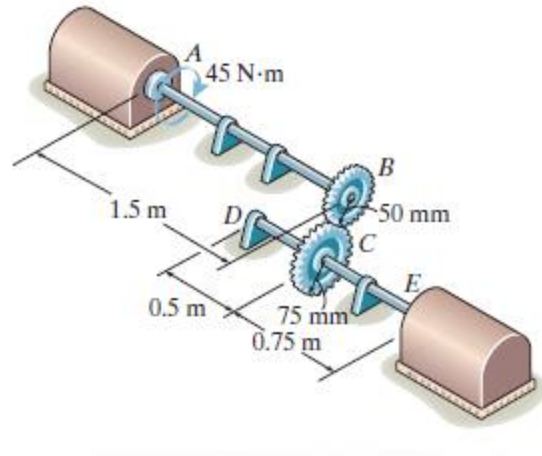
Example 2.

The two shafts are made of A-36 steel (G=75 GPa). Each has a diameter of 25 mm, and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end B when the torques are applied to the assembly as shown.



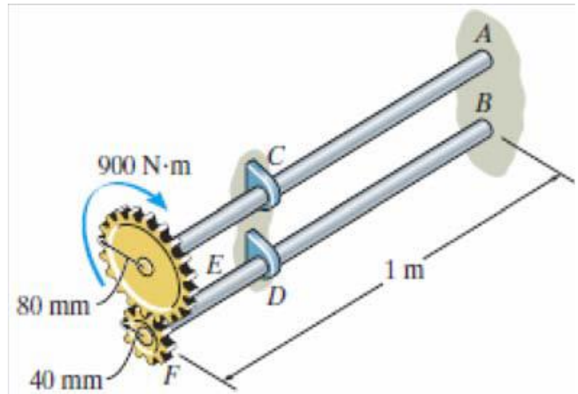
Example 3.

The 30-mm-diameter shafts are made of steel (G=75GPa). They are supported on journal bearings that allow the shaft to rotate freely. If the motor at A develops a torque of on the shaft AB, <u>while the turbine at E is fixed from turning</u>, determine the amount of rotation of gears B and C.



Example 4.

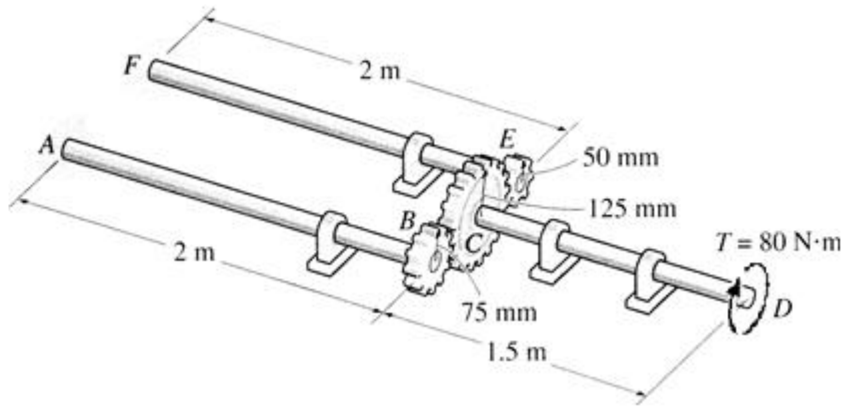
The two 1-m-long shafts are made of aluminum (G=27GPa). Each has a diameter of 30 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 900 N·m is applied to the top gear as shown, determine the maximum shear stress in each shaft.



Example 5.

The two shafts AB and EF are fixed at their ends, and fixed connected to gears that are in mesh with a common gear at C, which is fixed connected to shaft CD If a torque of T = 80 N·m is applied to end D, determine the angle of twist of end D.

Each shaft has a diameter of 20 mm, and made from A-36 steel. G = 75 GPa



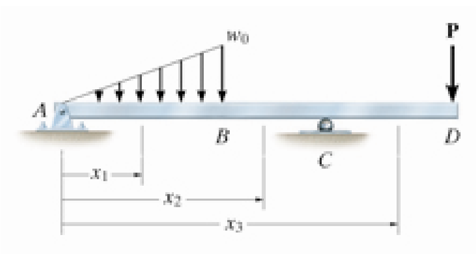
CHAPTER 6. BENDING

OUTLINE

CHAPTER 6. BENDING (*EĞİLME*)

6.1-6.2: Construction of shear and bending diagrams for beams

Beams: Members that are slender and support loads that are applied perpendicular to their longitudinal axis

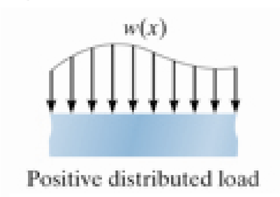


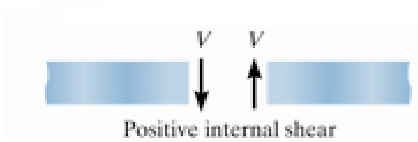
Beams might be subjected to point and/or distributed loads.

Beam design requires calculation of the shear and moment variation over the length of the beam.

- V(x) and M(x) diagrams

Sign convention





The graphical method to draw V(x) and M(x) diagrams

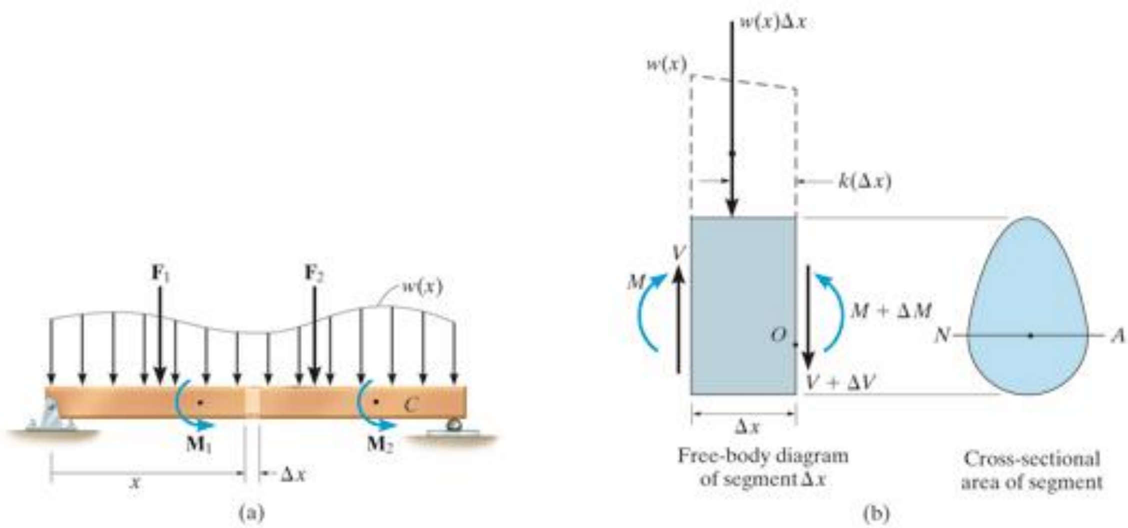


Fig. 6–10

Applying equilibrium equations (∑Fy=0 and ∑M$_o$=0) to Fig. 6-10b we get

$$\frac{dV}{dx} = -w(x), \quad \frac{dM}{dx} = V \qquad\qquad \Delta V = -\int w(x)\,dx, \quad \Delta M = \int V(x)\,dx$$

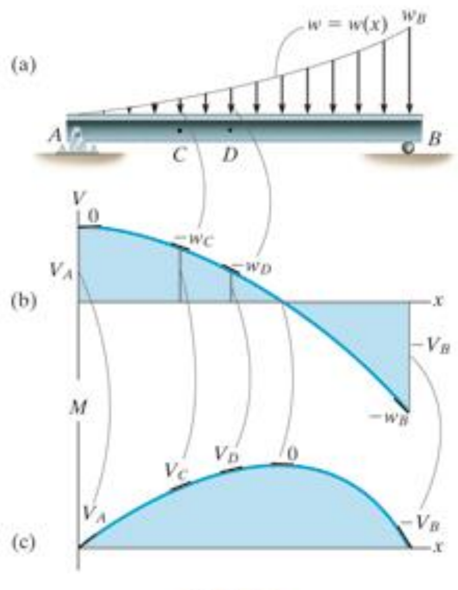


Fig. 6–11

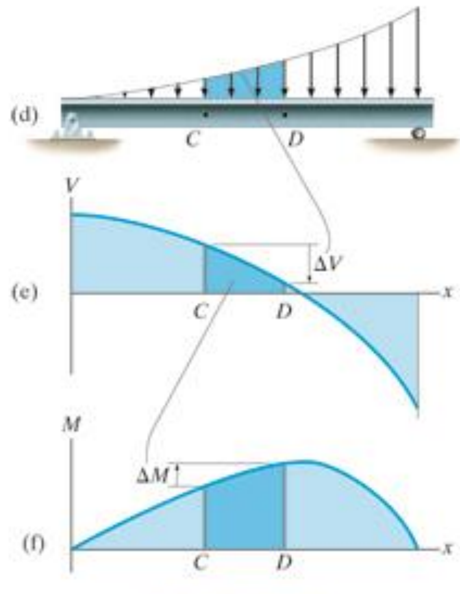
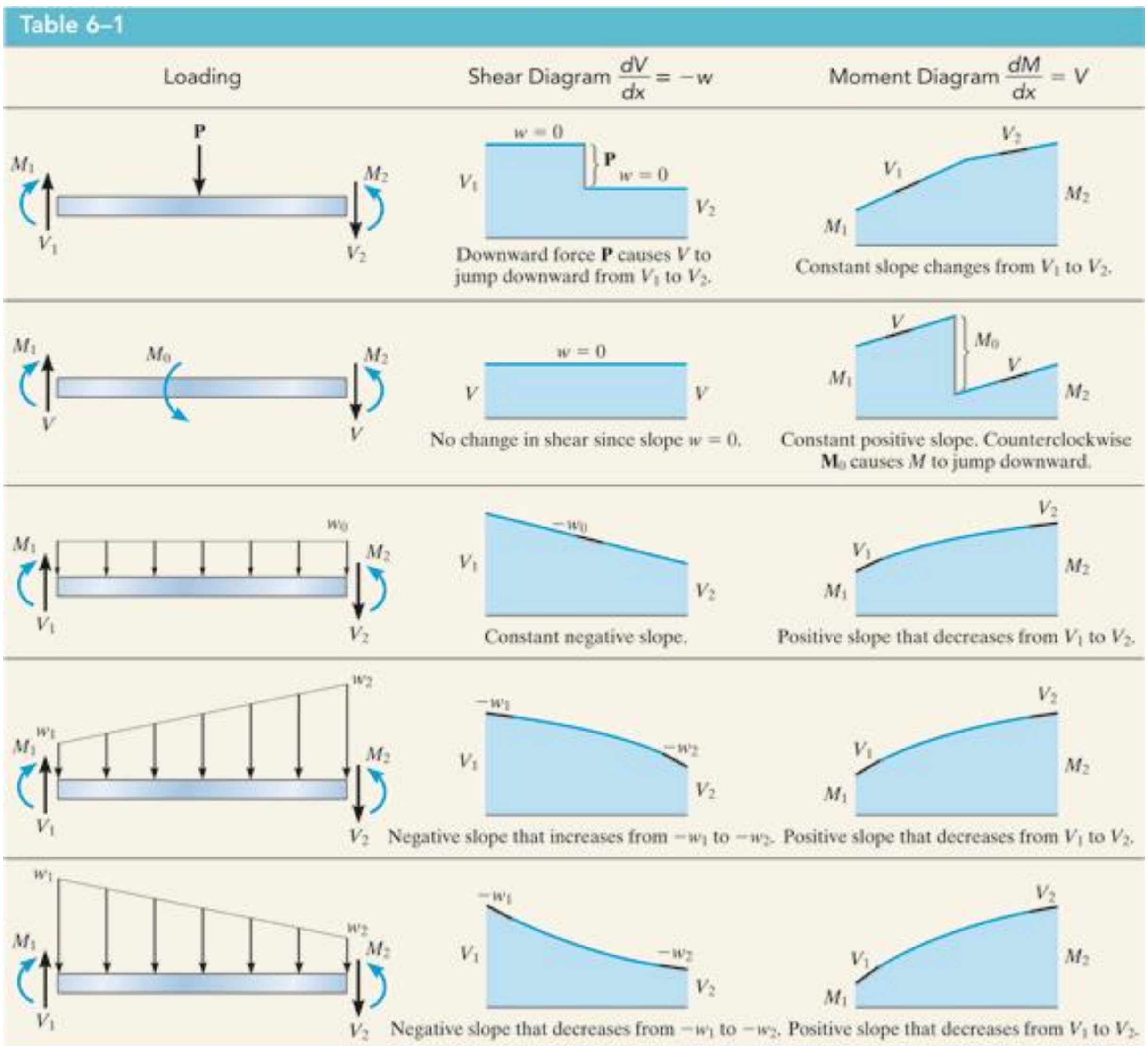


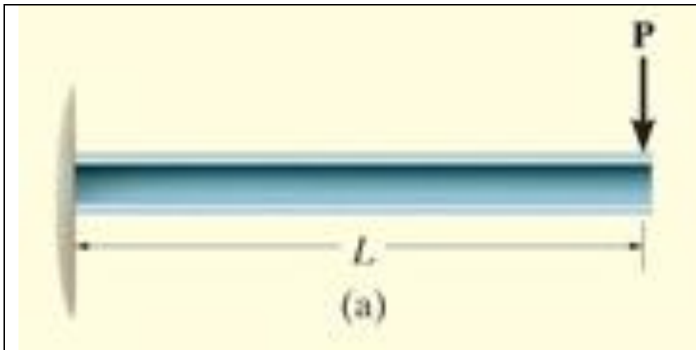
Fig. 6–11 (cont.)

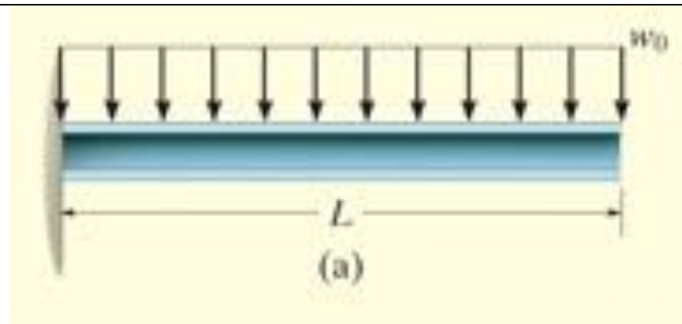
Table 6-1: Application of the graphical method to some common loading cases



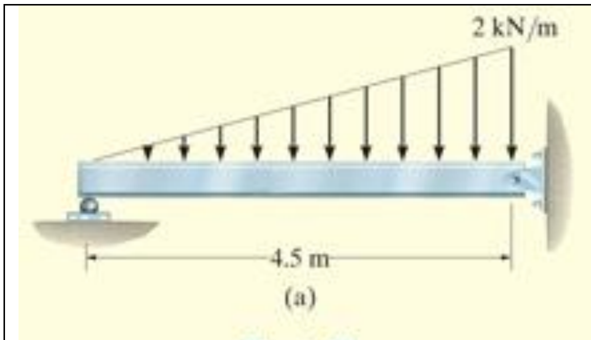
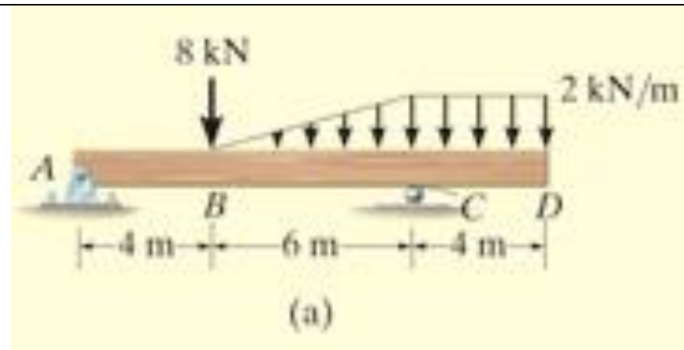
Examples

Draw the shear and moment diagrams of the followings by using the graphical method





Draw the shear and moment diagrams of the followings by using the graphical method

 Fig. 6–17	

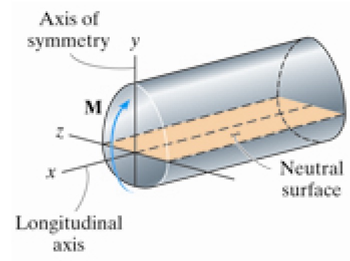
6.3. Bending deformation of a Straight Member

- Beams made of **homogenous** materials

(composite materials: Section 6.6, we do not cover)

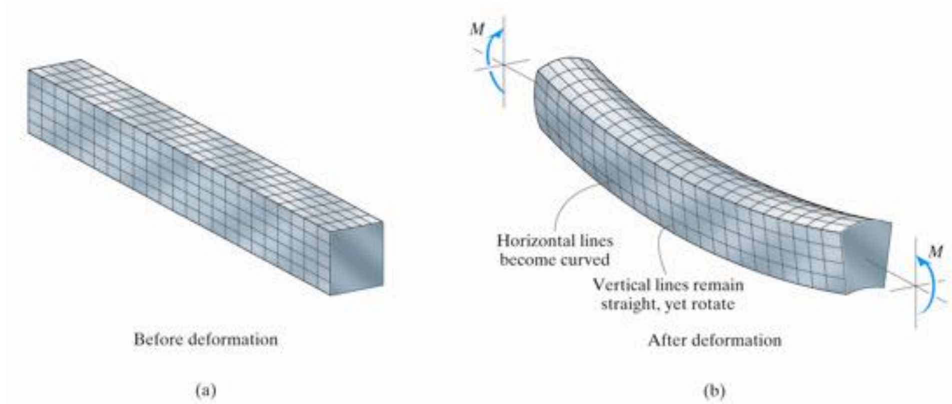
- Cross sectional area is **symmetric** with respect to an axis and bending moment is applied about an axis **perpendicular** to this axis of symmetry

(unsymmetric bending: Section 6.5, we will cover)



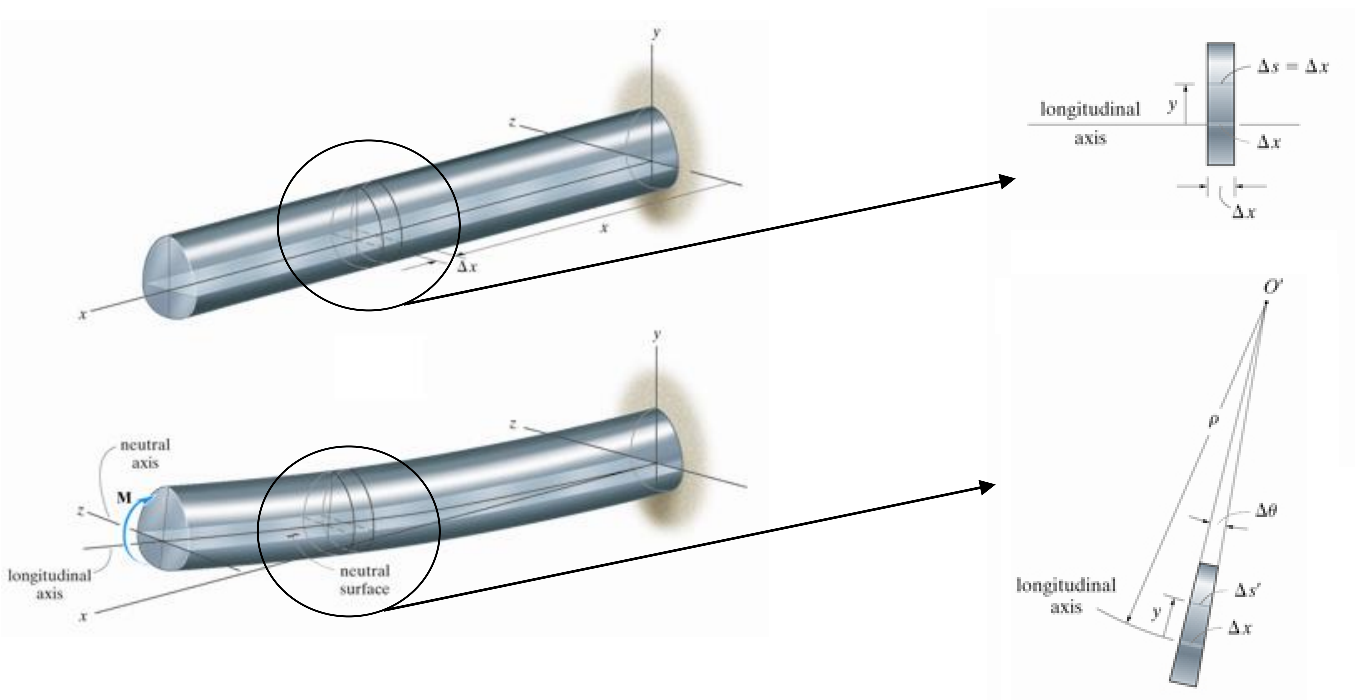
When bending moment is applied,

- Horizontal lines become curved
- Vertical lines remain straight, but they rotate

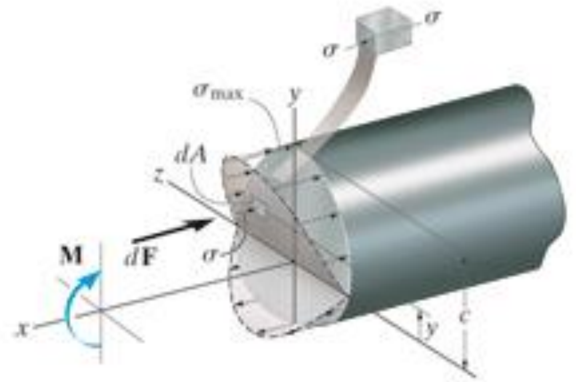


<u>Three assumptions</u>

1. **No change in length** on longitudinal axis within the neutral surface
2. All cross sections **remain plane**
3. **No deformation of cross section** within its own plane



6.4. The Flexure Formula



Bending stress variation

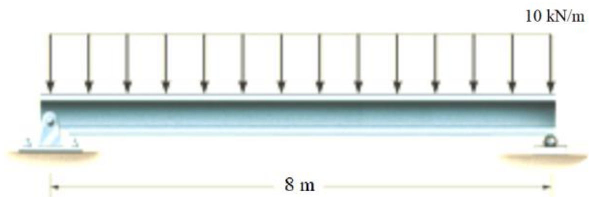
(c)

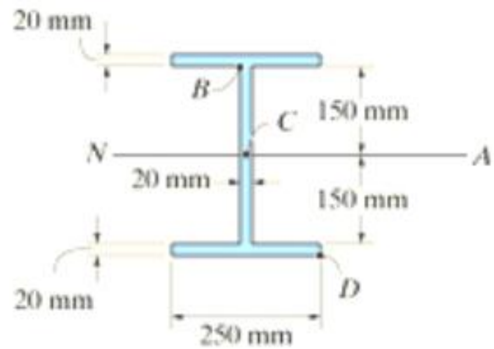
Fig. 6–26 (cont.)

Example 1.

Compute the factor of safety for the aluminum beam shown.

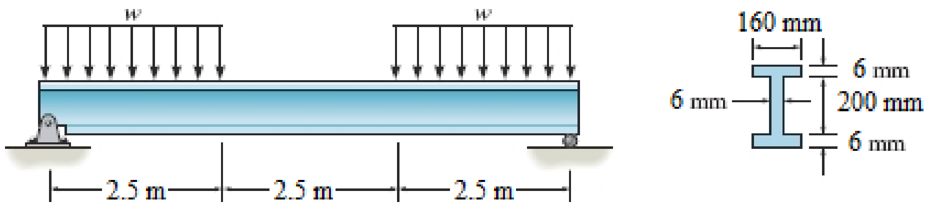
Take σ_Y=414 MPa.





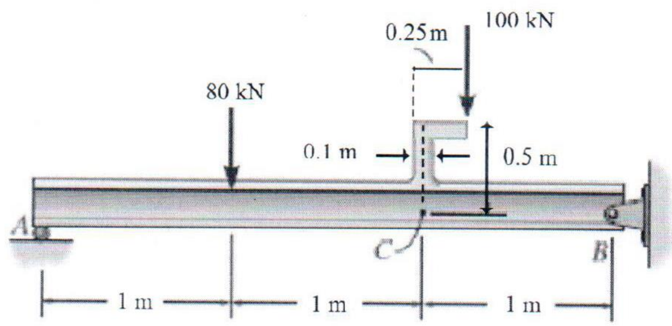
Example 2.

Determine the absolute maximum bending stress in the beam when w=7.5 kN/m.



Example 3.

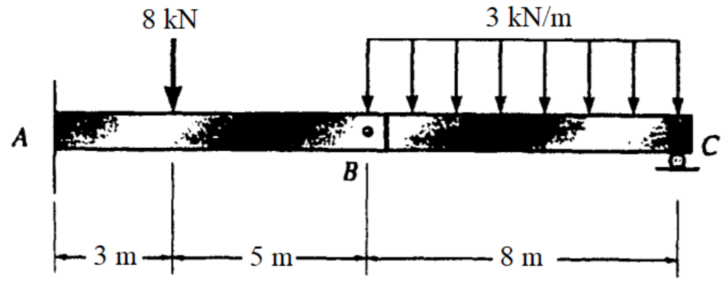
The beam shown has a square cross section of *b* mm on each side. If the allowable bending stress is 400 MPa, determine the smallest value of *b*.



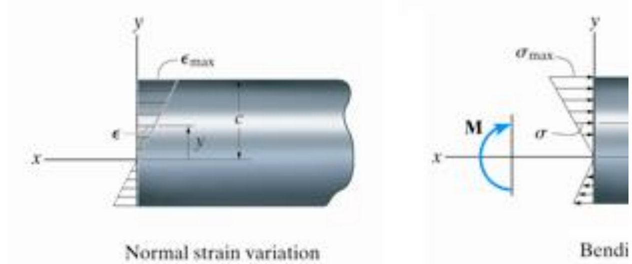
Example 4.

Determine the absolute maximum bending stress developed.

Each segment has a rectangular cross section with a base of 100 mm and height of 200 mm.



6.5. Unsymmetric Bending



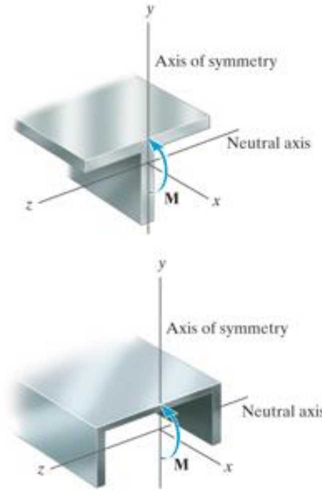


Fig. 6–31

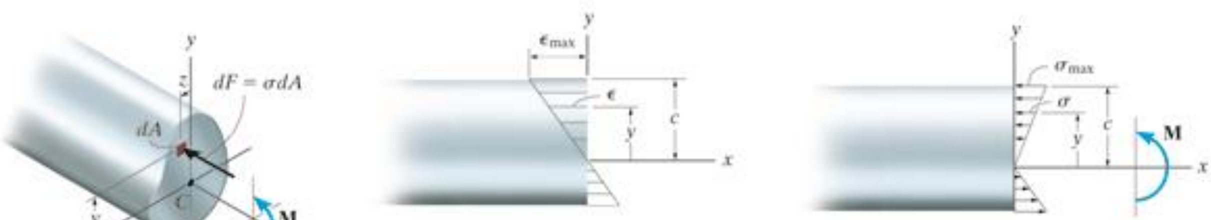
While developing $\sigma = -\frac{M\,y}{I}$ (flexure formula), we imposed the following conditions

- The cross sectional area should be symmetric about an axis perpendicular to the neutral axis
- The resultant internal moment M should act along the neutral axis

In this section, we will show how the flexure formula can also be applied to

- a beam having a cross sectional area of any shape
- a beam having a resultant internal moment that acts in any direction

A) Beam having a cross sectional area of any shape



(a) Normal-strain distribution (profile view) (b) Bending-stress distribution (profile view) (c)

Fig. 6–32

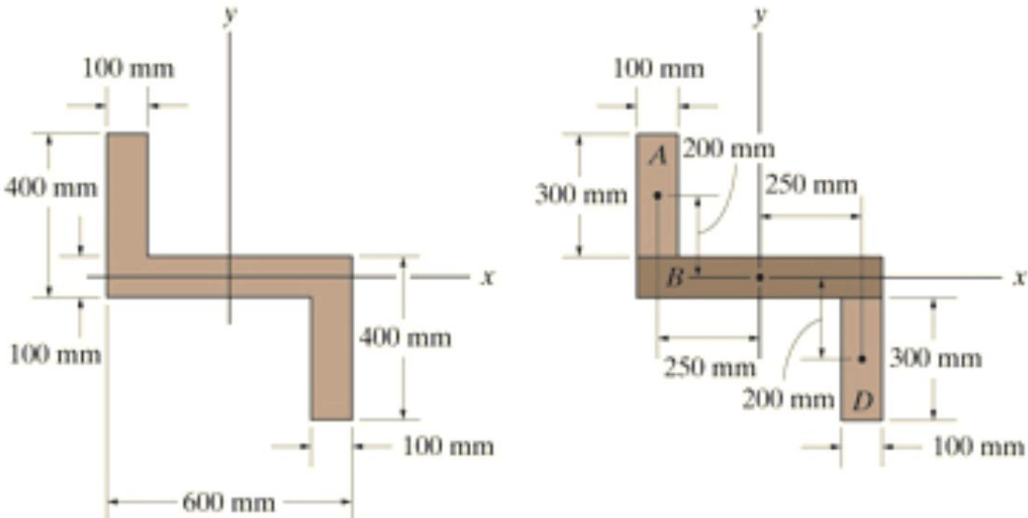
<u>Equilibrium equations</u>

Positioning the principal axes

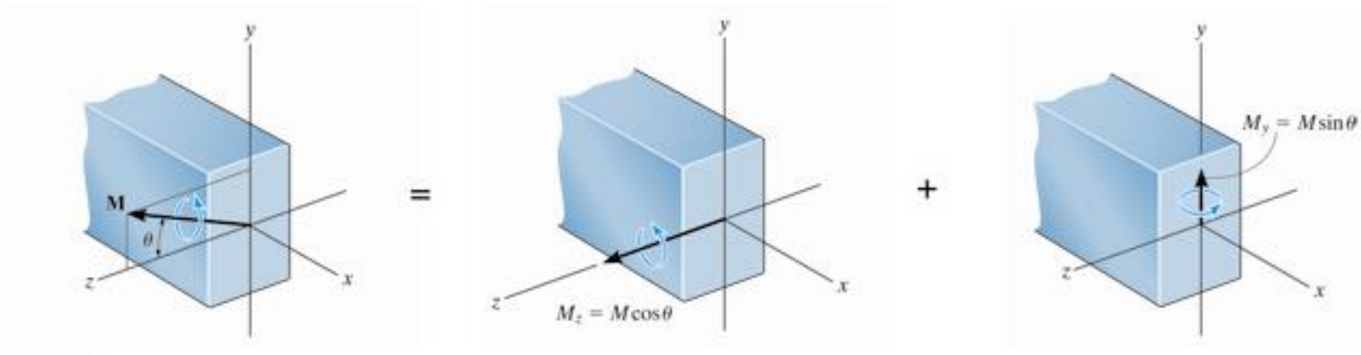
- If the cross section is symmetric, y and z axes are naturally principal axes.
- If the cross section is unsymmetric, the principal axes can be positioned using **Mohr's circle** or **transformation equations** (see Appendix A of Hibbeler's book).

Example.

Find the orientation of the principal axes for the cross section shown.



B) Resultant internal moment acts in any direction



After resolving the moment into its components, we have $\sigma = -\dfrac{M_z\,y}{I_z} + \dfrac{M_y\,z}{I_y}$

(DO NOT MEMORIZE THE PLUS AND MINUS SIGNS!! TRY TO UNDERSTAND)

__**Orientation of the Neutral Axis**__

Normal stress is zero on the neutral axis → $-\dfrac{M_z\,y}{I_z} + \dfrac{M_y\,z}{I_y} = 0$

$M_z = M\cos\theta$ and $M_y = M\sin\theta$ → $\boxed{\tan\alpha = \dfrac{I_z}{I_y}\tan\theta}$ where $\tan\alpha = y/z$

Note: The angles θ and α are measured positive from +z axis toward +y axis.

My = 12.99 kN·m

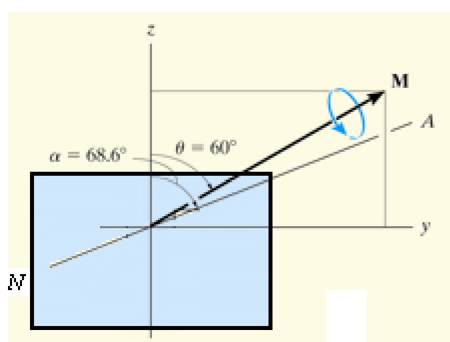
Mz = 7.50 kN·m

Iz = 20.53×10^{-6} m^4

Iy = 13.92×10^{-6} m^4

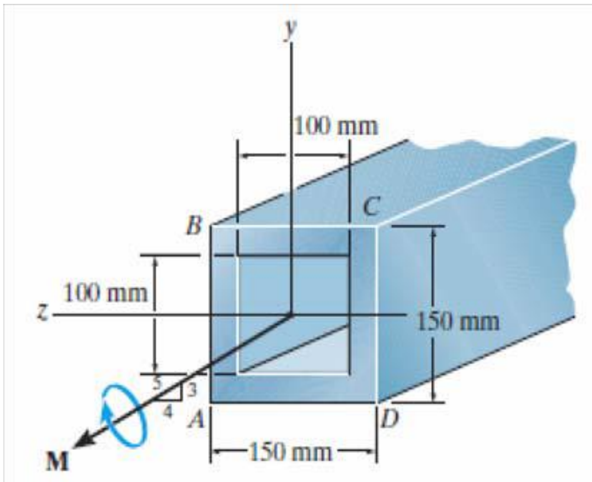
tan θ = 12.99/7.50 → θ = 60˚

$\tan\alpha = \dfrac{20.53}{13.92}\tan 60°$ → α = 68.6˚



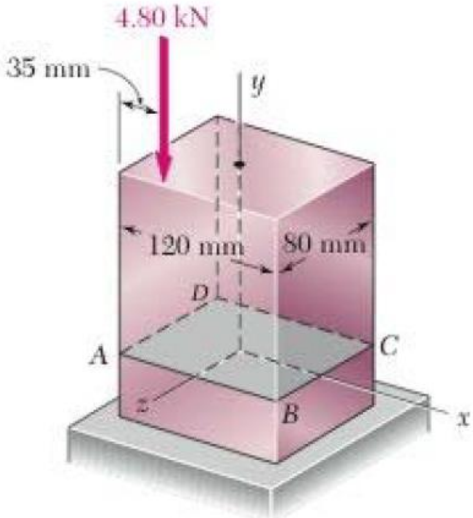
Example 1.

Determine the maximum magnitude of the bending moment **M** so that the bending stress in the member does not exceed 100 MPa.



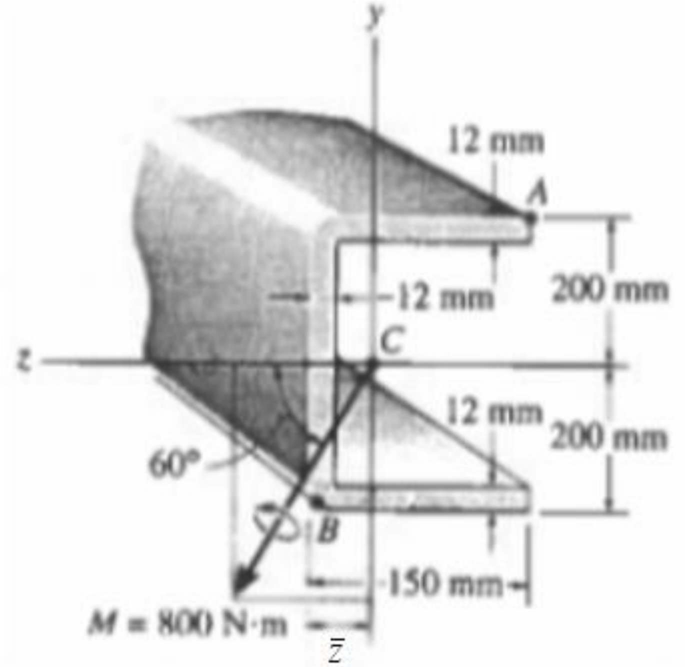
Example 2.

Determine the stresses at points A, B, C and D.



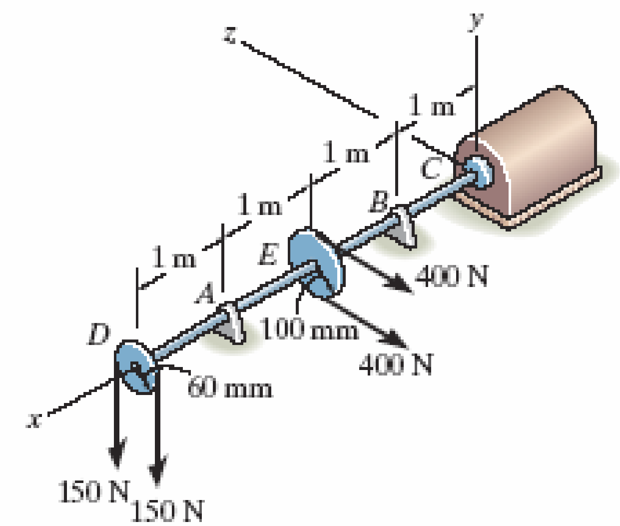
Example 3.

The resultant moment acting on the cross section of the aluminum strut has magnitude of = 800 N-m and is directed as shown. Determine the maximum bending stress in the strut.
Note: The location of the centroid C of the struts cross-sectional area must be determined first.



Example 4.

The 30-mm-diameter shaft is subjected to the vertical and horizontal loadings of two pulleys as shown It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the maximum bending stress developed in the shaft.



CHAPTER 7. TRANSVERSE SHEAR

OUTLINE

7.1. Shear in Straight Members

7.2. The Shear Formula

7.3. Shear Stress Distribution in Beams

- Rectangular Cross Section
- Wide-Flange Beam

CHAPTER 7: TRANSVERSE SHEAR (*ENİNE KAYMA*)

7.1. Shear in Straight Members

- Beams generally support both shear and moment loadings
- Due to the complementary property of shear, transverse shear stress is also associated to longitudinal shear stress.

Fig. 7–1

Why shear stress develops on the longitudinal planes of the beam ?

Consider the beam made from three boards that are not bonded together (Fig. 7-2). The application of a the force P will cause the boards to <u>slide</u> over each other.

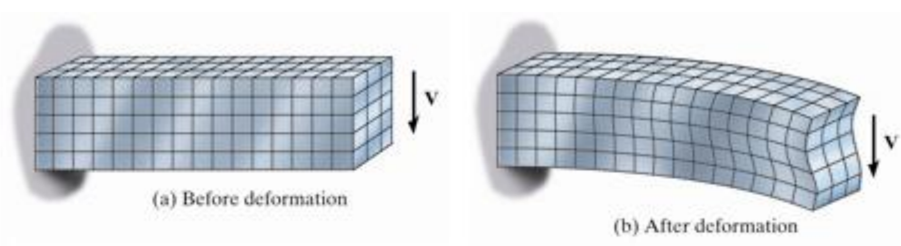
If the boards are bonded, then the longitudinal shear stress will prevent sliding of the boards.

Fig. 7–2

As a result of the shear stresses, **shear strains** are developed.

These shear strains are in a complex manner (they are **not linearly varying** as in bending and torsion).

This non-uniform shear strain variation will cause the cross section to warp (çarpılma).



For axial loading, the strains are constant. Similarly, for bending and torsion, the strains are linear. Thus, we were able to start from strain distributions to compute stresses.

For transverse shear, on the other hand, the strains are nonlinear and cannot be easily expressed mathematically. Therefore, we will not start from strains to compute stresses.

Instead, we will compute shear stresses using the **V = dM/dx** formula.

7.2. The Shear Formula

Consider a beam under external loading.

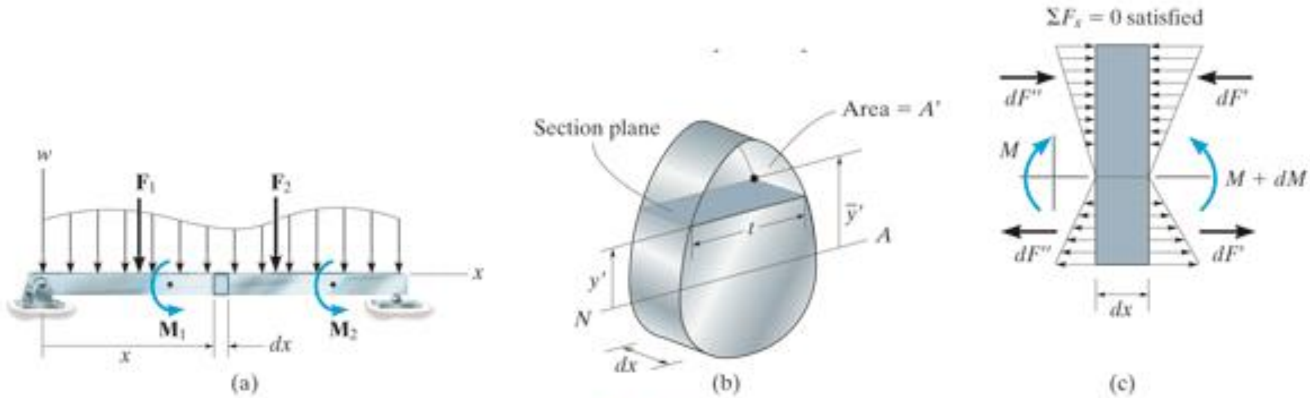


Fig. 7–4

Concentrate on a section that is y’ distance away from the neutral axis.

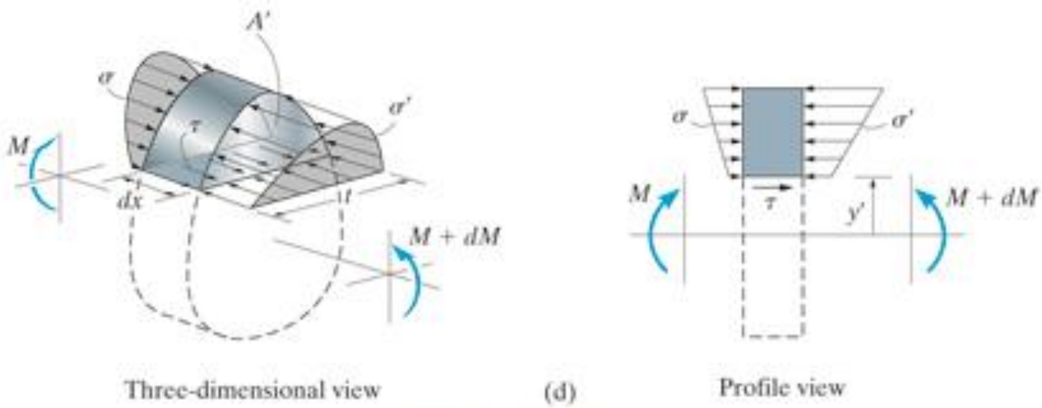
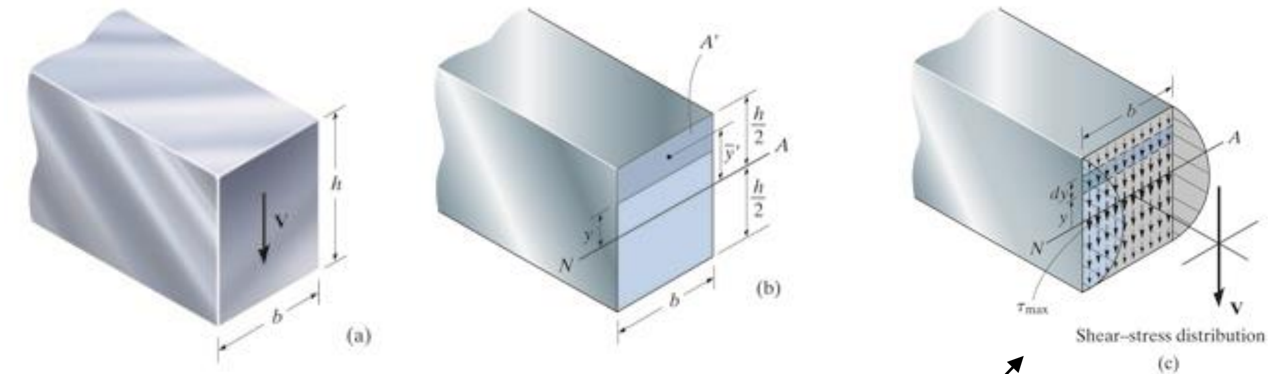


Fig. 7–4 (cont.)

<u>Equilibrium Equation</u>

7.3. Shear Stress Distribution in Beams

Rectangular Cross Section



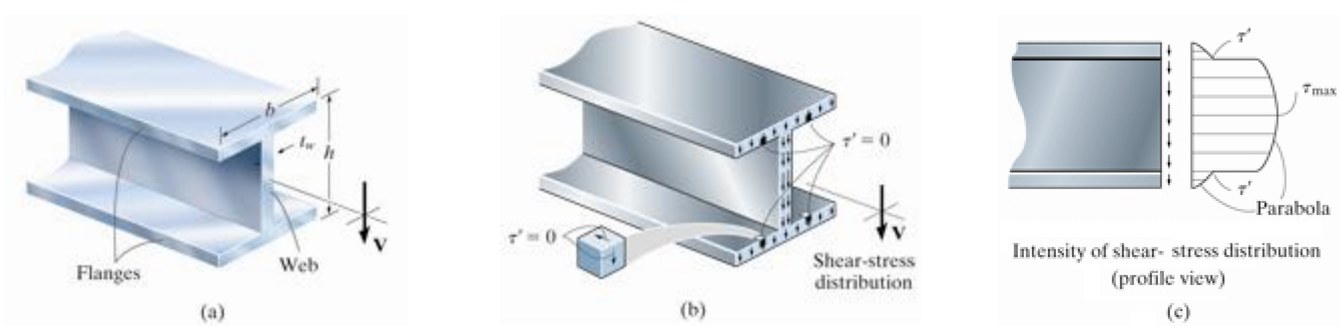
$$Q=\bar{y}'A'=\left[y+\frac{1}{2}\left(\frac{h}{2}-y\right)\right]\left(\frac{h}{2}-y\right)b=\frac{1}{2}\left(\frac{h^2}{4}-y^2\right)b$$

$$\tau=\frac{VQ}{It}=\cdots=\frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right)$$ **(parabolic distribution)**

The shear stress is MAXIMUM on the neutral plane $\tau_{\max}=1.5\dfrac{V}{A}$

(Recall that the normal stress is zero on the neutral plane)

Wide-Flange Beam (Geniş Flanşlı Kiriş)



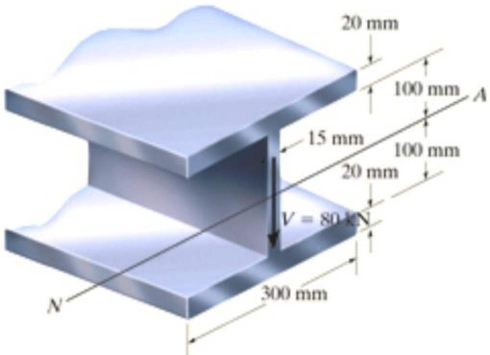
As the thickness changes instantly
A **jump** occurs in shear stress distribution

Example 1: For the cross section shown,

(a) Plot the shear-stress distribution over the cross section

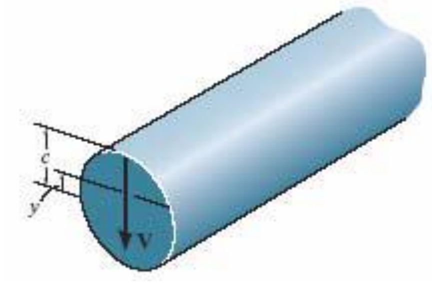
(b) Determine how much of the shear load is carried by the web

(c) Determine how much of the bending moment (if acts) is carried by the flanges

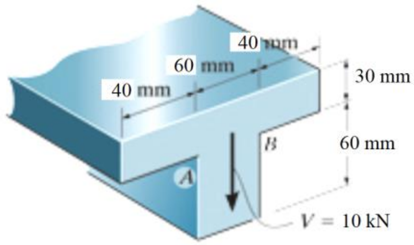


Example 1 (continued):

Example 2: Plot the shear-stress distribution over the cross section of a rod that has a radius c.

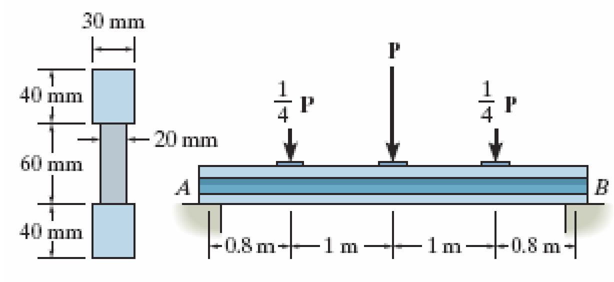


Example 3: Calculate the shear stress distribution over the section shown



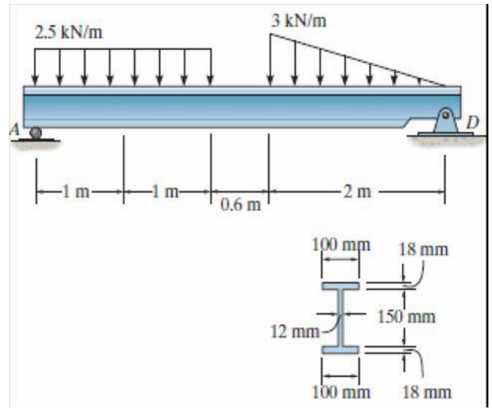
Example 4:

The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load *P* that can be applied without causing the glue to lose its bond.



Example 5:

Determine the maximum shear stress acting in the beam at the critical section.



CHAPTER 8. COMBINED LOADING

OUTLINE

8.1. Thin-Walled Pressure Vessels

8.2. State of Stress Caused by Combined Loadings

CHAPTER 8: COMBINED LOADING (BİLEŞİK YÜKLEME)

8.1. Thin-Walled Pressure Vessels

- Cylindrical or spherical vessels are commonly used as boilers or tanks
- For thin walled vessels, $r/t \geq 10$.
  - ➢ If $r/t = 10$, then $\sigma_{thin-walled} \approx 0.96\sigma_{actual}$
  - ➢ The stress distribution throughout the thickness do not change significantly, so it is assumed to be uniform (or constant)

<u>Cylindrical Vessels</u> - an element of the material will be subjected to a **biaxial** state of stress

For Fig. 8-1(b), $\sum F_x = 0,$

For Fig. 8-1(c), $\sum F_y = 0,$

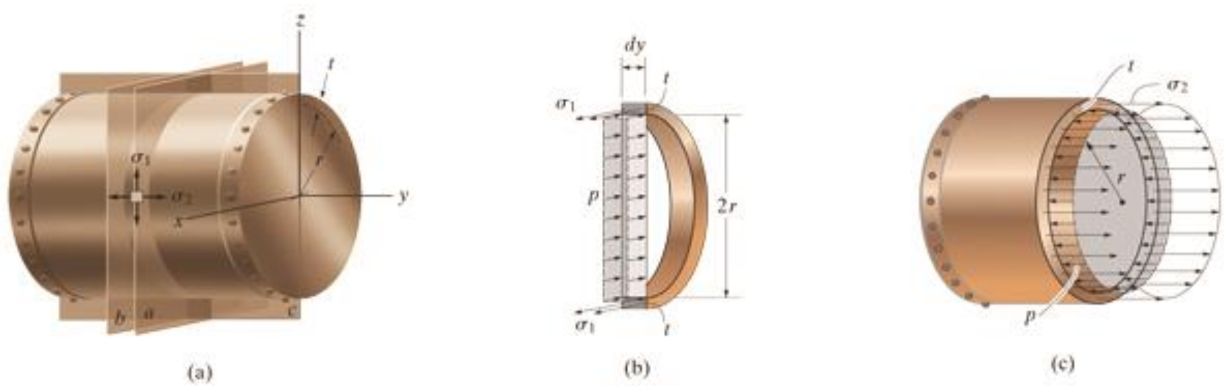


Fig. 8–1

<u>Spherical Vessels</u>

For Fig. 8-2(b), $\sum F_y = 0,$

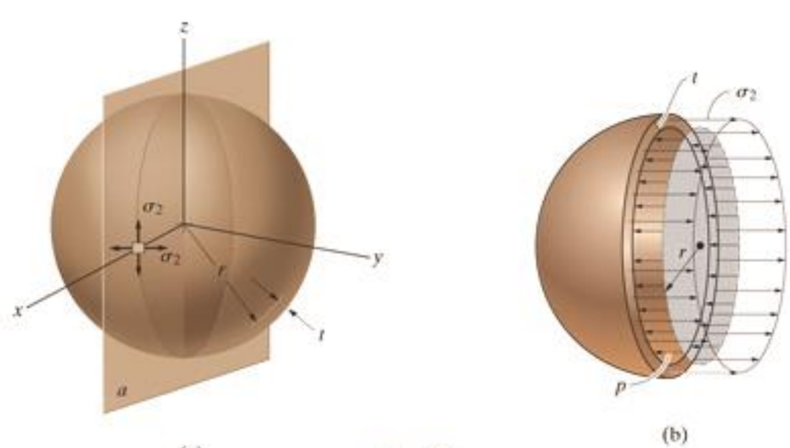
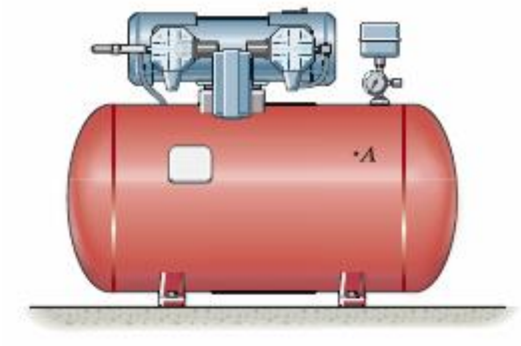


Fig. 8–2

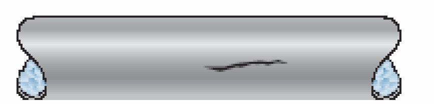
Example 1:

The tank of the air compressor is subjected to an internal pressure of 0.63 MPa. If the internal diameter of the tank is 550 mm, and the wall thickness is 6 mm, determine the stress components acting at point *A*.



Example 2:

The **open-ended** pipe has a wall thickness of 2 mm and an internal diameter of 40 mm. Calculate the pressure that ice exerted on the interior wall of the pipe to cause it to burst in the manner shown. The maximum stress that the material can support at freezing temperatures is σ_{max} = 360 MPa.



8.2. State of Stress Caused by Combined Loadings

- Axial loading (N)
- Shear loading (V)
- Bending moment (M)
- Torsional moment (T)
- Internal pressure (p)

Several of these forces may act together

The method of superposition can be used to determine the resultant stress distribution

Procedure for Analysis

The following procedure provides a general means for establishing the normal and shear stress components at a point in a member when the member is subjected to several different types of loadings simultaneously. It is assumed that the material is homogeneous and behaves in a linear elastic manner. Also, Saint-Venant's principle requires that the point where the stress is to be determined is far removed from any discontinuities in the cross section or points of applied load.

Internal Loading.

- Section the member perpendicular to its axis at the point where the stress is to be determined and obtain the resultant internal normal and shear force components and the bending and torsional moment components.
- The force components should act through the *centroid* of the cross section, and the moment components should be computed about *centroidal axes*, which represent the principal axes of inertia for the cross section.

Stress Components.

- Determine the stress component associated with *each* internal loading. For each case, represent the effect either as a distribution of stress acting over the entire cross-sectional area, or show the stress on an element of the material located at a specified point on the cross section.

Normal Force.

- The internal normal force is developed by a uniform normal-stress distribution determined from $\sigma = P/A$.

Shear Force.

- The internal shear force in a member is developed by a shear-stress distribution determined from the shear formula, $\tau = VQ/It$. Special care, however, must be exercised when applying this equation, as noted in Sec. 7.2.

Bending Moment.

- For *straight members* the internal bending moment is developed by a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. This stress distribution is determined from the flexure formula, $\sigma = My/I$. If the member is *curved*, the stress distribution is nonlinear and is determined from $\sigma = My/[Ae(R - y)]$.

Torsional Moment.

- For circular shafts and tubes the internal torsional moment is developed by a shear-stress distribution that varies linearly from the central axis of the shaft to a maximum at the shaft's outer boundary. This stress distribution is determined from the torsional formula, $\tau = T\rho/J$.

Thin-Walled Pressure Vessels.

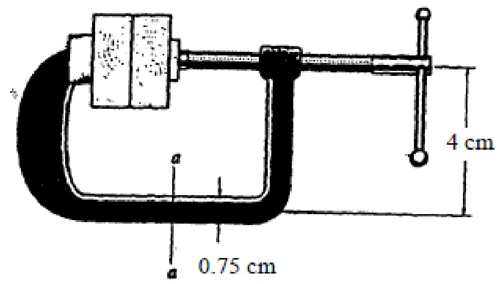
- If the vessel is a thin-walled cylinder, the internal pressure p will cause a biaxial state of stress in the material such that the hoop or circumferential stress component is $\sigma_1 = pr/t$ and the longitudinal stress component is $\sigma_2 = pr/2t$. If the vessel is a thin-walled sphere, then the biaxial state of stress is represented by two equivalent components, each having a magnitude of $\sigma_2 = pr/2t$.

Superposition.

- Once the normal and shear stress components for each loading have been calculated, use the principle of superposition and determine the resultant normal and shear stress components.
- Represent the results on an element of material located at the point, or show the results as a distribution of stress acting over the member's cross-sectional area.

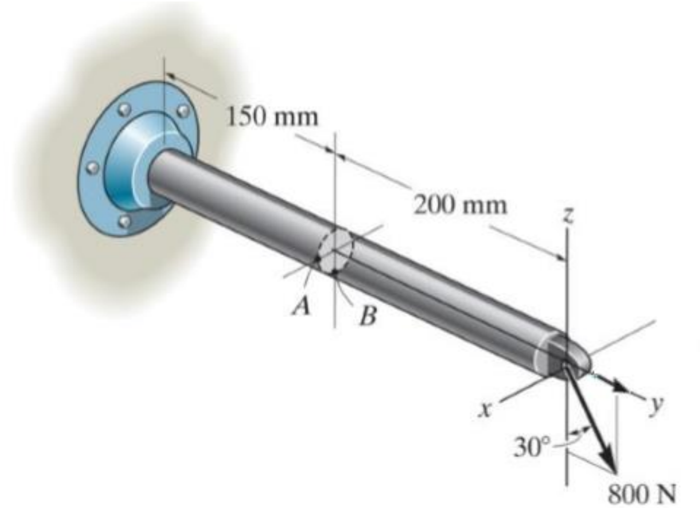
Example 1:

The screw of the clamp exerts a compressive force of 500 N on the wood blocks. Sketch the stress distribution along section *a–a* of the clamp. The cross section there is rectangular, 0.75 cm by 0.50 cm.



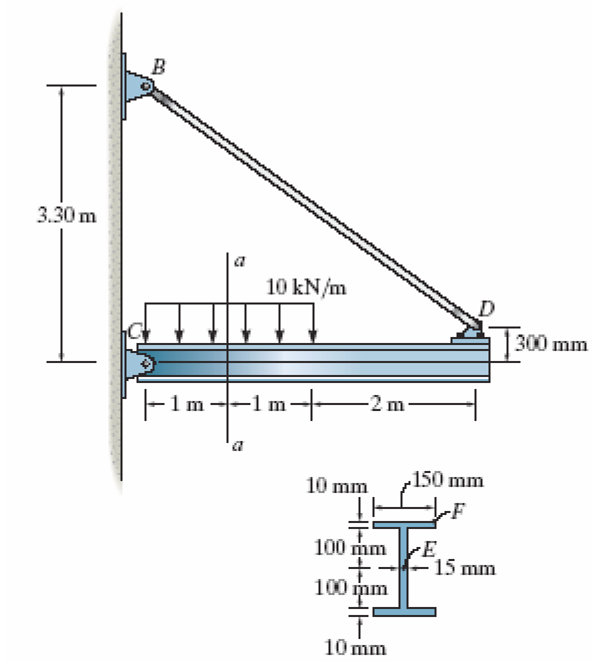
Example 2:

The bar has a diameter of 40 mm. If it is subjected to the loadings as shown, determine the stress components that act at points A and B.



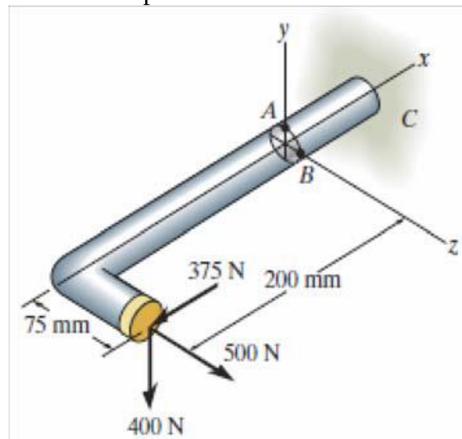
Example 3:

Determine the state of stress at points E and F at section a-a.



Example 4:

The 25-mm-diameter rod is subjected to the loads shown. Determine the state of stress at point B.



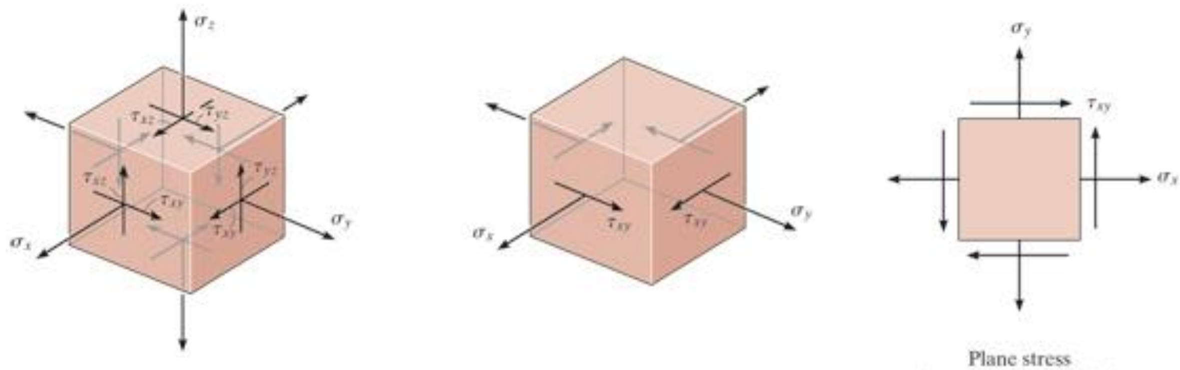
CHAPTER 9. STRESS TRANSFORMATION

OUTLINE

CHAPTER 9: STRESS TRANSFORMATION

9.1. Plane-Stress Transformation

- The general state of stress is characterized by 6 independent stress components (Fig. 9-1a).
- If there is no load on the surface of a body, then the normal and shear stress components are zero on the face of an element that lies on the surface (Fig. 9-1b).
- This state of stress is called plane-stress and the body can be analyzed in a single plane.
- Plane-stress state can be represented by 3 stress components (σ_x, σ_y, τ_{xy}) (see Fig. 9-1c).

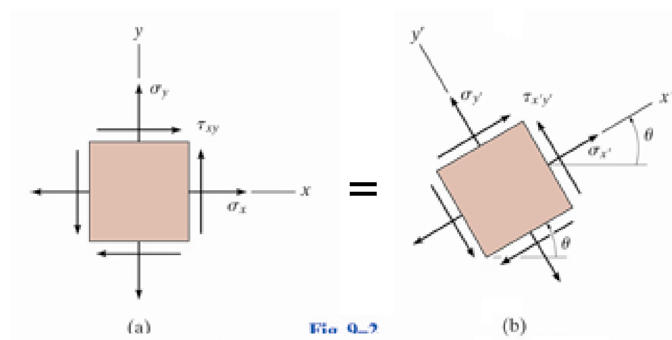


General state of stress (a) — Plane stress (b) — Plane stress (two dimensional view) (c)

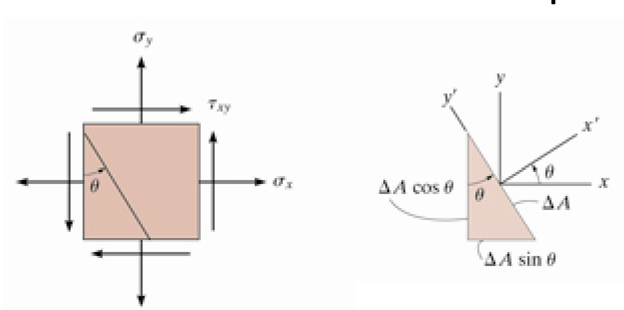
Fig. 9–1

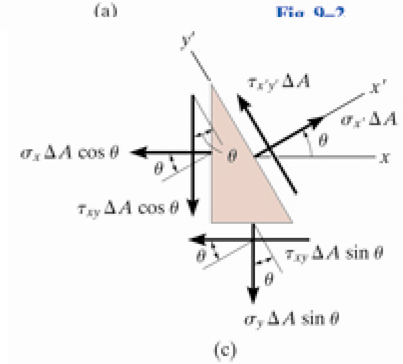
If the orientation of coordinate axes changes, the stresses components on the new orientation also changes.
(σ_x, σ_y, τ_{xy}) → ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$)

- ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$) stress components can be expressed in terms of (σ_x, σ_y, τ_{xy}) components using equilibrium equations.



9.2. Plane-Stress Transformation Equations

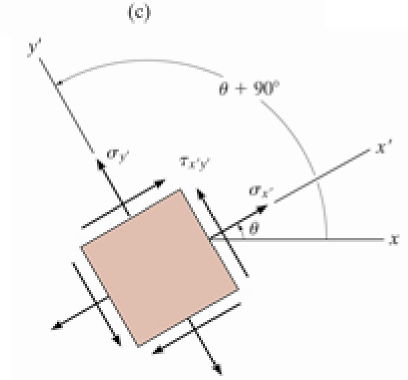




Equilibrium Equations:

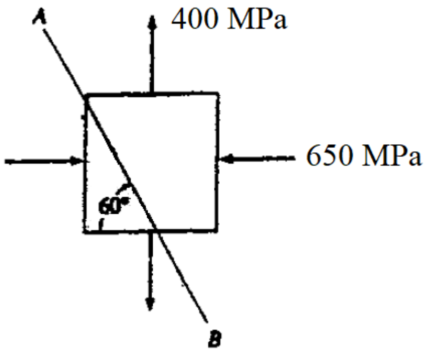
$$\left.\begin{aligned}\sum F_{x'} &= 0\\ \sum F_{y'} &= 0\end{aligned}\right\} \quad \begin{aligned}\sigma_{x'} &= \frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta\\ \tau_{x'y'} &= -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta+\tau_{xy}\cos 2\theta\end{aligned} \quad (*)$$

$$\theta=\theta+90^\circ \;\rightarrow\; \sigma_{y'}=\frac{\sigma_x+\sigma_y}{2}-\frac{\sigma_x-\sigma_y}{2}\cos 2\theta-\tau_{xy}\sin 2\theta$$



Example:

The state of stress at a point in a machine element is shown. Determine the stress components acting on the inclined plane AB using stress transformation equations.



9.3. Principal Stresses and Maximum Shear Stress

- ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$) stress components depend on the orientation (θ angle).
- In engineering practice, it is often important to determine the orientation of the planes that causes the normal stress to be a maximum or minimum, and the shear stress to be maximum.

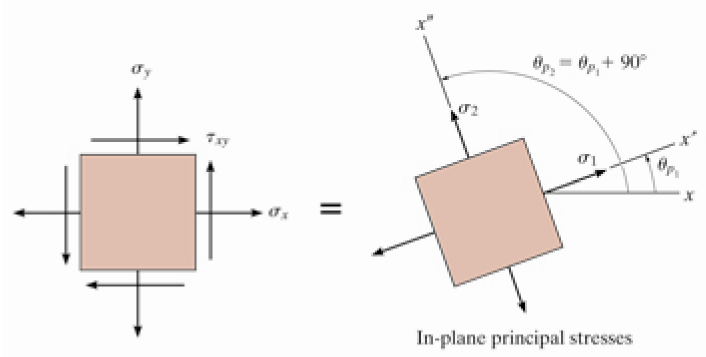
For maximum normal stress: $\frac{d\sigma_{x'}}{d\theta} = 0 \rightarrow \tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$

Two roots of this equation are θ_{p1} and θ_{p2}, and they are 90° apart.

Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

($\sigma_1 \geq \sigma_2$)



No shear stress acts on the principal planes.

$\tau_{xy} = 0$

For maximum shear stress: $\frac{d\tau_{x'y'}}{d\theta} = 0 \rightarrow \tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$

Two roots of this equation are θ_{s1} and θ_{s2}, and they are 90° apart.

Comparing the orientation of principal stresses and maximum shear stress, we see that

$\tan 2\theta_s = -\frac{1}{\tan 2\theta_p}$ Thus, the angle between $2\theta_s$ and $2\theta_p$ is 90°.

The angle between θ_s and θ_p is 45°.

(The planes for maximum shear stress can be determined by orienting an element 45° from the position of an element that defines the planes of principal stress.)

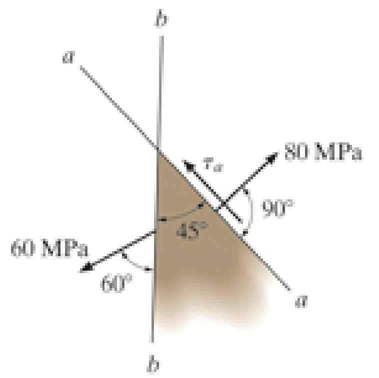
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Note that the normal stresses are not zero on planes of maximum shear stress!!. $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

Example:

The stress acting on two planes at a point is indicated (**realize that stress is not a point property, you need to specify a plane to define it**).

Determine the shear stress on plane a-a, and principal stresses at the point.



9.4. Mohr Circle (for state of Plane-Stress)

Mohr circle presents a graphical solution for stress transformation equations.

Construction of circle

- Establish a coordinate system where abscissa represents the normal stress σ (positive to the right), and the ordinate represents the shear stress τ (positive downward).
- Locate the center of the circle C, which lies on σ axis at a distance $\sigma_{avg} = \left(\sigma_x + \sigma_y\right)/2$ from the origin.
- Locate a reference point A, which has coordinates $A\left(\sigma_x, \tau_{xy}\right)$.
- Connect the points C and A, and compute the distance CA (the radius of the circle) by trigonometry.

Principal stresses

- The circle intersect the σ axis at two points (B and D). They are the principal stresses $\sigma_1 \geq \sigma_2$.
- The angle between CA and CB is $2\theta_{p1}$, and the angle between CA and CD is $2\theta_{p2}$.
- A rotation of 2θ in the circle corresponds to a rotation of θ in the element.

Maximum shear stress

- The radius of circle is equal to the maximum shear stress value.
- The angle between CA and CE is $2\theta_{s1}$, and the angle between CA and CF is $2\theta_{s2}$.
- Again, a rotation of 2θ in the circle corresponds to a rotation of θ in the element.

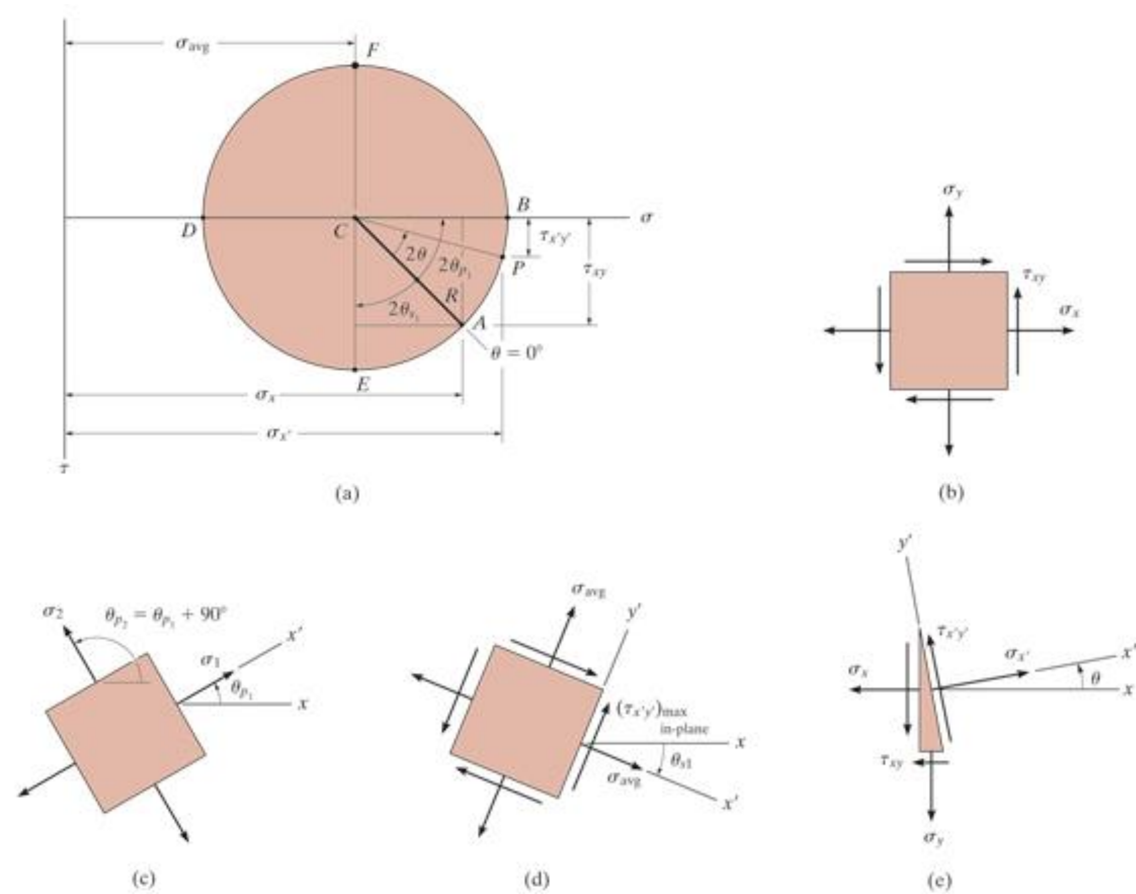
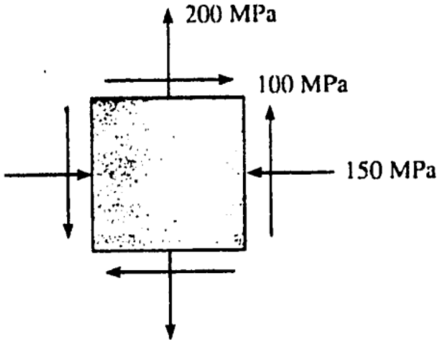


Fig. 9–17

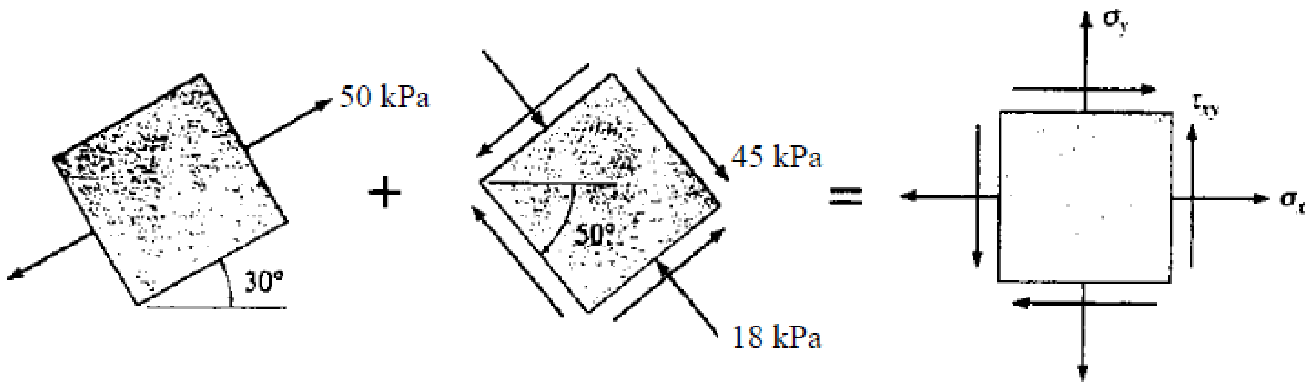
Example 1.

By using Mohr's circle, determine the principal stresses and maximum shear stress for the element shown.



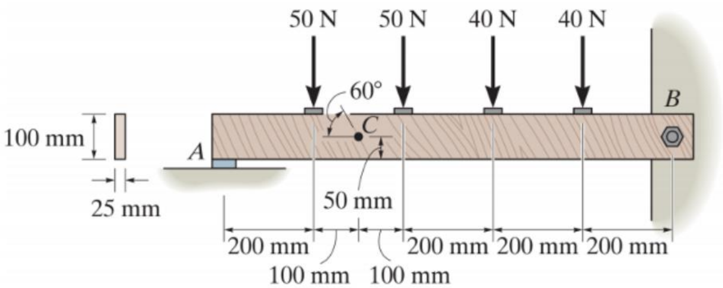
Example 2.

A point on a thin plate is subjected to two successive states of stress as shown.
Determine the resulting state of stress.

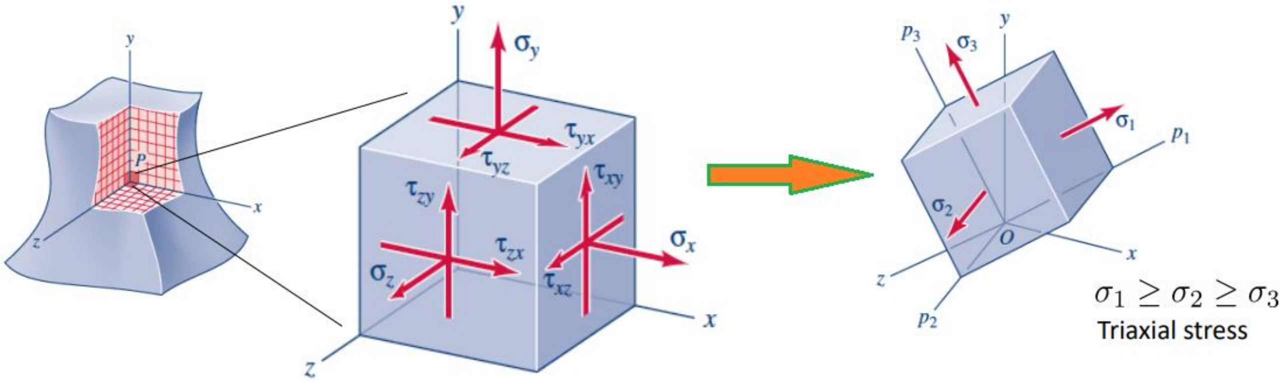


Example 3.

The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively.



9.5. Absolute Maximum Shear Stress



- For 3-D problems, all of the 6 independent stress components may exist.
- It is possible to rotate a 3D plane so that there are no shear stresses on that plane.
- Then the three normal stresses at that orientation would be the three principal normal stresses: σ_1, σ_2, σ_3.
- These three principal stress can be found by solving the following cubic equation

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

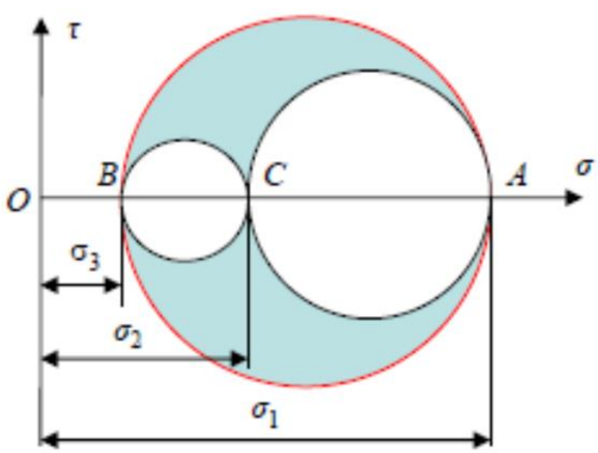
where

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} + \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$

- Now we have 3 circles.
- The radius of the largest circle is equal to the absolute maximum shear stress



The absolute maximum shear stress and the corresponding average stress are calculated from

$$\tau_{\max}^{\text{abs}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

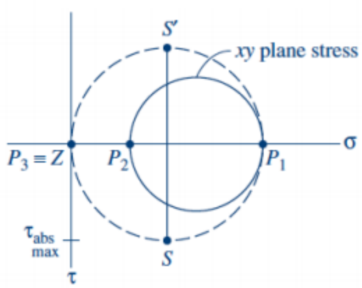
Plane stress

For plane stress, one principal stress is always zero.

We have 3 cases to consider:

$$\boxed{\sigma_3 = 0}$$

$$\sigma_1 \geq \sigma_2 \geq 0$$

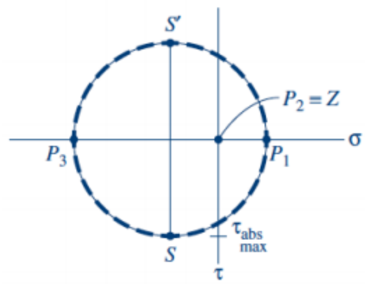


$$\tau_{\max}^{\text{abs}} = \frac{\sigma_1}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_1}{2}$$

$$\boxed{\sigma_2 = 0}$$

$$\sigma_1 \geq 0 \geq \sigma_3$$

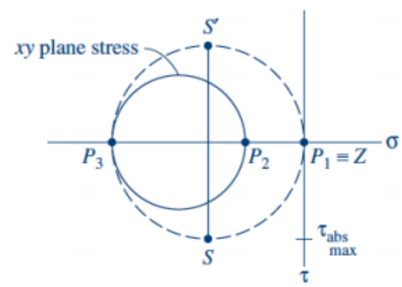


$$\tau_{\max}^{\text{abs}} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_1 + \sigma_3}{2}$$

$$\boxed{\sigma_1 = 0}$$

$$0 \geq \sigma_2 \geq \sigma_3$$

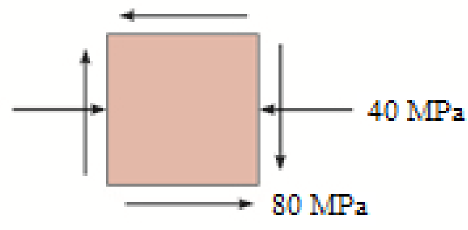


$$\tau_{\max}^{\text{abs}} = \frac{-\sigma_3}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_3}{2}$$

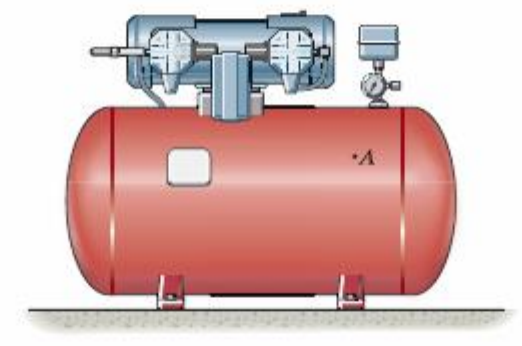
Example 1.

Compute the absolute maximum shear stress for the element shown.



Example 2.

The tank of the air compressor is subjected to an internal pressure of 0.5 MPa. If the internal diameter of the tank is 400 mm, and the wall thickness is 5 mm, determine the maximum absolute shear stress at point *A*.



CHAPTER 12. DEFLECTION OF BEAMS AND SHAFTS

OUTLINE

CHAPTER 12: DEFLECTION OF BEAMS AND SHAFTS

12.1. Elastic Curve

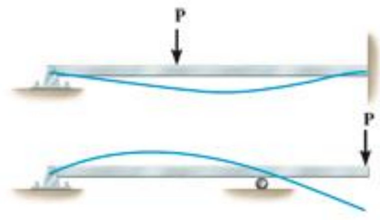
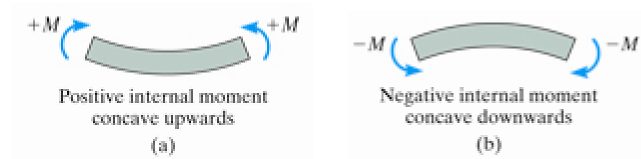


Fig. 12–1

The deflection diagram of the longitudinal axis that passes through the centroid of each cross sectional area of the beam is called the **elastic curve**. It is often helpful to sketch the deflected shape of the elastic curve to "visualize" the computed results and partially check these results.

If the moment diagram is known, the elastic curve can be constructed without much difficulty. (Recall that if the beam is slender, moment is more influential than shear.)



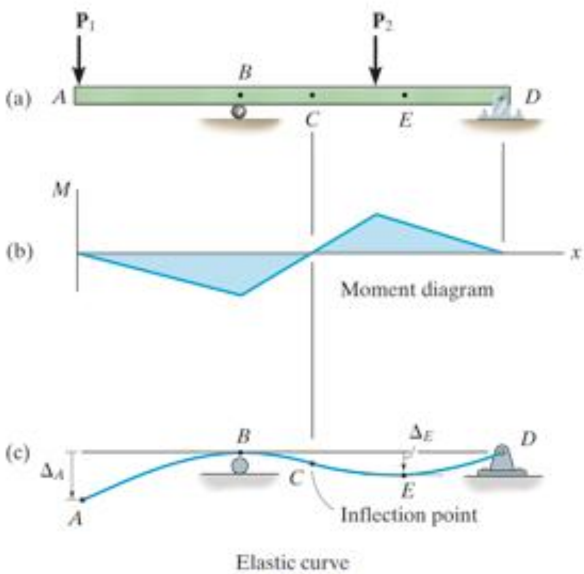


Fig. 12–3

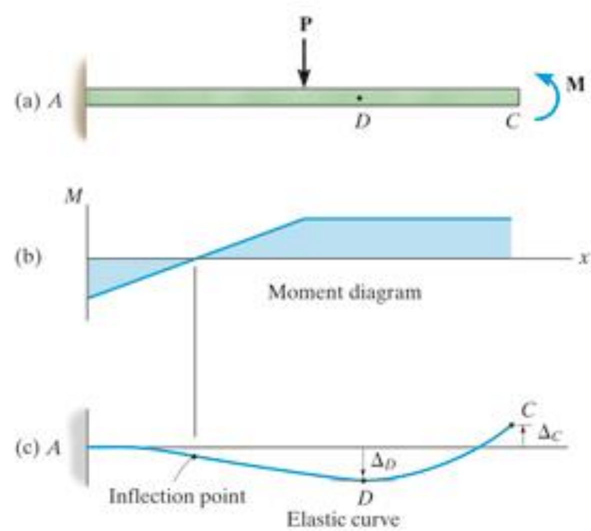


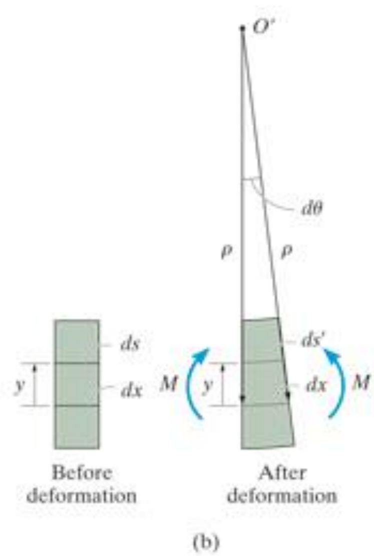
Fig. 12–4

Moment-Curvature Relationship

$$\varepsilon = \frac{ds' - ds}{ds} = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta} = \frac{-y}{\rho} \quad \text{or} \quad \frac{1}{\rho} = -\frac{\varepsilon}{y}$$

Use Hooke's $\varepsilon = \sigma / E$ and flexure formula $\sigma = -My / I$

So we have $\frac{1}{\rho} = \frac{M}{EI}$. EI: flexural rigidity (eğilme esnemezliği)



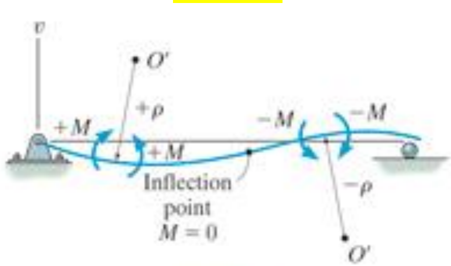


Fig. 12–6

If M is (+), ρ extends <u>above</u> the beam If M is (-), ρ extends <u>below</u> the beam

12.2. Slope and Displacement Calculation by Integration

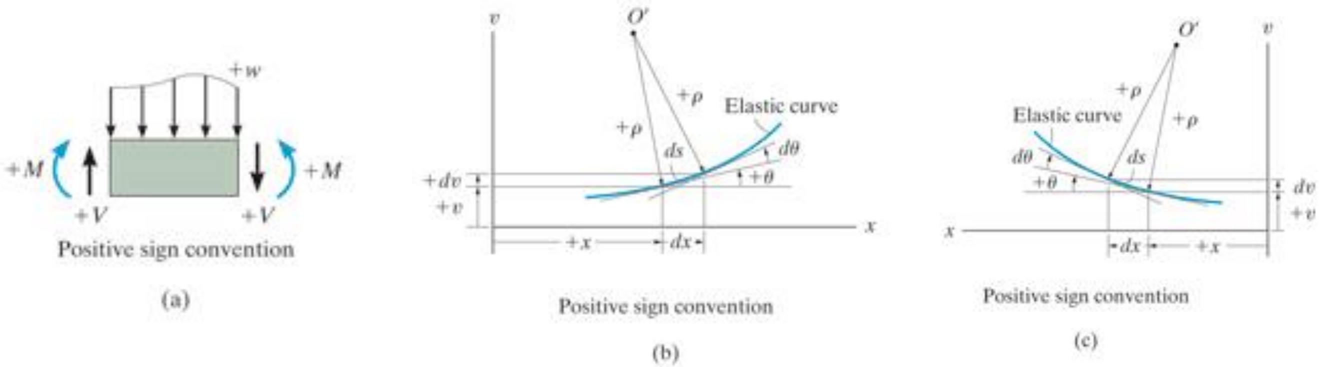
The equation for the elastic curve $v = f(x)$. $\frac{1}{\rho} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}} = \frac{M}{EI}$

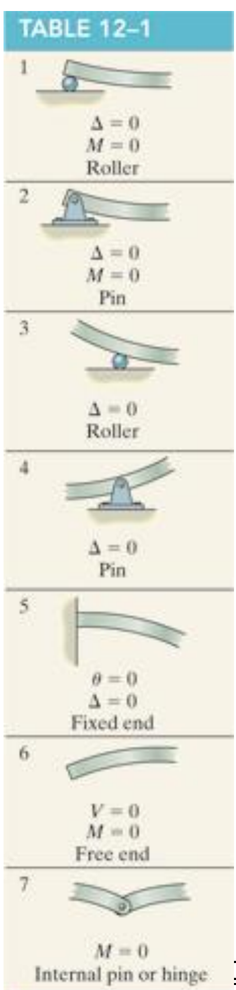
$$dv/dx \ll 1 \quad \rightarrow \quad \frac{d^2v}{dx^2} = \frac{M}{EI} \quad \rightarrow \quad EI\frac{d^2v}{dx^2} = M(x)$$

$$V = dM/dx \quad \rightarrow \quad \frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right) = V(x) \quad \rightarrow \quad EI\frac{d^3v}{dx^3} = V(x) \qquad (*)$$

$$-w = dV/dx \quad \rightarrow \quad \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = -w(x) \quad \rightarrow \quad EI\frac{d^4v}{dx^4} = -w(x)$$

Sign Convention

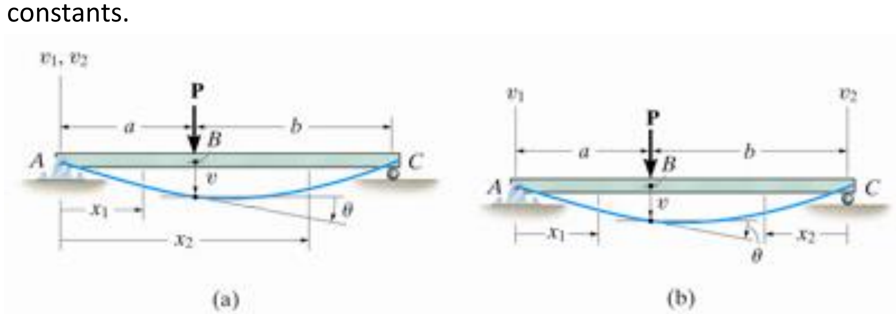




Before solving the above differential equations (*), w(x) or M(x) is first calculated.
Often we choose to calculate M(x) as it leads to two integration constants.

Solution of any of these equations requires successive integrations. For each integration, it is necessary to introduce **integration constants**.
To evaluate the integration constants, it is necessary to know the values of *v*(x), w(x), V(x) or M(x) at some particular locations. → **BOUNDARY CONDITIONS (Table 12-1)**
(Do Not Memorize! Try To Understand)

Sometimes it is not possible to use a single x coordinate to express the equation for the slope or the elastic curve. In that case, **continuity conditions** must be used to evaluate some of the integration constants.

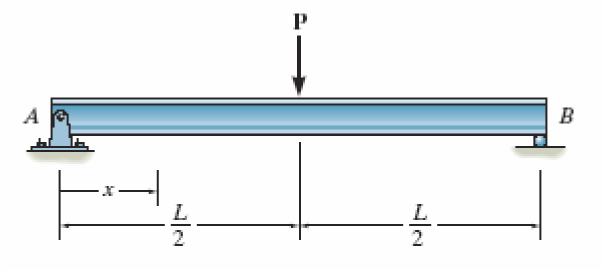


$\theta_1(a) = \theta_2(a)$ $\qquad\qquad$ $\theta_1(a) = -\theta_2(b)$

$v_1(a) = v_2(a)$ $\qquad\qquad$ $v_1(a) = v_2(b)$

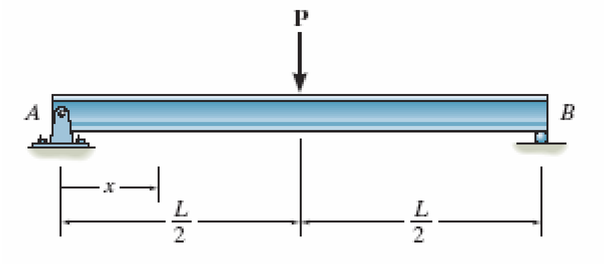
Example 1.

Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



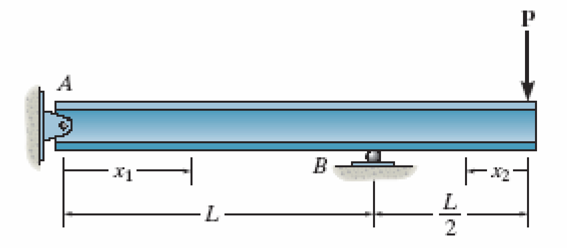
Example 2.

Solve Example 1 by using symmetry boundary conditions.



Example 3.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. *EI* is constant.



12.5. Method of Superposition

The differential equation $EI\frac{d^4v}{dx^4} = w(x)$ satisfies two necessary requirements:

- The load w(x) is linearly related to the deflection v(x)
- The load does not significantly change the original geometry

<u>Consider the example shown:</u>

Determine the displacement at point C and the slope at the support A of the beam shown in Fig. 12–28*a*. EI is constant.

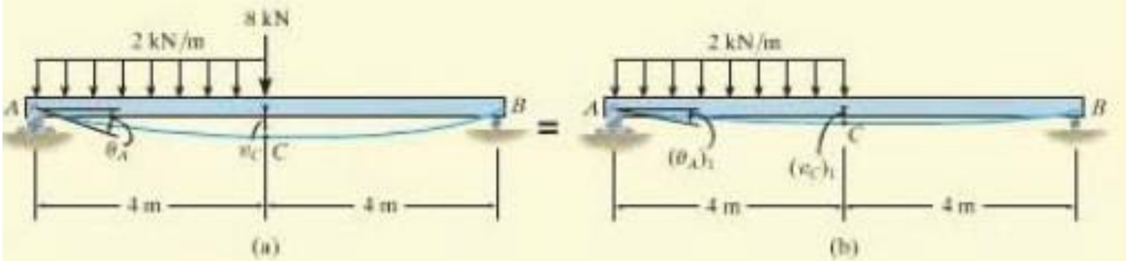
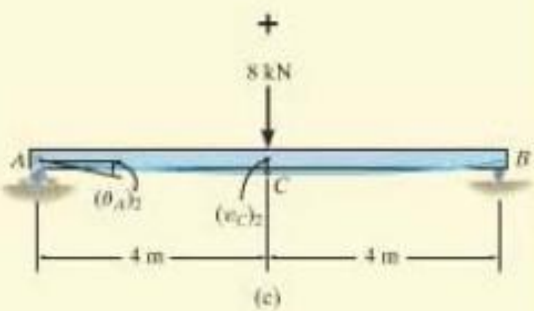


Fig. 12–28



SOLUTION

The loading can be separated into two component parts as shown in Figs. 12–28*b* and 12–28*c*. The displacement at C and slope at A are found using the table in Appendix C for each part.

For the distributed loading,

$$(\theta_A)_1 = \frac{3wL^3}{128EI} = \frac{3(2\text{ kN/m})(8\text{ m})^3}{128EI} = \frac{24\text{ kN}\cdot\text{m}^2}{EI}\curvearrowright$$

$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(2\text{ kN/m})(8\text{ m})^4}{768EI} = \frac{53.33\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

For the 8-kN concentrated force,

$$(\theta_A)_2 = \frac{PL^2}{16EI} = \frac{8\text{ kN}(8\text{ m})^2}{16EI} = \frac{32\text{ kN}\cdot\text{m}^2}{EI}\curvearrowright$$

$$(v_C)_2 = \frac{PL^3}{48EI} = \frac{8\text{ kN}(8\text{ m})^3}{48EI} = \frac{85.33\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

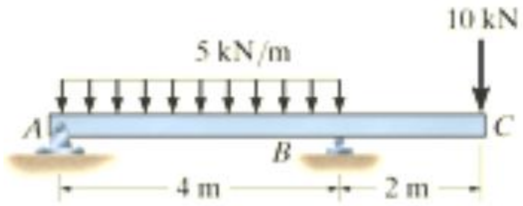
The displacement at C and the slope at A are the algebraic sums of these components. Hence,

$$(+\curvearrowright) \qquad \theta_A = (\theta_A)_1 + (\theta_A)_2 = \frac{56\text{ kN}\cdot\text{m}^2}{EI}\curvearrowright \qquad \textit{Ans.}$$

$$(+\downarrow) \qquad v_C = (v_C)_1 + (v_C)_2 = \frac{139\text{ kN}\cdot\text{m}^3}{EI}\downarrow \qquad \textit{Ans.}$$

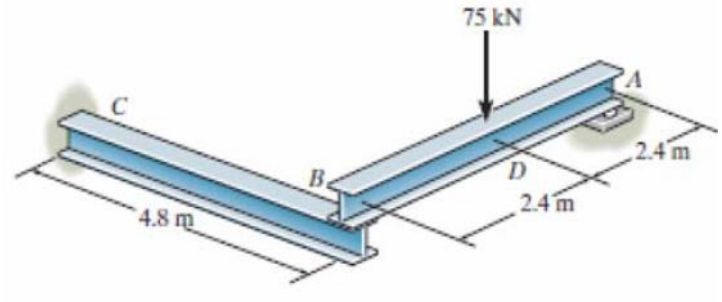
Example 1.

Compute the deflection at end C.



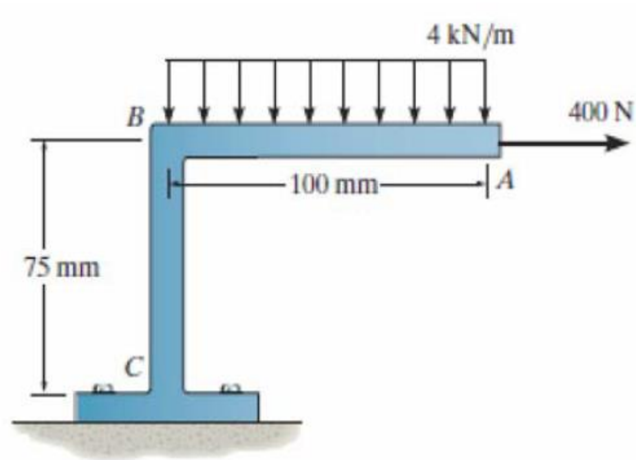
Example 2.

The assembly consists of a cantilevered beam *CB* and a simply supported beam *AB*. If each beam is made of A-36 steel (E=200 GPa) and has a moment of inertia about its principal axis of $I_x = 46(10^6)$ mm^4, determine the displacement at *D*.



Example 3.

Determine the vertical deflection and slope at the end *A* of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment *AB*. *EI* is constant.

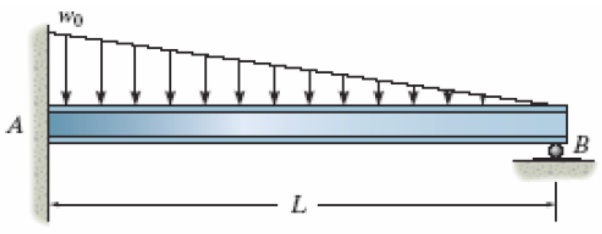


12.6. Statically Indeterminate Beams and Shafts

- For bars, we used displacements $\delta = \dfrac{PL}{EA}$ in compatibility equations
- For torque problems, we used angles of twist $\phi = \dfrac{TL}{GJ}$ in compatibility equations
- Now, for beams, we will use **deflections and rotations** in compatibility equations

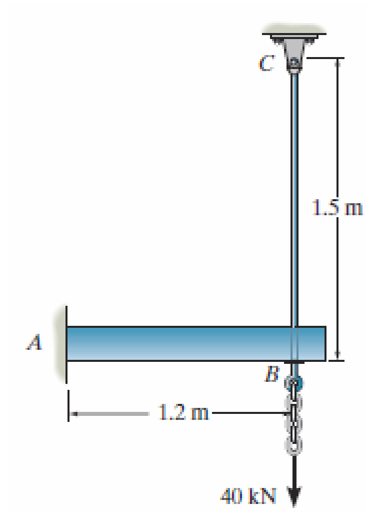
Example:

Determine the reactions at the supports *A* and *B*. *EI* is constant.



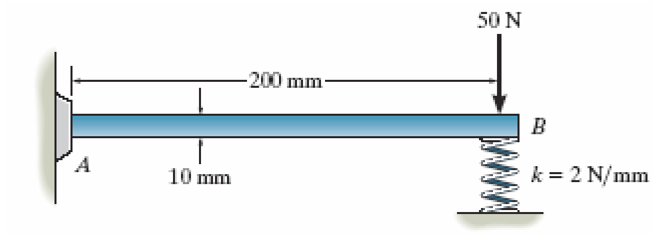
Example 2.

The A-36 steel beam (E=200 GPa) and rod are used to support the load of 40 kN. The diameter of the rod is 20 mm. The beam is rectangular, having a height of 125 mm and a thickness of 75 mm.
Compute the deflection at B and the stress in the rod.



Example 3.

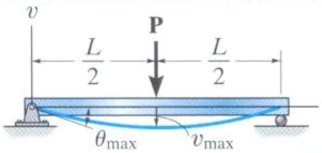
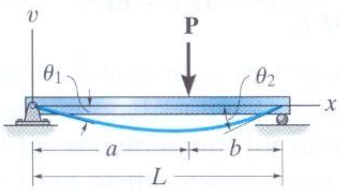
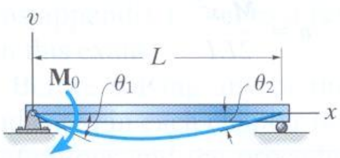
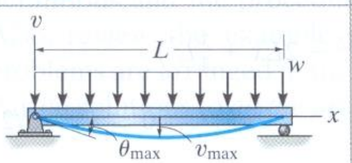
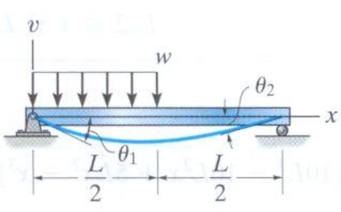
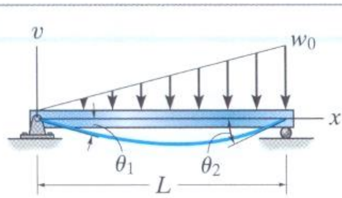
Determine the deflection at the end *B* of the clamped A-36 steel strip. The spring has a stiffness of k = 2 N/mm. The strip is 5 mm wide and 10 mm high.



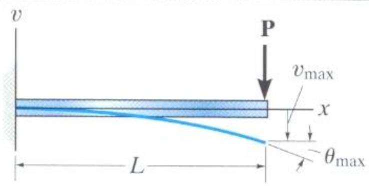
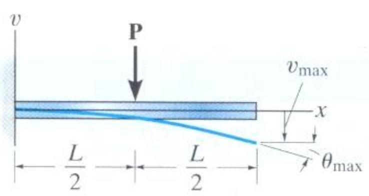
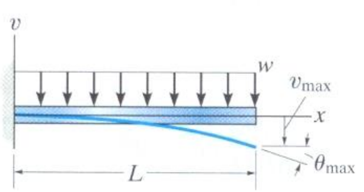
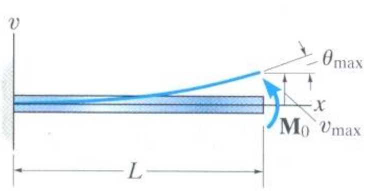
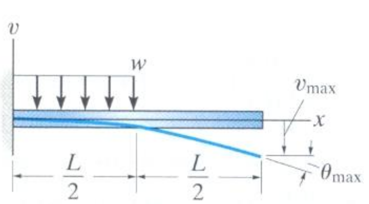
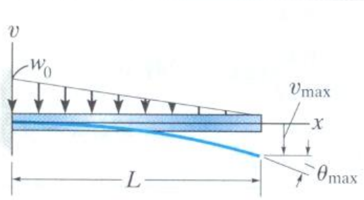
APPENDIX

C Slopes and Deflections of Beams

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0L}{3EI}$ $\theta_2 = \dfrac{M_0L}{6EI}$	$v_{max} = \dfrac{-M_0L^2}{\sqrt{243}EI}$	$v = \dfrac{-M_0x}{6EIL}(x^2 - 3Lx + 2L^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0L^3}{360EI}$ $\theta_2 = \dfrac{w_0L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}\left(\frac{3}{2}L - x\right)$ $0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}\left(3x - \frac{1}{2}L\right)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0L}{EI}$	$v_{max} = \dfrac{M_0L^2}{2EI}$	$v = \dfrac{M_0x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x - L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$